

Omega3P

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Introduction

Omega3P is a parallel finite-element electromagnetic code for high-fidelity modeling of cavities. It calculates the resonant frequencies and other rf parameters of a cavity, as well as damping effects due to external couplings. It uses Nedelec-type hierarchical vector basis up to 6th order with quadratic (10-points) tetrahedral elements for improved solution accuracy.

Mathematical Modeling

Lossless cavities

Maxwell's equations in the frequency domain for a perfectly conducting cavity can be written as the following PDE,

$$\begin{aligned} \nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{E} \right) - k^2 \epsilon \mathbf{E} &= 0 \quad \text{on } \Omega \\ \mathbf{n} \times \mathbf{E} &= 0 \quad \text{on } \Gamma \end{aligned}$$

$$\begin{aligned} \nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{E} \right) - k^2 \epsilon \mathbf{E} &= 0 \quad \text{on } \Omega \\ \mathbf{n} \times \mathbf{E} &= 0 \quad \text{on } \Gamma \end{aligned}$$

We use Nedelec-type vector basis functions to discretize the electric field:

$$\mathbf{E} = \sum_{j=1}^N \mathbf{e}_j$$

$$\frac{\begin{matrix} \text{Unknown macro: \{latex\}} \\ \backslash begin \\ \text{Unknown macro: \{mathbf Kx\}} \\ = k^2 \\ \text{Unknown macro: \{mathbf Mx\}} \\ \quad \quad \quad \backslash quad \text{ where } \& \\ \text{Unknown macro: \{mathbf K\}} \\ - \\ \text{Unknown macro: \{ij\}} \\ = \int _ \\ \text{Unknown macro: \{\Omega\}} \\ \left(\backslash nabla \backslash times \backslash vec \\ \text{Unknown macro: \{mathbf N\}} \\ _i \right) \backslash cdot \backslash frac \\ \text{Unknown macro: \{1\}} \\ \text{Unknown macro: \{\mu\}} \\ \left(\backslash nabla \backslash times \backslash vec \\ _j \right) d\Omega \& \\ \text{Unknown macro: \{mathbf M\}} \\ - \\ = \int _ \\ \text{Unknown macro: \{\Omega\}} \\ \backslash vec \\ \text{Unknown macro: \{mathbf N\}} _i \backslash cdot \backslash \\ _j d\Omega \& \\ \backslash end \end{matrix}}{\quad}$$

Waveguide loaded cavities

Absorbing BC

$$\vec{n} \times \frac{1}{\mu} \nabla \times \vec{E} + i\sqrt{k^2 - k_{c3}^2} \vec{n} \times \vec{n} \times \vec{E} = 0$$

Absorbing BC

$$\vec{n} \times \frac{1}{\mu} \nabla \times \vec{E} + i\sqrt{k^2 - k_{c2}^2} \vec{n} \times \vec{n} \times \vec{E} = 0$$

Absorbing BC

$$\vec{n} \times \frac{1}{\mu} \nabla \times \vec{E} + i\sqrt{k^2 - k_{c1}^2} \vec{n} \times \vec{n} \times \vec{E} = 0$$

Open Cavity

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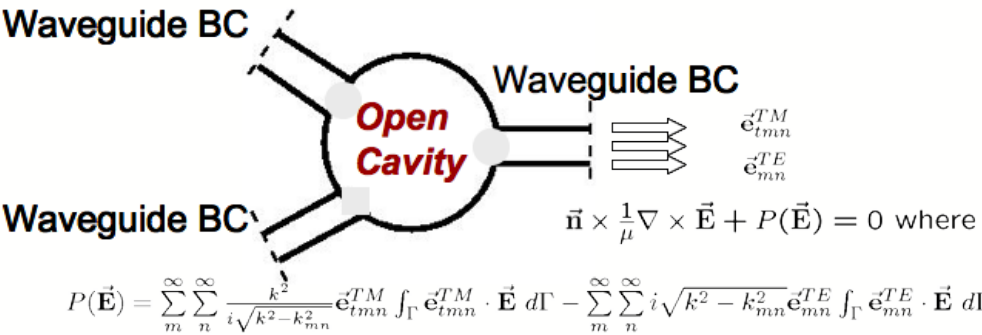
[

$$\begin{aligned}
 & \mathbf{K} \mathbf{x} \\
 & + i \sum_j \sqrt{k^2 - k_{cj}^2} \\
 & \mathbf{W} \\
 & \mathbf{x} \\
 & = k^2 \\
 & \mathbf{M} \mathbf{x} \\
 & \mathbf{J}
 \end{aligned}$$

where the damping matrix $\mathbf{W}$ is

$$\begin{aligned}
 & [\\
 & \mathbf{W} \\
 &] \\
 & \int_{\Gamma} \\
 & = \int_{\Gamma} \\
 & \Gamma \\
 & \mathbf{n} \\
 & \times \mathbf{e} \\
 & \mathbf{N} \\
 & \mathbf{n} \cdot \mathbf{e} \\
 & \times \mathbf{e} \\
 & \mathbf{N} \\
 & \mathbf{n} \cdot d\Gamma
 \end{aligned}$$

The following picture illustrates a cavity with waveguides, which are modeled with WBC.



The corresponding discretized system becomes a complex nonlienaar eigenvalue problem,

$$\begin{aligned}
 & [\\
 & \mathbf{K} \\
 & + i \sum_{m,n} \\
 & \sqrt{k^2 - (k_{mn}^2)} \\
 & \mathbf{W}
 \end{aligned}$$

$$- \frac{1}{2} \sum_{\mathbf{m}, \mathbf{n}} \mathbf{W}_{\mathbf{m}, \mathbf{n}} \mathbf{x}_{\mathbf{m}} \mathbf{x}_{\mathbf{n}}^*$$

$$+ i \sum_{\mathbf{m}, \mathbf{n}} \frac{\mathbf{W}_{\mathbf{m}, \mathbf{n}}}{\sqrt{k^2 - (k_{\mathbf{m}}^2 + k_{\mathbf{n}}^2)}}$$

$$- \frac{1}{2} \sum_{\mathbf{m}, \mathbf{n}} \mathbf{W}_{\mathbf{m}, \mathbf{n}} \mathbf{x}_{\mathbf{m}} \mathbf{x}_{\mathbf{n}}^*$$

$$= k^2 \mathbf{M}_{\mathbf{x}} \mathbf{x}$$

where the waveguide matrices are

$$\mathbf{W}_{\mathbf{m}, \mathbf{n}} = \int_{\Gamma} \mathbf{e}_{\mathbf{m}} \cdot \mathbf{e}_{\mathbf{n}} \, d\mathbf{x}$$

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$$i\,d\Gamma\Gamma\int t$$

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$$\vec{}$$

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$$^{}$$

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$$-$$

$$\cdot\vec{}$$

Unknown macro: {mathbf N}

$$\int_j d\Gamma\Gamma$$

$$\left[\right.$$

$$^{}$$

$$-$$

Unknown macro: {mn}

$$\left. \right)$$

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$$=\int t$$

Unknown macro: {Gamma}

$$\vec{}$$

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$$\cdot\vec{}$$

Unknown macro: {mathbf N}

$$i\,d\Gamma\Gamma\int t$$

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$$\vec{}$$

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$$^{\text{TM}}-$$

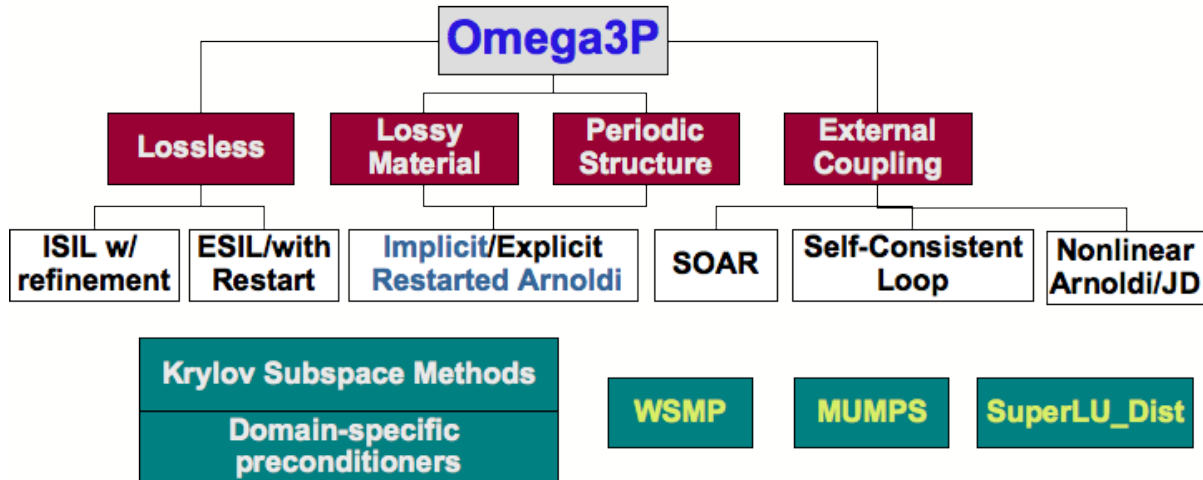
$$\cdot\vec{}$$

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$$\int_j d\Gamma\Gamma$$

$$\left. \right]$$

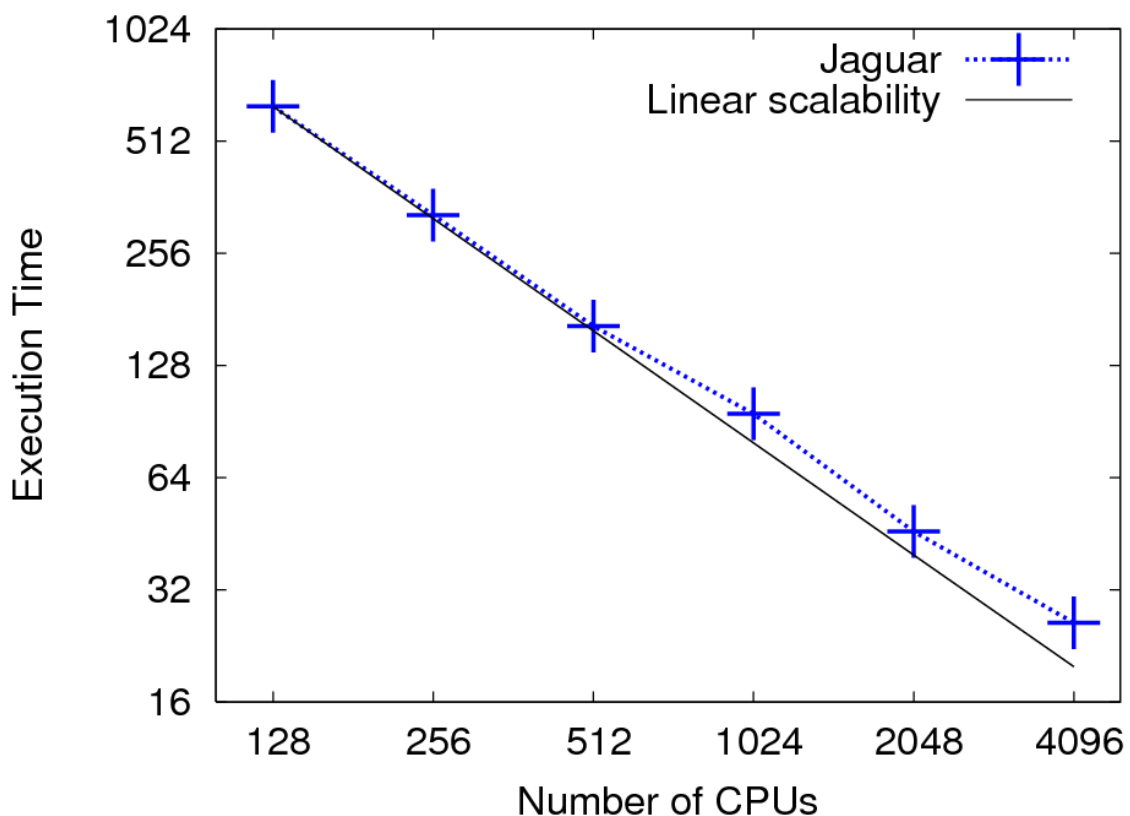
The following is a graphical description of the four different types of physics problems that Omega3P can solve and the solver options that can be used.



Parallel Performance

Omega3P parallel performance is heavily dependent on the solver used. In general, using sparse direct solvers for shifted linear systems often yields higher FLOPS than using Krylov subspace iterative methods. On the other hand, the latter has much better scalability.

The following is a strong scalability plot for Omega3P running on ORNL Jaguar (Cray XT). It used 1.5 million higher-order tetrahedral elements for the RF gun of the LCLS. . The number of the degrees of freedom is about 9.6 millions, and the number of nonzeros in the resulting matrix about 506 millions. It is shown that Omega3P scales to 4096 number of CPUs with more than 95% of parallel efficiency.



Past Accomplishments