

TULIP Algorithm Trilateration

TULIP Trilateration Algorithm

$\text{minRTT} = \text{propagation delay} + \text{extra delay (due to extra circular routes, congestion and router delays)}$

$\text{delta}(T)_{\text{measured}} = \text{delta}(t) + \text{delta}(t_0)$

(Pseudo-distance)

$\text{PD} = \text{delta}(T)_{\text{measured}} \cdot \alpha$

(Actual distance)

$D = \text{delta}(T) \cdot \alpha$

$\text{PD} = (\text{delta}(T) + \text{delta}(T_0)) \cdot \alpha$

$\text{PD} = D + \text{delta}(T_0) \cdot c \dots\dots\dots(1)$

$D = \text{actual distance from the landmark.}$

$C = \text{speed of light}$

$\alpha = X(c)$ i.e. Speed of digital info in fiber optic cable

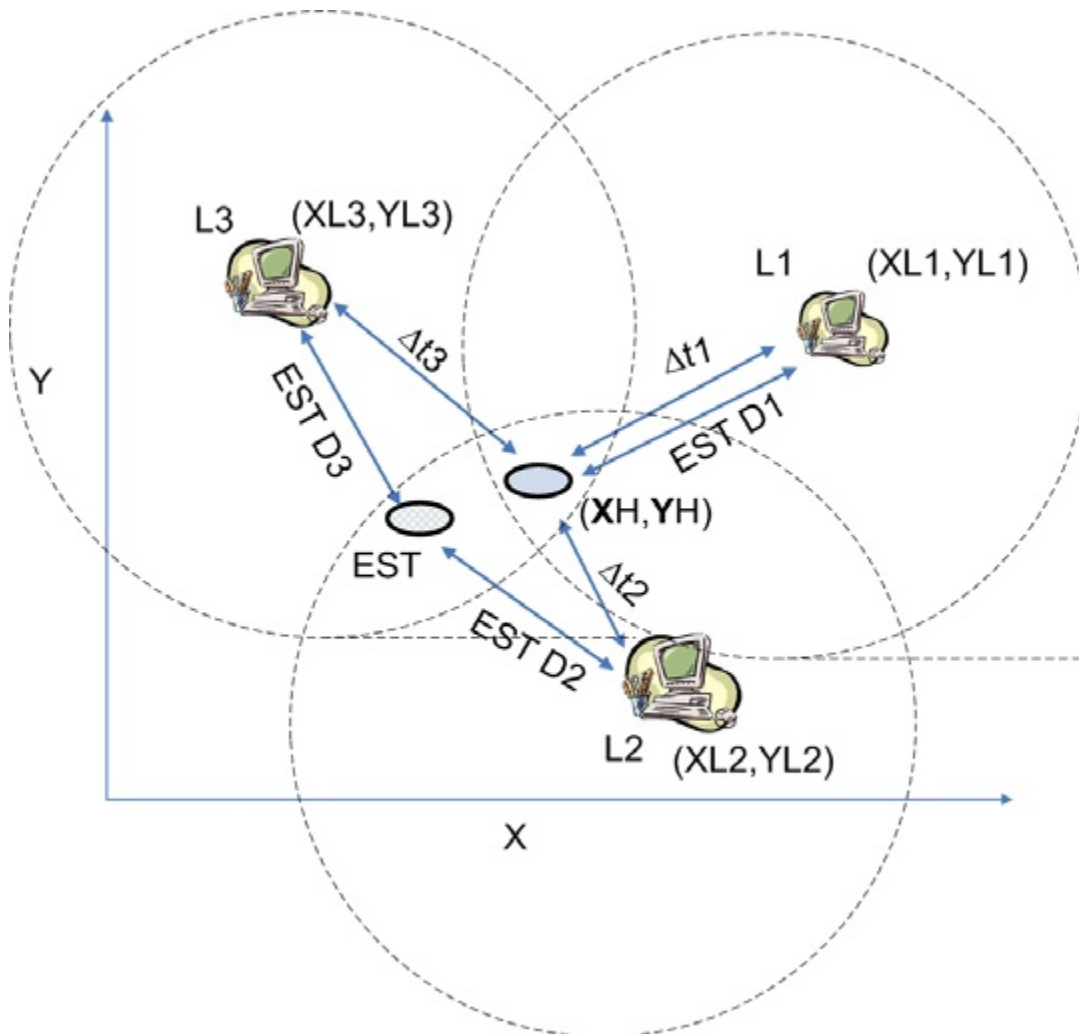
$X = \text{factor of } c \text{ with which digital info travels in fiber optic cable.}$

$\text{delta}(T) = \text{actual propagation delay along the greater circle router/paths.}$

$\text{delta}(T_0) = \text{the extra delay causing overestimation.}$

$\text{PD} = \text{pseudo distance}$

Graphically,



H: host
L1: Landmark 1
L2: landmark 2
L3: landmark 3

Using distance formula:
 $D1 = \sqrt{(XL1 - XH)^2 + (YL1 - YH)^2}$ (2)
 FROM (1) & (2)

$PD1 = (\sqrt{(XL1 - XH)^2 + (YL1 - YH)^2} + \Delta t1) \cdot \alpha$ (A)

Similarly for other 2 landmarks:

$PD2 = (\sqrt{(XL2 - XH)^2 + (YL2 - YH)^2} + \Delta t2) \cdot \alpha$ (B)

$PD3 = (\sqrt{(XL3 - XH)^2 + (YL3 - YH)^2} + \Delta t3) \cdot \alpha$ (C)

We need to linearize (A), (B) & (C) to solve them

Using Taylor Series:

$$f(x) \approx f(x_0) + f'(x_0) \cdot (x - x_0)$$

$$f(x) = f(x_0) + \frac{f'(x_0) \cdot (x - x_0)}{1!} + \frac{f''(x_0) \cdot (x - x_0)^2}{2!} + \dots$$

Considering the simplified part first

$$f(x) \approx f(x_0) + f'(x_0) \cdot (x - x_0)$$

$$\Delta t(x) = \Delta t(x_0) + \Delta t'(x_0) \cdot (x - x_0)$$

$$f(x) = f(x_0) + f'(x_0) \cdot \Delta t(x) \dots\dots\dots (3)$$

Hence to compute the original value of X an arbitrary value x_0 is required, this is done by simple trilateration

We know that:

$$H_x = X_{est} + \Delta x$$

$$H_y = Y_{est} + \Delta y$$

Also

$$Est\ Di = (L_{hi} - X_{est}) + (H_y - Y_{est}) \dots\dots\dots(4)$$

From (3) and (4)

$$d(EstDi) \cdot \Delta x + d(EstDi) \cdot \Delta y$$

$$PD_i = Est\ Di + \frac{(X_{est} - X_{li})}{dX} + \frac{(Y_{est} - Y_{li})}{dY}$$

After carrying out partial differentiation

$$(X_{est} - X_{li}) \quad (Y_{est} - Y_{li})$$

$$OD_i = Est\ Di + \frac{\dots\dots\dots}{dX} \cdot \Delta x + \frac{\dots\dots\dots}{dY} \cdot \Delta y + c \cdot \Delta t$$

Now we need to solve ($x, y, \Delta t$)

$$\begin{bmatrix} PD_1 - E_{st}D_1 \\ PD_2 - E_{st}D_2 \\ PD_3 - E_{st}D_3 \end{bmatrix} = \begin{bmatrix} \frac{(X_{Est} - X_{l1})}{E_{st}D_1} & \frac{(Y_{Est} - Y_{li})}{E_{st}D_1} & c \\ \frac{(X_{Est} - X_{l2})}{E_{st}D_2} & \frac{(Y_{Est} - Y_{l2})}{E_{st}D_2} & c \\ \frac{(X_{Est} - X_{l3})}{E_{st}D_3} & \frac{(Y_{Est} - Y_{l3})}{E_{st}D_3} & c \end{bmatrix} \times \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta t \end{bmatrix}$$

Solution from (4) is put in eq(D) to get new estimations.

H_x, H_y becomes the new estimated position

Reference

<http://www.ece.cmu.edu/research/publications/2003/CMU-ECE-2003-038.pdf>