

# T3P

T3P is a 3D parallel finite-element time-domain solver to calculate transient field response of a electromagnetic structure to imposed fields, and dipole or beam excitations.

In our approach, Ampere's and Faraday's laws are combined and integrated over time to yield the inhomogeneous vector wave equation for the time integral of the electric field  $\mathbf{E}$ :

$$\left( \epsilon \frac{\partial^2}{\partial t^2} + \sigma_{eff} \frac{\partial}{\partial t} + \nabla \times \mu^{-1} \nabla \times \right) \int^t \mathbf{E} d\tau = -\mathbf{J}$$

with permittivity  $\epsilon = \epsilon_0 \epsilon_r$  and permeability  $\mu = \mu_0 \mu_r$ . In the current implementation, a constant value of the effective conductivity  $\sigma_{eff} = \epsilon_0 \omega \cdot 2\pi f \cdot \tan \delta$  is assumed by fixing a frequency  $f$ , and the losses are specified by the loss tangent  $\tan \delta$ . As is common for Wakefield computations of rigid beams, the electric current source density  $\mathbf{J}$  is given by a one-dimensional Gaussian particle distribution, moving at the speed of light along the beam line.

The computational domain is discretized into curved tetrahedral elements and  $\int^t \mathbf{E} d\tau$  in Eq. (1) is represented as an expansion in hierarchical Whitney vector basis functions  $\mathbf{N}_i(\mathbf{r})$

$$\int^t \mathbf{E}(\mathbf{r}, \tau) d\tau = \sum_{i=1}^{N_e} \mathbf{e}_i(t) \cdot \mathbf{N}_i(\mathbf{r})$$

up to order  $P$  within each element. For illustration, the numbers of basis functions for first, second and sixth order are  $N_1 = 6$ ,  $N_2 = 20$  and  $N_6 = 210$ , respectively. Higher order elements (both curved and with higher-order basis functions) not only significantly improve field accuracy and dispersive properties, but also generally lead to higher-order accurate particle-field coupling equivalent to, but much less laborious than, complicated higher-order interpolation schemes commonly found in finite-difference methods.