Particle transport and acceleration in SNR shock is studied.
Mechanisms of particle spectrum formation are discussed.

What can the nonlinear shock acceleration theory tell us about the RXJ 1713 gamma ray spectrum in sub-TeV range?

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Motivations

Hillas '05 review:

All round, the model of diffusive shock acceleration seems to become more persuasive, though the flatter spectrum predicted at high energies may yet turn out to be a severe Problem for cosmic rays.

As an alternative cause of a reduction of the TeV flux, a more steeply falling proton spectrum in the SNR would alleviate the isotropy problem for galactic cosmic rays, and a discussion by Gaisser et al [98] of gamma rays of lower energy observed by EGRET in two older SNRs indeed suggested that the best fit was with a spectrum $E^{-2.4}$

This, though, would involve a drastic change in the pressure balance of cosmic rays in current models of diffusive shock acceleration, in which the most energetic particles play a large role.

Diffusive Shock Acceleration

Trilogy

- Injection
- Acceleration
- Escape

A new twist to the old story :

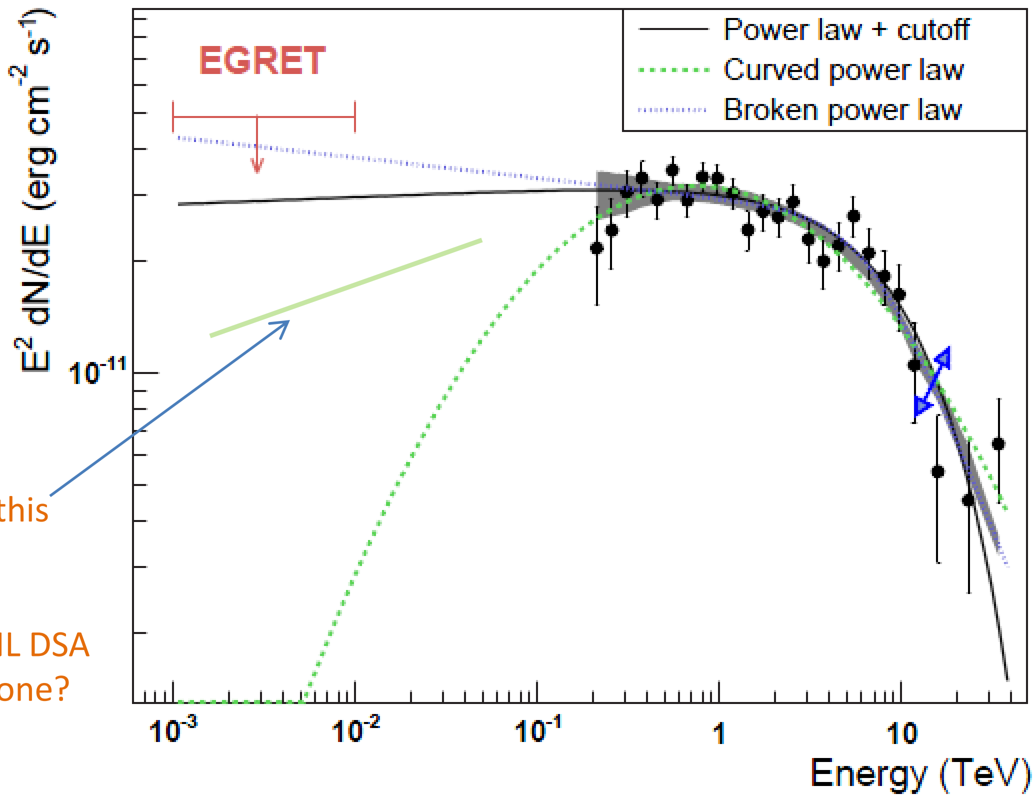
acceleration and escape overlap

Escape as a direct result of acceleration, not of external conditions

→ Phase space fragmentation

→ Spectral break?

Tentative evidence for the break



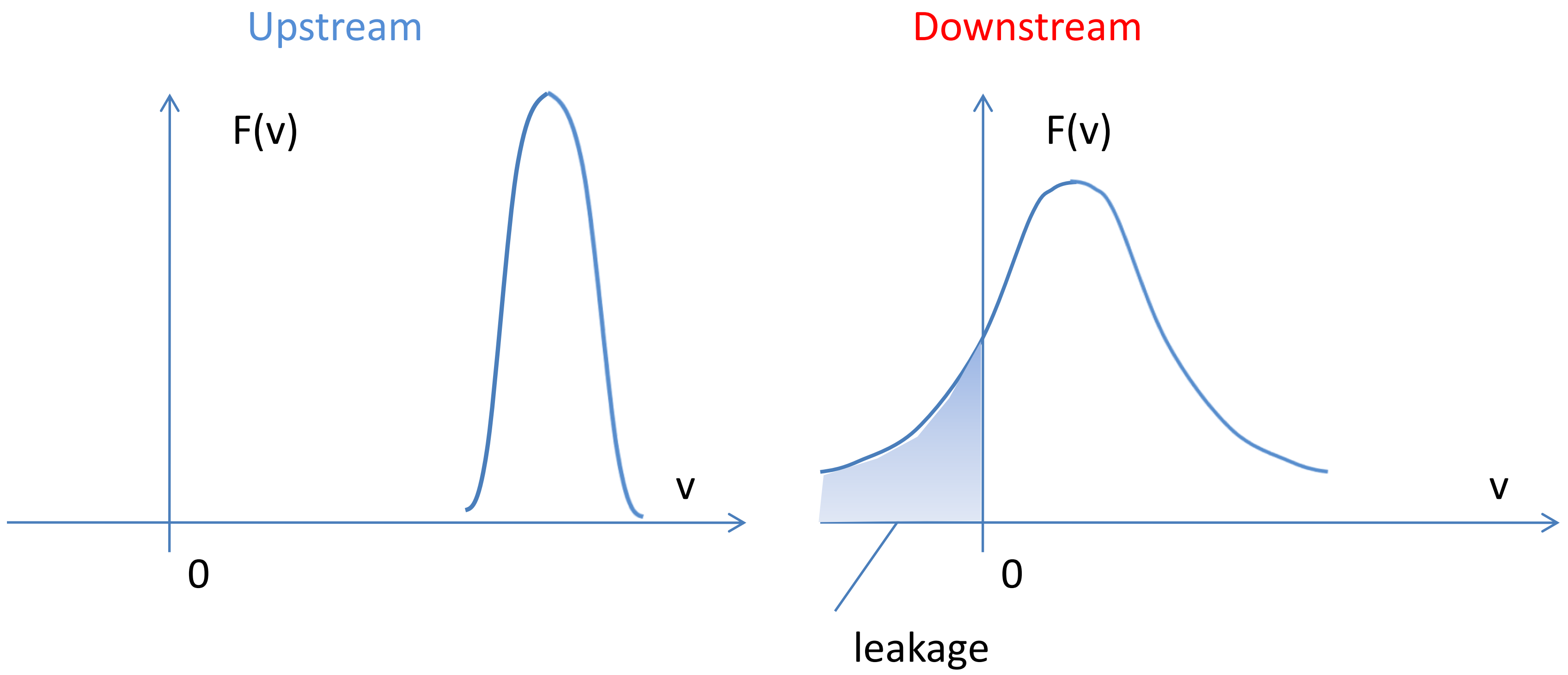
If Fermi reveals something like this For protons

→ triumph of NL DSA over the linear one?

Aharonian, F. and HESS team (2006). A detailed spectral and morphological study of the gamma-ray supernova remnant RX J1713.7-3946 with HESS. *Astronomy and Astrophys.*, 449:223–242.

Injection

Thermal leakage:



Injection

recipe to calculate its rate

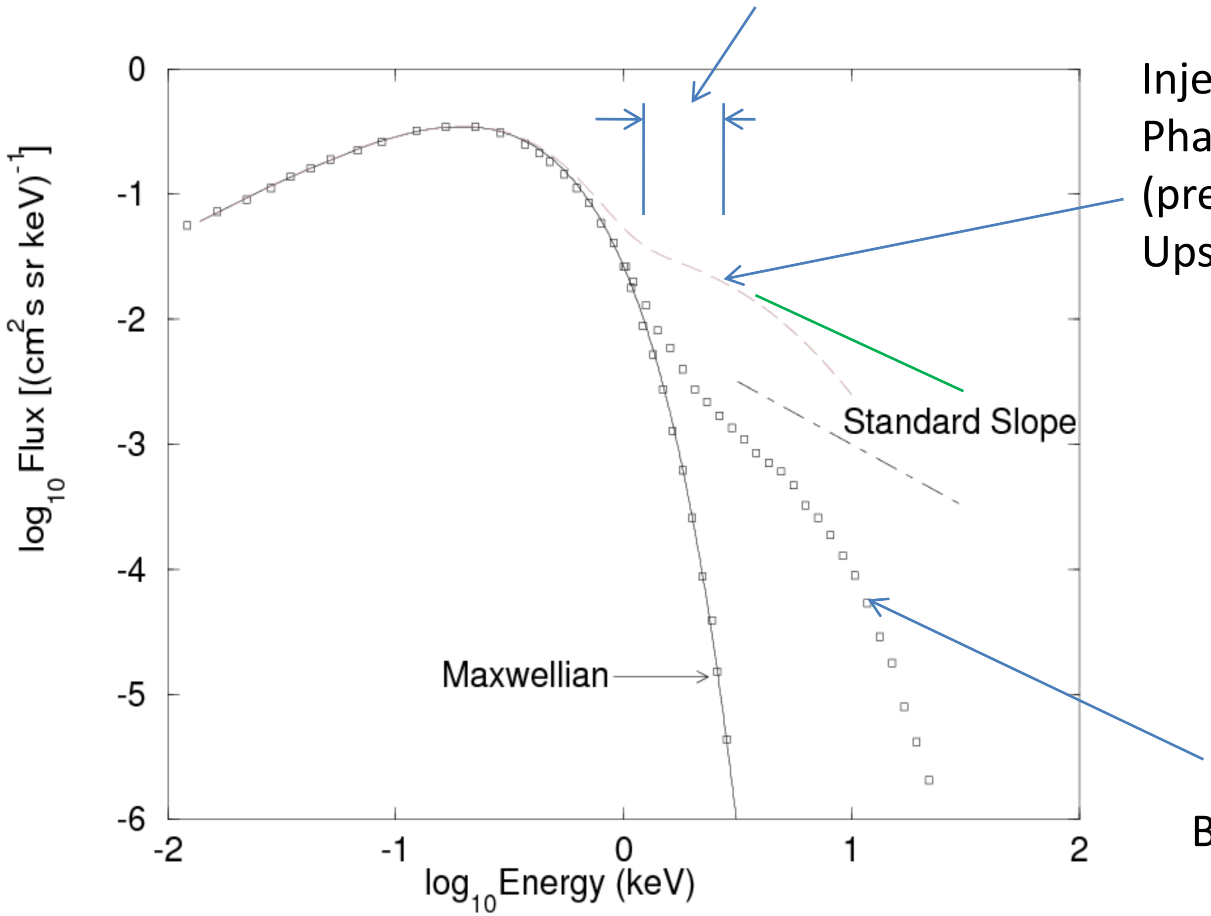
- start with a (unknown) distribution f(**v**) that leaks upstream from the downstream shocked plasma
- follow it as particles scattered by upstream waves till they return back downstream*
- follow that part of the distribution which is scattered back upstream*
- add thermally leaking and/or shock reflected particles
- make the distribution equal to what you started with: f(**v**)

$$f(\mathbf{v}) = L\, f(\mathbf{v}) + f_{Maxw}(v)$$

* Do not use the diffusion-convection equation: particle distribution is highly anisotropic

Injection

Overlap region (matching with the standard DSA)



Injection spectrum in random Phase broad band wave ensemble (prescribed pitch angle scattering Upstream and downstream `a la MC)

Slope is fine
Normalization is wrong
(by an order of magnitude!)

Bennett & Ellison 95 Hybrid code

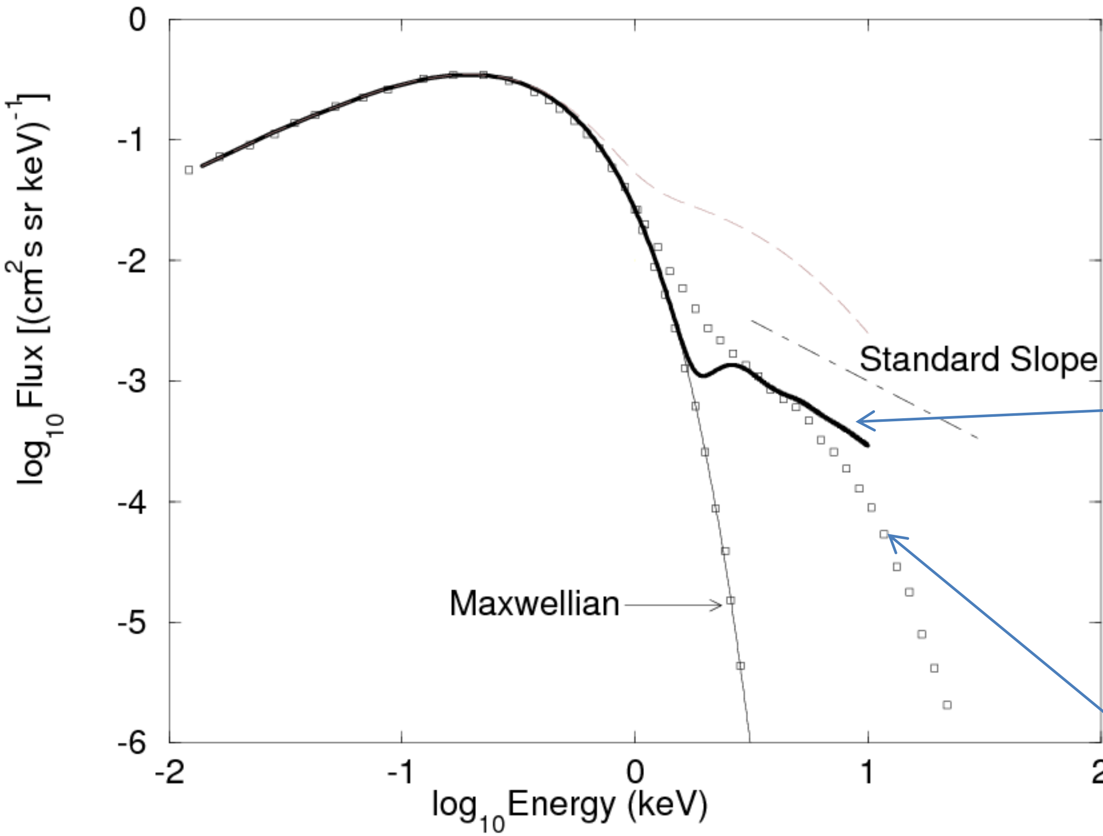
Injection: how to improve?

- ❑ Include back-reaction of the over-injected particles on the flow;
  modified flow → suppression of injection (MC scenario)

But: Earth's and IP shocks are not modified

- ✓ Abandon prescribed scattering field and calculate scattering **self-consistently**:
- ➢ Leaking particles drive a coherent, quasi-monochromatic A-wave upstream that being convected (and compressed) downstream traps and carries further downstream ~90% of the leaking particles
- ❑ Include electrostatic overshoot (beyond hybrid, PIC)

Injection



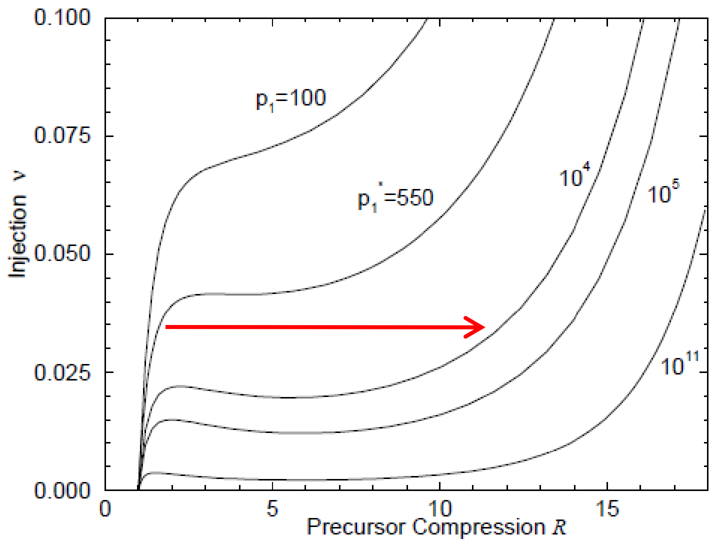
Quasi-monochromatic wave Driven by leaking particles (self-Consistent calculations)

Bennett & Ellison 95 Hybrid code

Injection bottom line

- Generates correct spectral slope (consistent with the standard DSA predictions at higher energies where the distribution becomes isotropic and the diffusion-convection equation may be applied)
- Broad overlapping with the standard DSA slope
- the notion of 'injection momentum' is not needed
- Successfully benchmarked to Hybrid simulation with no free parameters
- Clear self-regulation mechanism: too strong injection→ big wave, strong trapping → weaker injection
- Limitation: Q-parallel shock

NL shock response to particle injection/acceleration



- consider injection as a control parameter
- flow modification (acceleration efficiency) as an order parameter

FIG. 1.— The nonlinear response of an accelerating shock (characterized by the precursor compression R) to the thermal injection ν given in the form of the function $\nu(R)$ calculated for the fixed injection momentum $p_0 = 10^{-3}$, Mach number $M = 150$ and for different $p_1 = 100; 550; 10^4; 10^5; 10^{11}$. The critical value (see text) $p_1^* = 550$. The TP regime is limited to the region $R \simeq 1$.

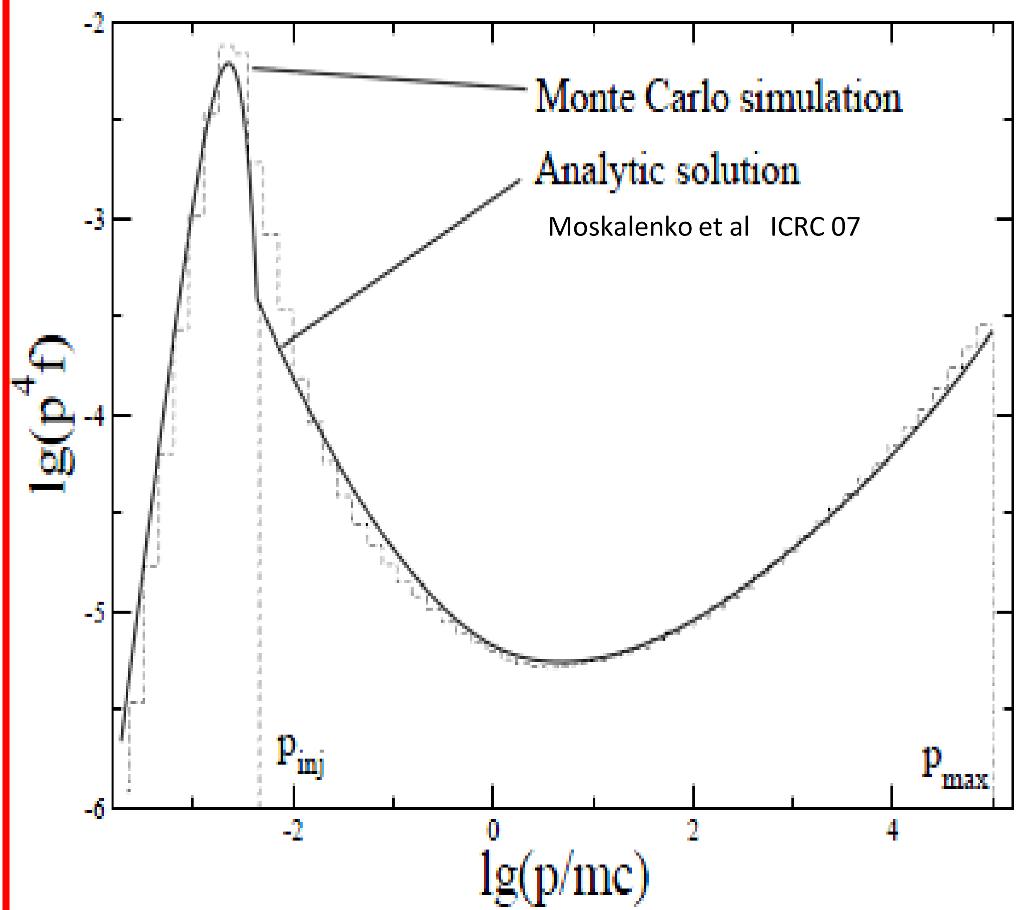
Can the calculated injection rate stay the same if the compression strongly increases? NO (sub-shock reduction)

Is solution multiplicity real? YES, if the injection is fixed (Contr. Par.)

Evidence #1

The same analytic solution that points at multiplicity and bifurcation, produces absolutely correct spectrum

MC Simulations: Ellison, D. C., Berezhko, E. G., & Baring, M. G. 2000, *Astrophys. J.* , 540, 292



Analytic solution: NL integral equation

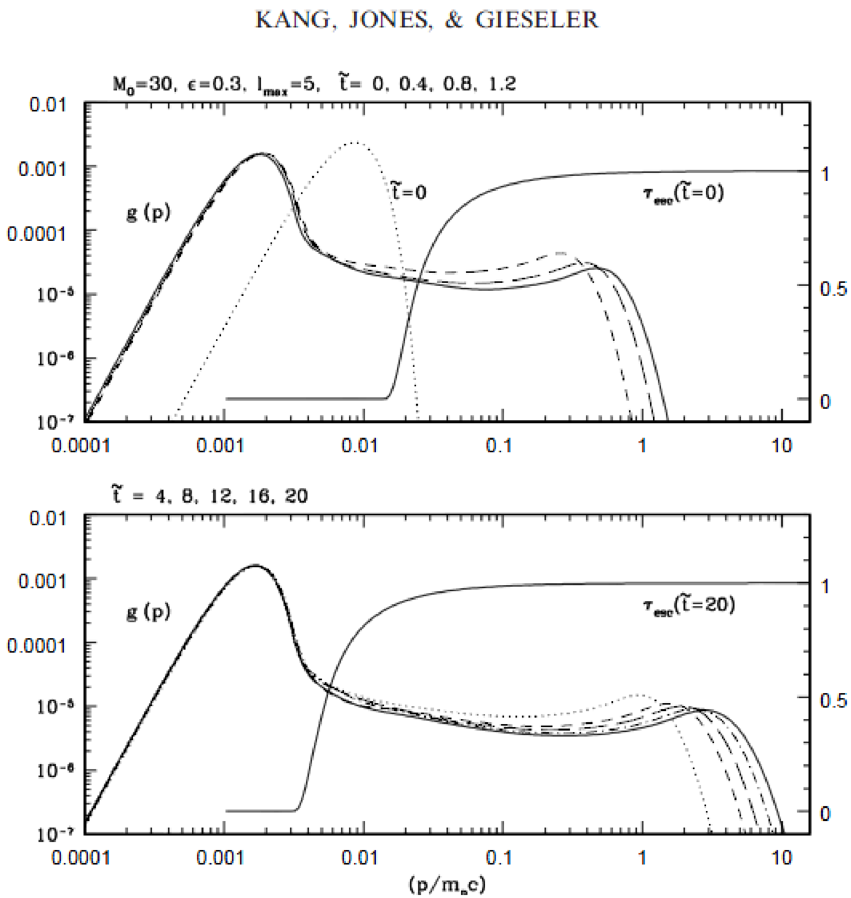
→treats particle spectrum and the shock flow structure self-consistently MM '97

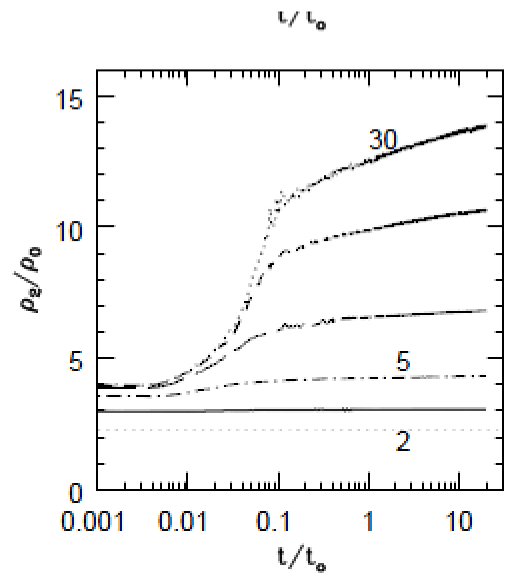
Different approximations

→ similar results: Blasi, Gabici, Amato, Reville, Kirk, Duffy 2002-2009

Evidence #2

Bifurcation of the acceleration regime (phase transition) in time dependent numerical solutions

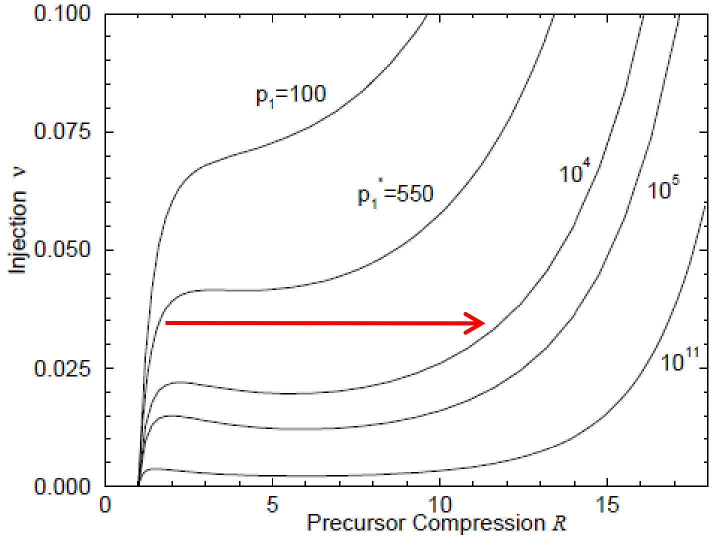




NL shock response to particle injection/acceleration

Self-organization of acceleration/shock structure

→ *~50% acceleration efficiency (CR/shock ram pressure)*



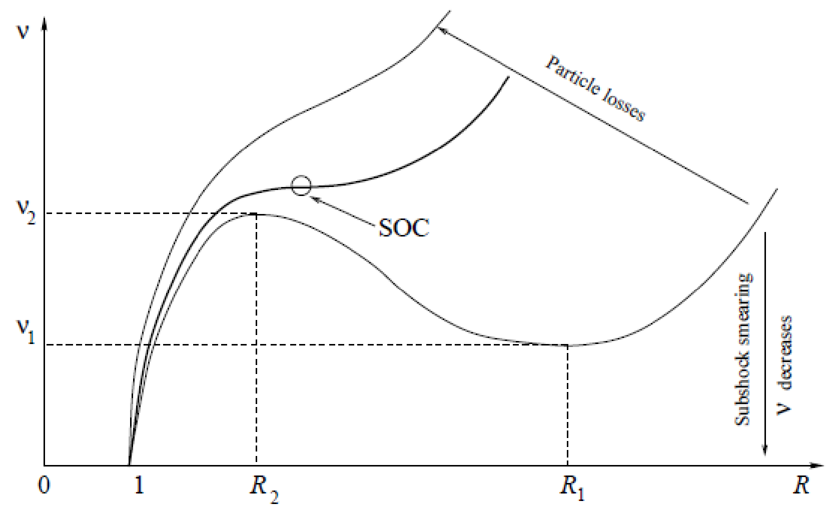


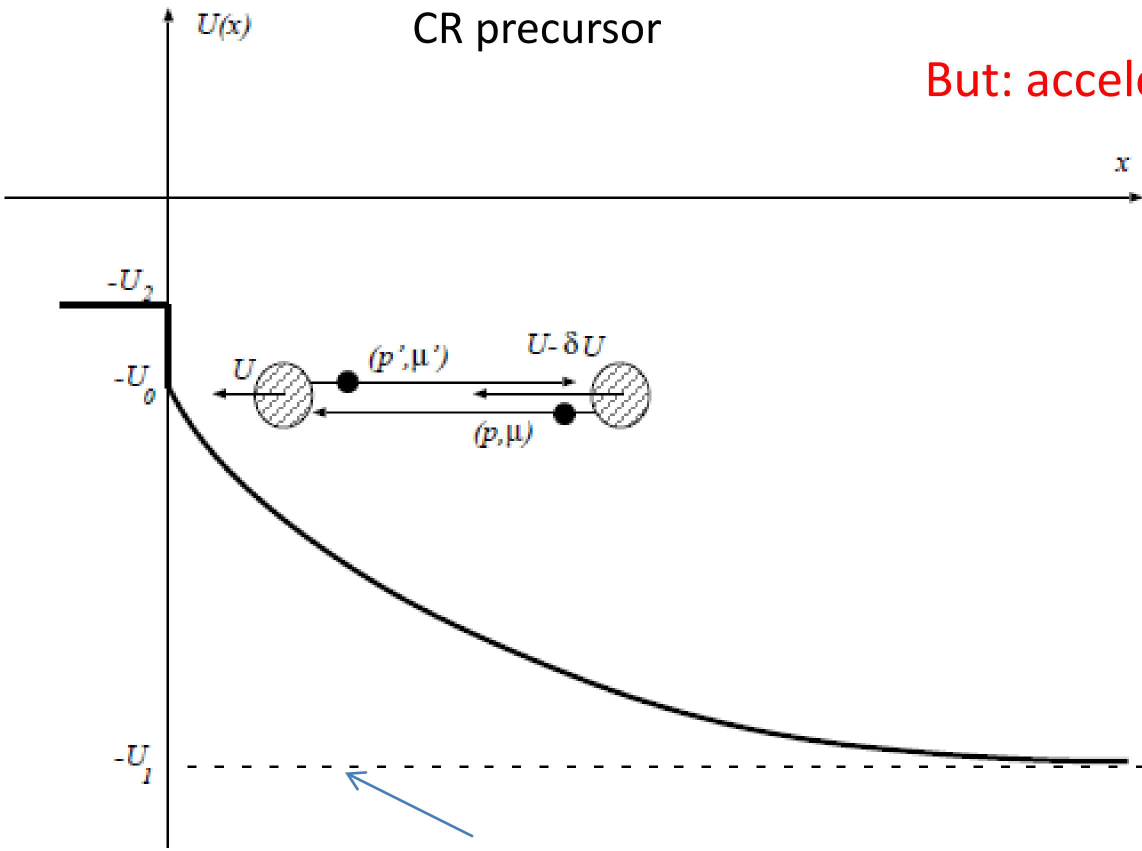
FIG. 1.— The nonlinear response of an accelerating shock (characterized by the precursor compression R) to the thermal injection ν given in the form of the function $\nu(R)$ calculated for the fixed injection momentum $p_0 = 10^{-3}$, Mach number $M = 150$ and for different $p_1 = 100; 550; 10^4; 10^5; 10^{11}$. The critical value (see text) $p_1^* = 550$. The TP regime is limited to the region $R \simeq 1$.

Can the calculated injection rate stay the same if the compression strongly increases? NO (sub-shock reduction) acceleration mechanism, same as in

Krymsky ‘77, Axford et al ‘77
Bell 78
Blandford and Ostriker 78

CR precursor

But: acceleration in the CR precursor

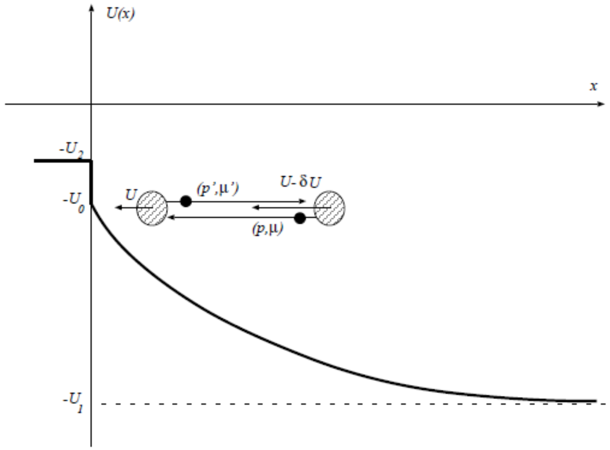


Unmodified classical shock

CR communicate information upstream (akin to ionizing front, radiative shock)

Two basic ways of communication
- upstream plasma instabilities
- upstream flow modification

Momentum gain



$$\delta U \ll c$$ (+ isotropy in pitch-angle μ)

after one shock crossing cycle

$$\frac{\langle \delta p \rangle}{p} \simeq 2\frac{\delta U}{c}\langle \mu \rangle = \frac{4}{3}\frac{\delta U}{c}$$

Discontinuity crossing

$$\frac{\Delta p}{p} = \frac{4}{3}\frac{(u_1 - u_2)}{c}$$

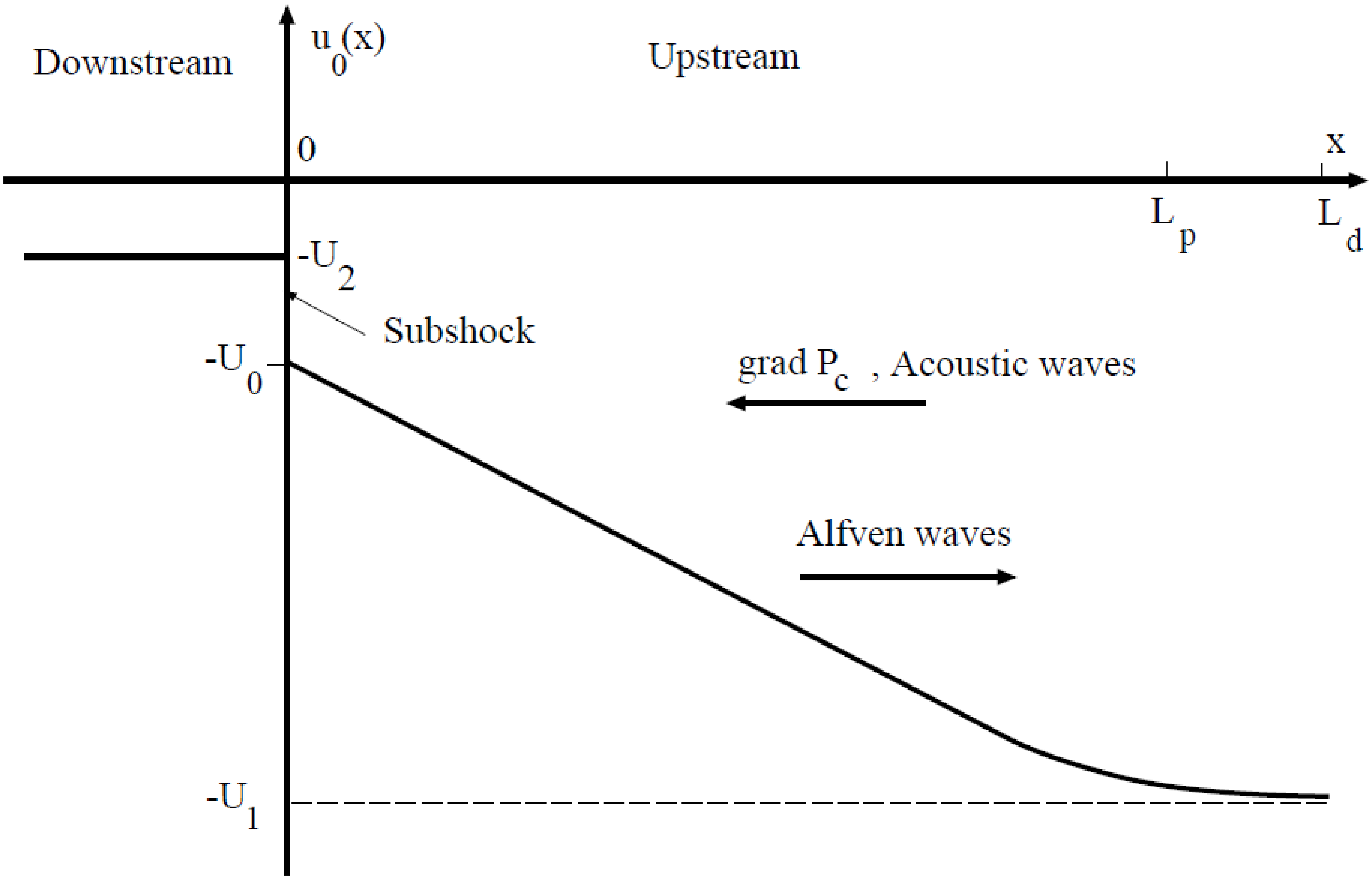
$$\frac{dp}{dt} \equiv \dot{p} \simeq \frac{\langle \delta p \rangle}{\langle \delta t \rangle} = \frac{1}{3} p \frac{\delta U}{\lambda}$$

$$\delta U \simeq -\lambda \frac{\partial U}{\partial z}$$

$$\dot{p} \simeq -\frac{1}{3} p \frac{\partial U}{\partial z}$$

$$\frac{\partial f}{\partial t} + U\frac{\partial f}{\partial z} - \frac{\partial}{\partial z}\kappa\frac{\partial f}{\partial z} = \frac{1}{3}\frac{\partial U}{\partial z} p \frac{\partial f}{\partial p}$$

Instabilities, important for particle transport in CRP



Instabilities

- cyclotron resonance, Alfven waves, $k_{\|} v_{\|} \approx \Omega_c(p)$, i.e., $k \sim r_g^{-1}(p)$

→Bell '78

- nonresonant (firehose): maximum growth in a very short wave range (not good for particle scattering)

→Achterberg '83, Shapiro and Quest '98, Bell and Lucek '01, 04, Reville et al 08

- →hydrodynamic, CR pressure gradient driven (Drury's) instability

→Drury 84, Drury and Falle 86, Zank et al 90, Kang et al 92…

Advantages:

- drive all wave numbers, $\gamma(k) \approx const$
- insensitive to CR distribution function
- staibilizes only nonlinearly (not quasi-linearly)
- long scales, much needed for particle confinement are naturally produced

→ Diamond and MM 07

Shock precursor equilibrium and its stability against generation of acoustic waves

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\rho u = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial}{\partial x}\left(P_c + P_g\right) + 2\mu\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} - \kappa\left(p\right)\frac{\partial^2 f}{\partial x^2} = \frac{p}{3}\frac{\partial u}{\partial x}\frac{\partial f}{\partial p}$$

CR pressure:

$$P_c = \frac{4\pi}{3}mc^2 \int \frac{p^4}{\sqrt{1+p^2}} f\,dp$$

Weakly nonlinear theory

Instability driver

CR linear response

$$\left(\frac{\partial}{\partial t} - c_s \frac{\partial}{\partial \xi}\right)\left(\frac{\partial}{\partial t} + c_s \frac{\partial}{\partial \xi}\right)\tilde{\rho} = -\frac{1}{\rho_0}\frac{\partial \bar{P}_c}{\partial \xi}\frac{\partial \tilde{\rho}}{\partial \xi} + \frac{\partial^2 \tilde{P}_c}{\partial \xi^2} + c_s^2 \frac{\gamma_g - 2}{2\rho_0}\frac{\partial^2 \tilde{\rho}^2}{\partial \xi^2}$$
$$- \frac{\partial}{\partial \xi}\left(2\tilde{u}\frac{\partial \tilde{\rho}}{\partial t} + \tilde{\rho}\frac{\partial \tilde{u}}{\partial t}\right) - 2\mu\rho_0 \frac{\partial^3 \tilde{u}}{\partial \xi^3}$$

Used Lagrangian coordinate

$$d\xi = dx - u_0(x)\, dt$$

⟹ Burgers eq.

$$\frac{\partial \tilde{\rho}}{\partial t} - c_s \frac{\partial \tilde{\rho}}{\partial \xi} - \frac{\gamma + 1}{2\rho_0} c_s \tilde{\rho}\frac{\partial \tilde{\rho}}{\partial \xi} - \mu \frac{\partial^2 \tilde{\rho}}{\partial \xi^2} = \gamma \tilde{\rho}$$

where the acoustic instability growth rate is

$$\gamma = -\frac{1}{2\rho_0 c_s}\frac{\partial \bar{P}_c}{\partial \xi} - \frac{\bar{q}}{18}\frac{mc^2 p_*}{\kappa_*}\frac{n_c(x)}{\rho_0}$$

Traveling wave solution driven by acoustic instability

$$\frac{\partial \hat{\rho}}{\partial t} + \hat{\rho}\frac{\partial \hat{\rho}}{\partial \zeta} = \gamma \hat{\rho} + \mu \frac{\partial^2 \hat{\rho}}{\partial \zeta^2} \qquad \zeta = \xi + c_s \mathrm{t}$$

$$\hat{\rho} = \hat{\rho}\left(\zeta - Ct\right)$$

$$\frac{\hat{\rho}^2}{2\mu} - \frac{\partial \hat{\rho}}{\partial \zeta} - \gamma \ln\left(1 - \frac{1}{\gamma}\frac{\partial \hat{\rho}}{\partial \zeta}\right) = E = const$$

(C=0)

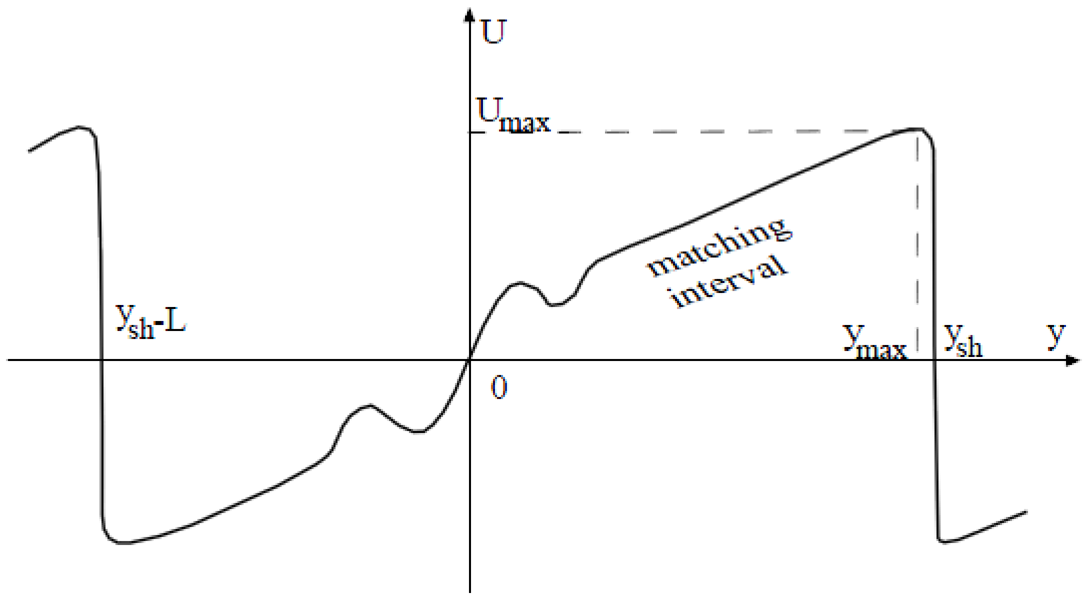
Exact shock train solution: periodic sequence of triangle waves

NL dispersion relation

$$L = \oint \frac{d\hat{\rho}}{(\partial \hat{\rho}/\partial \zeta)} = \sqrt{\frac{\mu}{2}} \oint \frac{d\psi}{(\gamma - \psi)\sqrt{E + \psi + \gamma \ln\left(1 - \psi/\gamma\right)}}$$

Traveling wave solution driven by acoustic and cyclotron instabilities

$$\frac{\partial \hat{\rho}}{\partial t} + \hat{\rho}\frac{\partial \hat{\rho}}{\partial \zeta} - \gamma\hat{\rho} - \mu\frac{\partial^2 \hat{\rho}}{\partial \zeta^2} = Q\left(\zeta - vt\right)$$

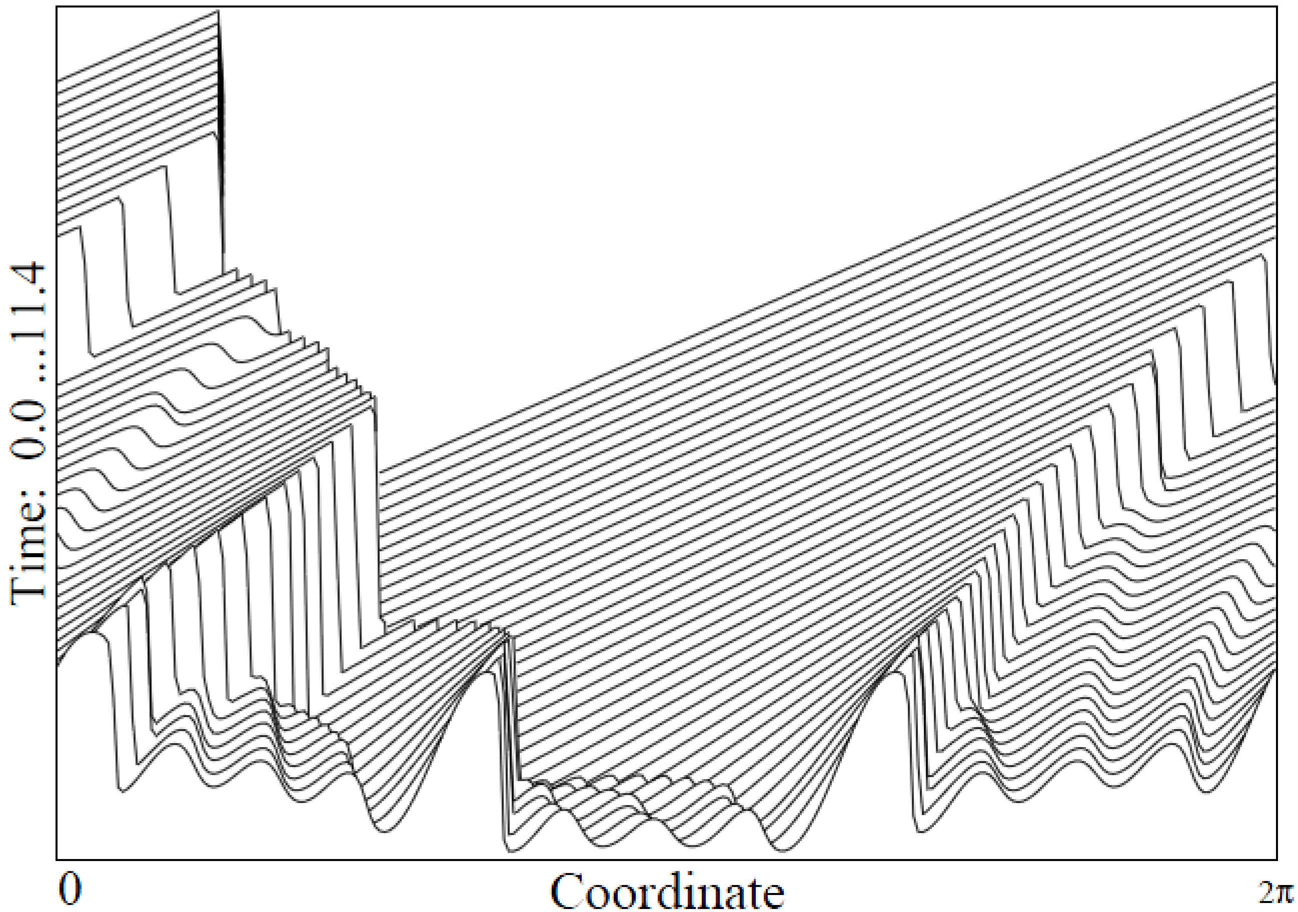


More general, ‘magnetic’ version of this solution but with a cyclotron-unstable driver only (no acoustic instability term)

→ Kennel et al JETP Let. ‘88,

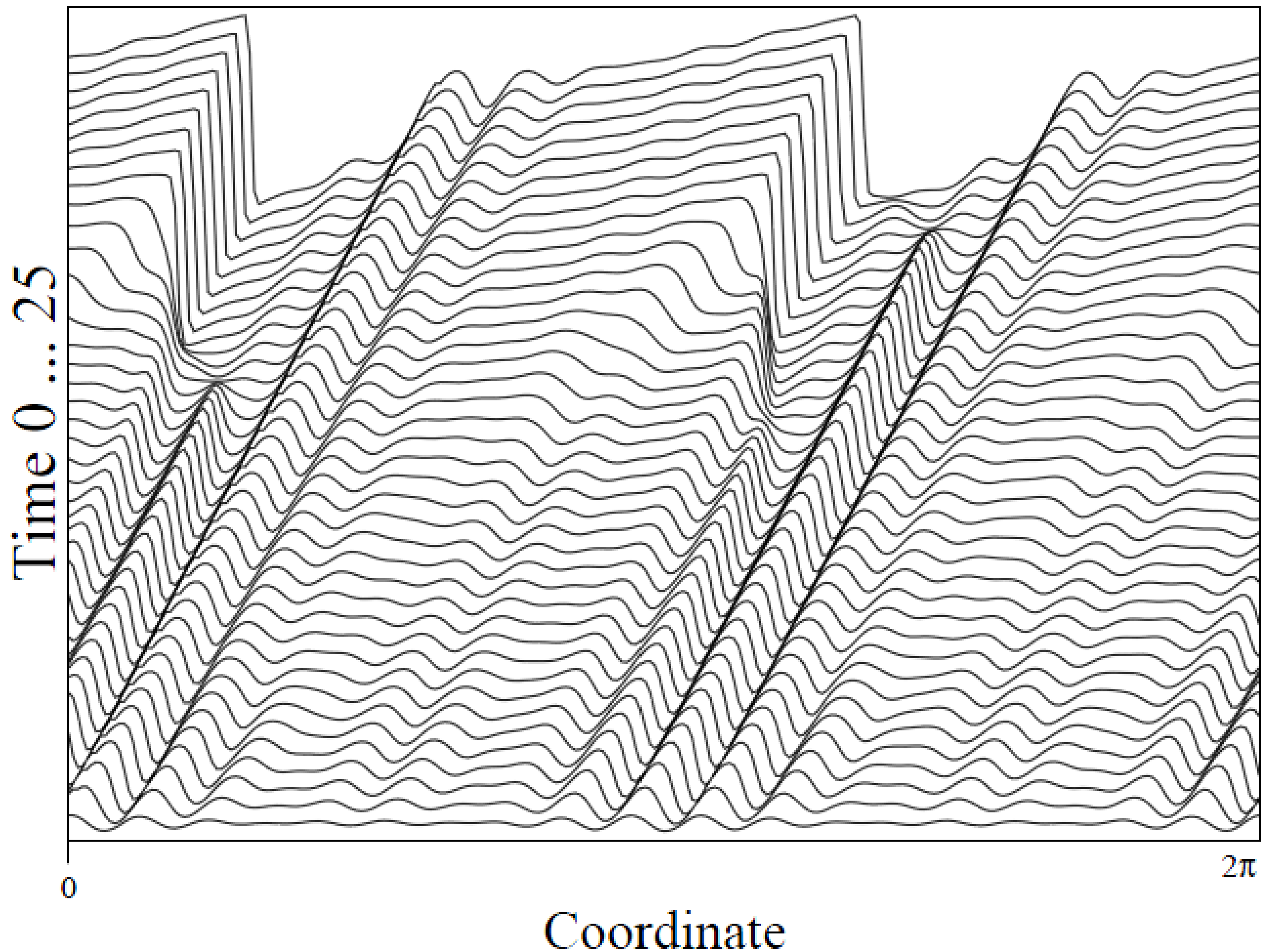
→ MM et al PFL ‘90

Numerical verification of the traveling wave solution (acoustic instability only)

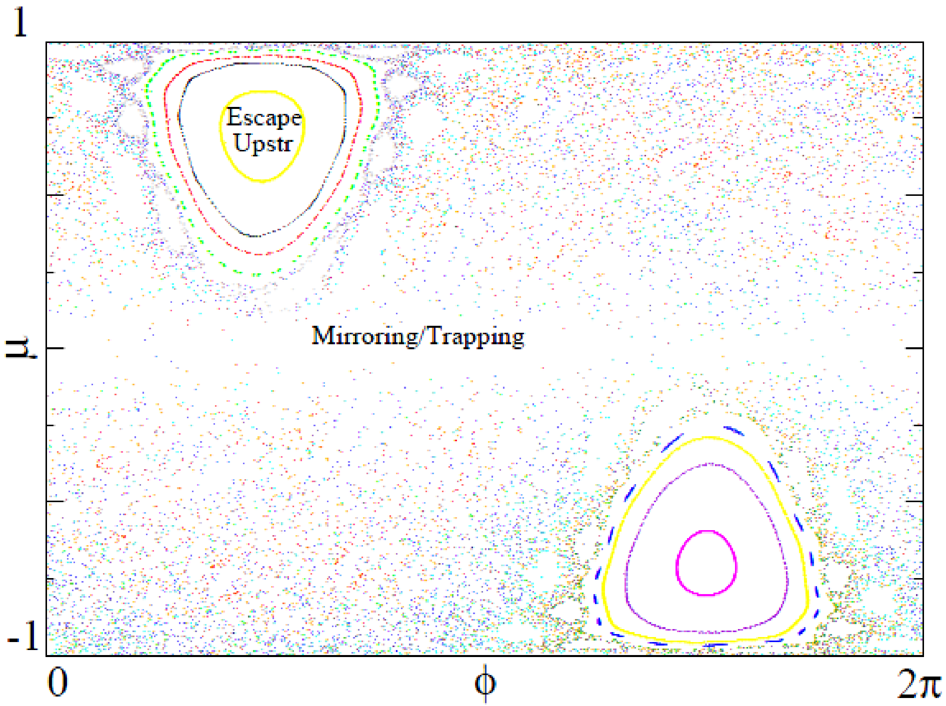


Initial perturbation profile steepens into 3 relatively weak shocks
They merge to form one strong shock

Numerical verification of the traveling wave solution (acoustic instability +IC instability)



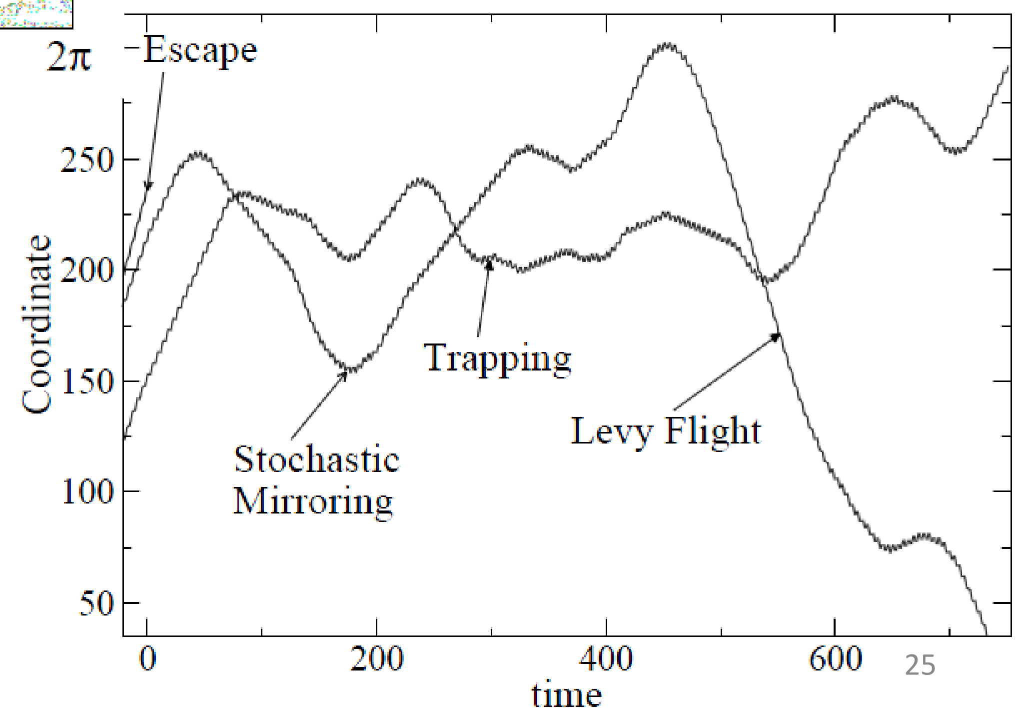
Particle dynamics and transport in a shock train inside of CR precursor



Pitch-angle/Gyro-phase
Poincare map
(Pitch-angle wrt shock normal, 45 deg here)

$r_g(p)/L \simeq 3$

Particle trajectories



Particle spectrum

For particles with momentum below the break $p=p_*$ the spectrum should be determined From nonlinear self-consistent solution of kinetic and HD equations.

Above the break at $p=p_*$ the spectrum can be approximated by a test particle solution (no significant contribution of those particles to the CR pressure)

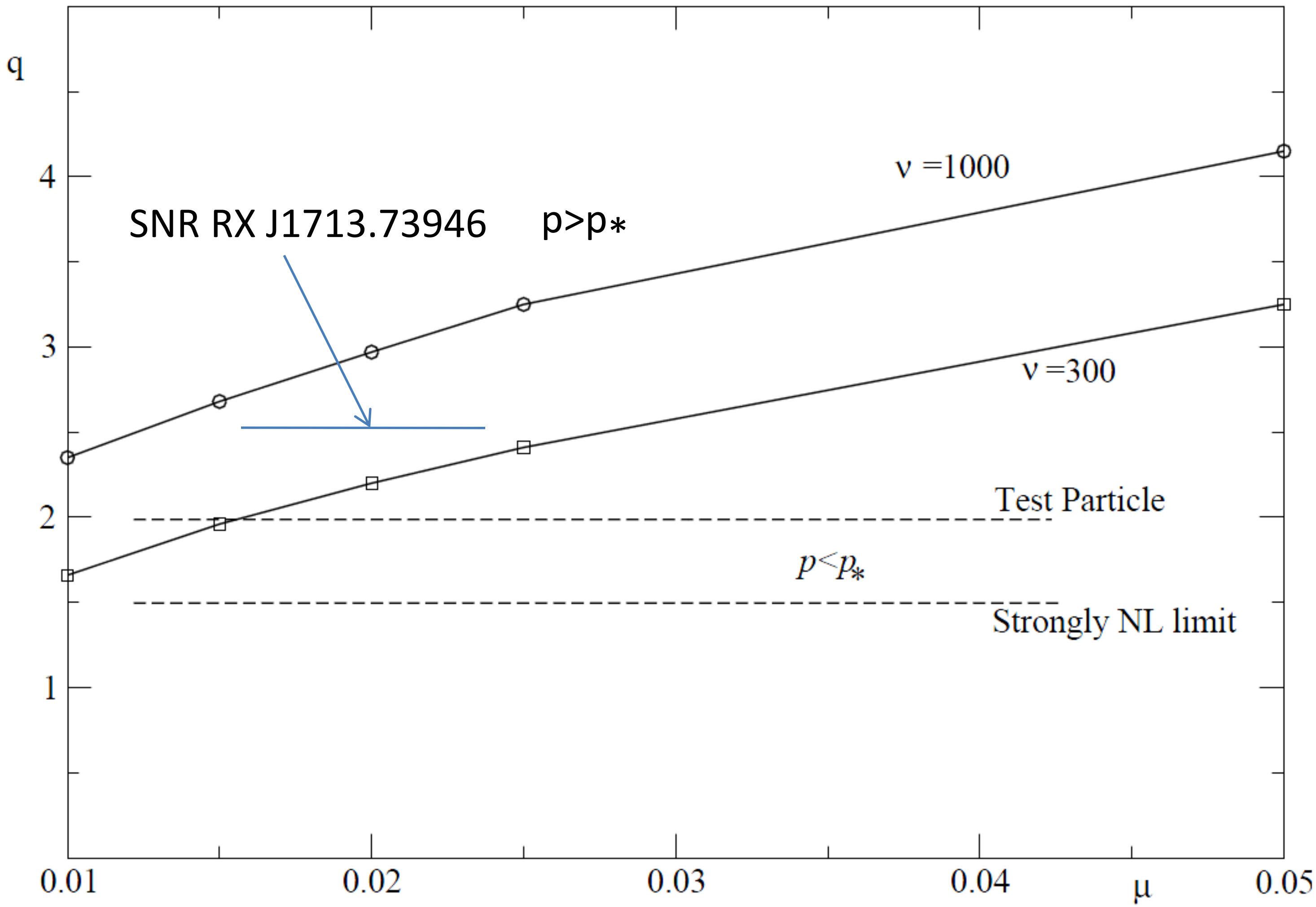
Fermi '49 general spectral index

$$q = 3 + \tau_{acc}/\tau_{conf}$$

$$q_e = 3 + \frac{\ln\left[(U_+/U_0)\left(\mathscr{P}_L^2/\mathscr{P}_{tr}\right)\right]}{K(\vartheta)\ln(1/\mathscr{P}_L)}$$

$\mathscr{P}_{tr}$ Trapping probability

$\mathscr{P}_L$ Detrapping probability (Levy flight)



$$\nu = \frac{U_+ \beta}{U_0 \alpha_+} \simeq \frac{U_+}{U_0} \frac{\tau_L \tau_L^+}{\tau_{tr}^2} \sim \frac{c}{U_0} \gg 1$$

$$\mu \equiv \frac{\beta \kappa_0}{2\pi \eta^2 p_0 p_* (U_0 - U_2)^2}.$$

Conclusions

• acoustic instability is robust (compared to cyclotron and firehose/mirror) in that it is hydrodynamic in nature and cannot be stabilized by kinetic (e.g. quasilinear) effects such as isotropization of particle distribution or particle trapping, or by the modulational or parametric instability of the Alfven waves

• magnetic shocktrain structures efficiently trap and mirror energetic particles. Usually, these processes quickly isotropize the energetic particle distribution, thus modifying and suppressing the growth rate of the cyclotron and fire-hose unstable Alfven waves

• shock merging in Burgers model naturally generates longer scales
– crucial for confinement of highest energy particles
– prevents the magnetic energy from rapid damping

Conclusions cont’d

• shock merging (beyond 1D) also generates vorticity which, in turn, amplifies the magnetic field in the precursor

• almost independent of the cyclotron instability, the acoustic instability creates a more efficient scattering environment which substantially improves particle confinement and enhances particle acceleration

• the spectrum of accelerated particles is softer than in a ’standard’ (resonant waves, QL) theory