SULI Final Report

Sextupole Magnets for Electron Beam Correction

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Abstract

Plasma Wakefield Acceleration is a relatively recently developed technique for accelerating charged particles that boasts the potential for a much higher accelerating gradient than traditional RF cavity acceleration . This concept is being explored by the FACET-II department at SLAC National Accelerator Laboratory [1]. In order to achieve a more efficient plasma interaction, incoming properties of the electron bunches that comprise the beam can be manipulated with magnets in the accelerating beamline. One specific type of magnet used are sextupole magnets, which can be used to perform higher order, energy-based corrections on the beam, manipulating properties such as the dispersion, chromaticity and waist location of the electron bunches.

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1 Introduction

Plasma Wakefield Acceleration (PWFA) represents a promising approach for accelerating electron beams to high energies using the strong electric fields generated in a plasma. The PWFA process relies on an initial electron bunch, known as the "drive" bunch, which generates a substantial electric field in a plasma that subsequently accelerates a "witness" bunch of electrons. However, achieving optimal beam performance in PWFA experiments requires precise control over the electron beam's properties, particularly in terms of its transverse dimensions and energy-dependent characteristics such as dispersion, waist location, and chromaticity.

Sextupole magnets play a crucial role in correcting these energy-dependent properties, making them essential for maintaining beam focus and minimizing chromatic aberrations. The importance of sextupole magnets in beam correction is underscored by their ability to manipulate the beam waist position, which is the point where the beam is most tightly focused in the transverse plane, as well as to control the chromaticity, which describes how the waist position varies with the energy of the particles in the beam. Misalignments in sextupole magnets can lead to significant aberrations, affecting the overall effectiveness of the PWFA process.

Previous studies, such as the alignment techniques used in the Stanford Linear Collider (SLC) final focus system, have demonstrated methods to achieve these beam corrections using sextupole magnets. These studies employed sextupole pairs to cancel chromatic aberrations and utilized specific alignment configurations to achieve precise control over the beam's waist and dispersion at the interaction point. This study first investigates if the alignment techniques used for the sextupole magnets in the SLC system are applicable for the FACET-II beamline and then focuses in how utilizing Multi-Objective Bayesian Optimization (MOBO) can be used to refine beam waist alignment. Through detailed simulations and experimental analyses, the study aims to minimize discrepancies in waist position from the nominal design and as well as to further optimize the strengths of both sextupole and quadrupole magnets to improve the overall efficiency of the PWFA process.

2 Sextupole Magnets

The magnetic fields present in an accelerator can be represented by the multipole expansion in cartesian coordinates [2],

$$B_y + iB_x = B_0 \sum (b_n + ia_n)(x + iy)^n \tag{1}$$

Taking any term gives the magnetic fields for a magnet with a fixed amount of poles, with n = 0 giving the field for a dipole magnet, n = 1 giving the field for a quadrupole magnet and n = 2 giving the expression for a sextupole magnet:

$$B_y = B_0(b_2x^2 - b_2y^2 - 2a_2yx) \tag{2}$$

$$B_x = B_0(a_2x^2 - a_22y^2 + 2b_2yx) \tag{3}$$

Setting $a_n = 0$ gives the fields of a normal sextupole ($b_2 = 0$ gives the skew sextupole fields). Additionally, these fields are often represented by the constant second derivative of the y-component of the field with respect to x, replacing the B_0 and b_n coefficients. For the normal sextupole fields this gives:

$$B_y = \frac{1}{2}\frac{\partial^2 B_y}{\partial x^2}(x^2 - y^2) \tag{4}$$

$$B_x = \frac{\partial^2 B_y}{\partial x^2}xy \tag{5}$$

An image of the field lines of a sextupole magnet can be observed in Figure 1.

In order to alter the magnetic fields that electron bunches entering the sextupole experience, the power to these magnets can be changed to change the field's magnitude or the sextupoles can be physically moved in either transverse dimension. It was this physical offset of the FACET-II sextupoles that was explored in this study. Because these sextupole magnets are utilized at locations where the beam has non-zero dispersion, meaning its energy spread is resolved across the transverse dimensions, these magnets allow for the alteration of energy-dependant properties of the beam including dispersion, waist location and chromaticity.

3 Simulation

The portion of FACET-II simulated in this work included the last bunch compression section (BC20) and the Final Focusing (FF) section of the accelerator. The offsets of four

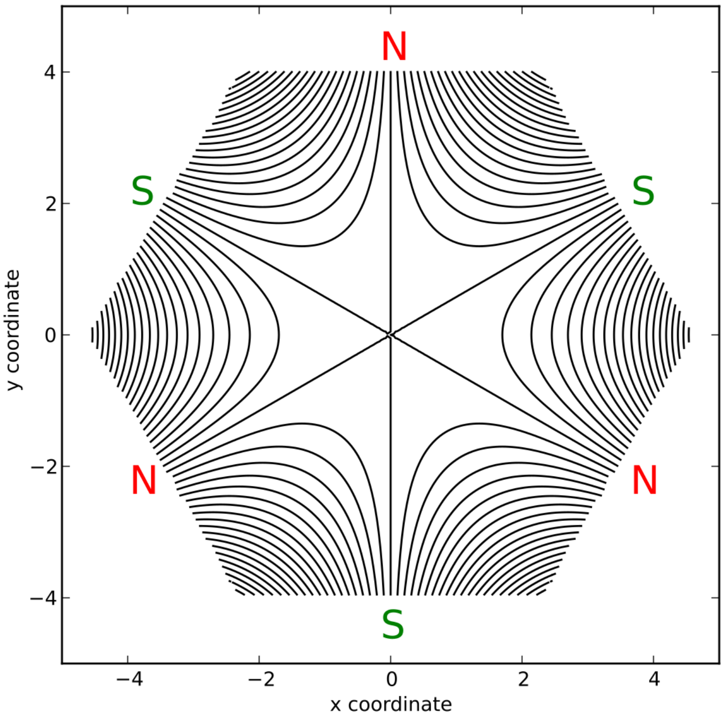

Figure 1: The magnetic field lines of an ideal sextupole magnet.

of the sextupole magnets in the bunch compressor were adjusted in order to alter the properties of the electron beam. This is illustrated in Figure 2.

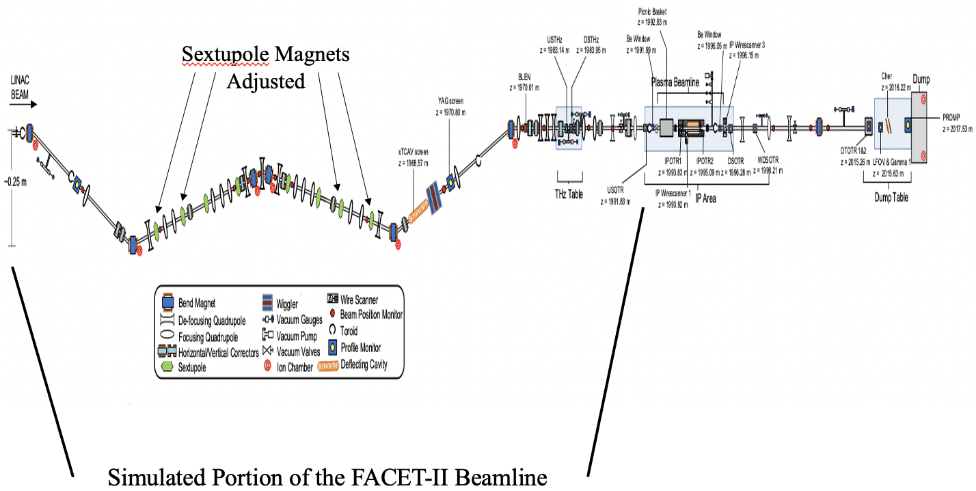

Figure 2: A depiction of the simulated portion of the FACET-II beamline and the sextupole magnets adjusted.

A single electron bunch whose creation had been simulated at the FACET-II photoinjector and which had been virtually propagated through the accelerator to the start of BC20 was used for this work. Its transverse distribution entering the bunch compressor is shown in Figure 3.

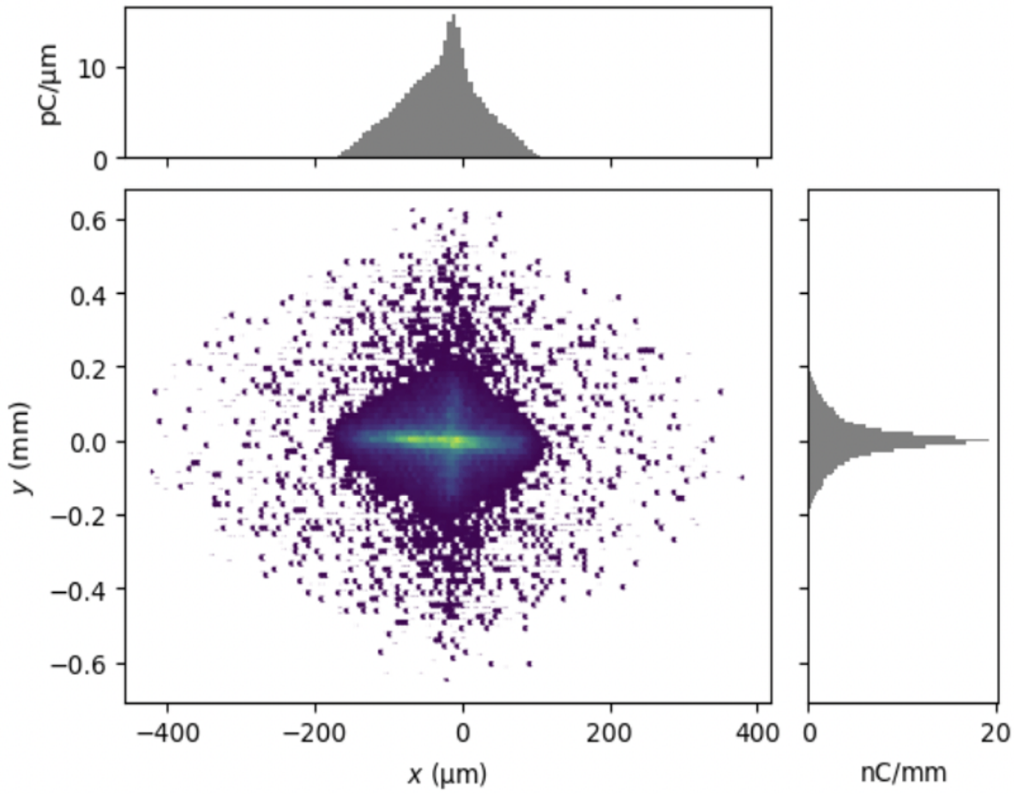

Figure 3: The transverse (xy) distribution of the electron bunch enetering BC20.

A model (AKA a lattice) of the FACET-II accelerator was loaded into the simulation toolkit BMAD [3] in order to simulate the propogation of this electron bunch through the BC20 and FF sections. A custom function was written to change the offsets of the desired sextupole magnets in the lattice.

3.1 Sextupole Alignment Components

One method for offsetting the sextupole magnets in the accelerator is based on a method which was used in the Slac Linear Collider (SLC) Final Focusing system [4]. Similar to the four sextupoles adjusted in the FACET-II bunch compressor, this beamline also contained four sextupole magnets. It was determined that the movement of two of these sextupoles in tandem could alter one of the properties of the beam without influencing others. These movements and the properties of the beam they influence are described in Figure 4. They will hereafter be referred to as alignments components 1-8.

To see if these components and their effects held true in the current accelerator, these sextupole movements were simulated on the four sextupoles in the FACET-II beamline. The dependant properties of the beam were plotted with respect to the degree of sextupole

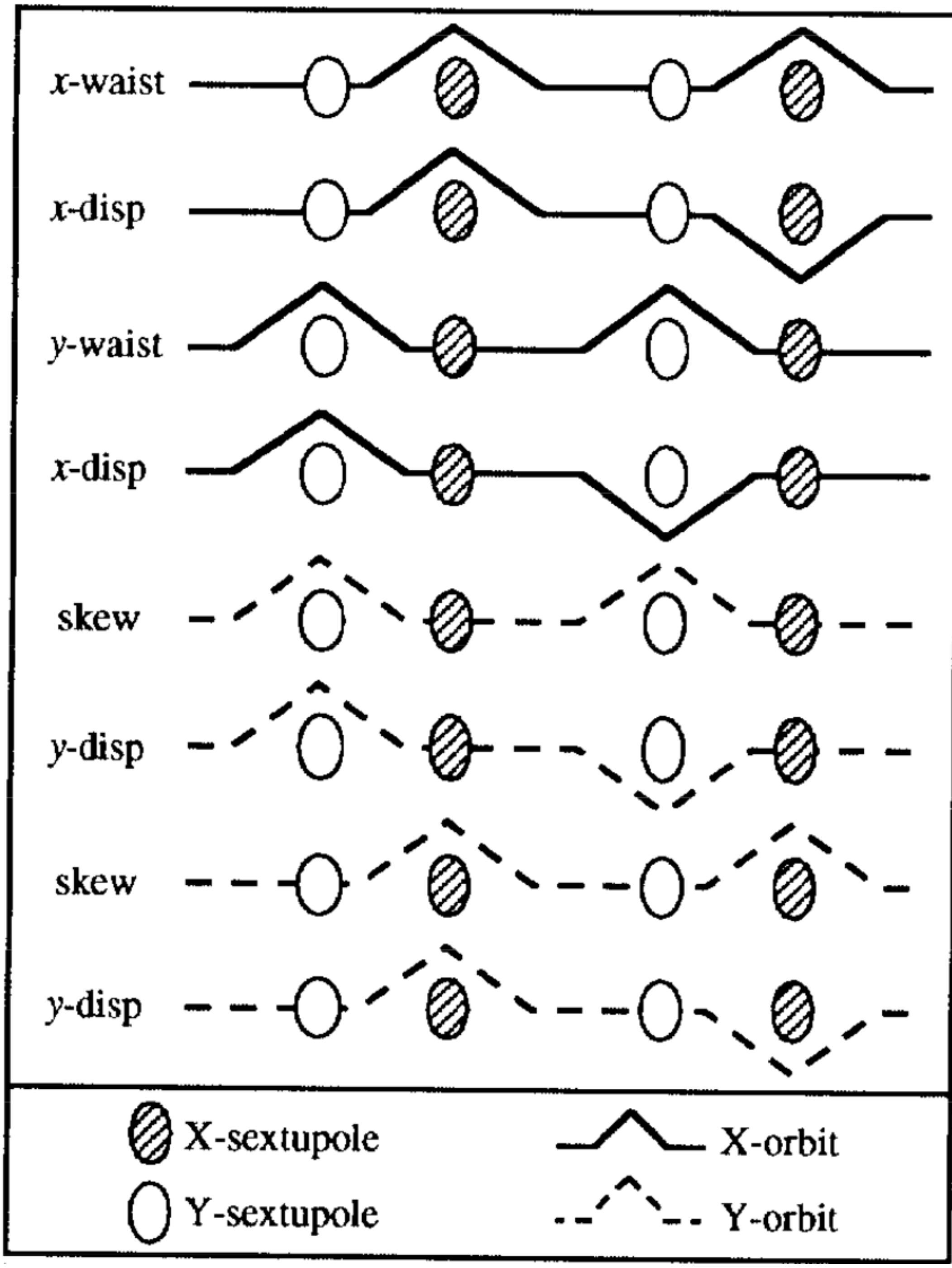

Figure 4: The Alignments of Sextupoles that influence certain properties of the electron beam in the SLC final focus system. An x-orbit component indicates motion of the sextupole in the x-direction and a y-orbit component indicates motion of the sextupole in the y-direction. These are referred to as components 1-8.

offset. Shown in figures 5, 6, 7, 8 are the effects of components 1, 2, 3, and 6, those that are supposed to alter the beam's x-waist, x-dispersion, y-waist, and y-dispersion respectively.

These plots offer a variety of information on the on the application of the alignment components in the FACET-II beamline. Overall, the effects of these components are definitively coupled, in that each alignment component affects multiple properties of the

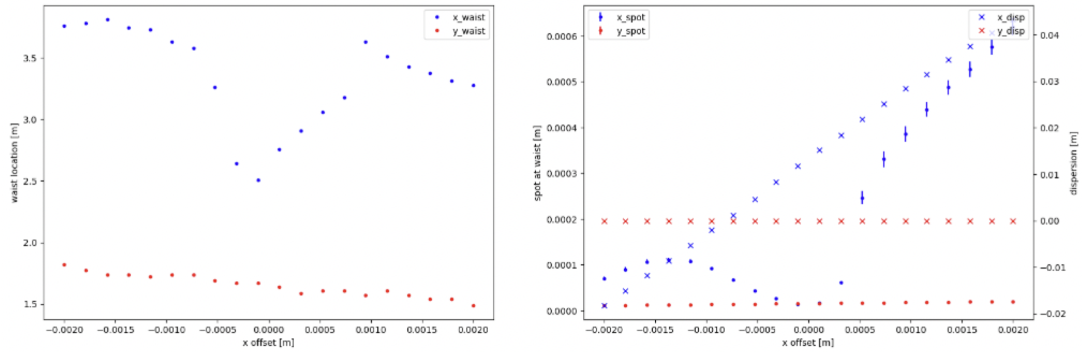

Figure 5: The effects on the electron bunch from component 1 (x-waist) in terms of sextupole offset. Dispersion was calculated at the Plasma Entrance.

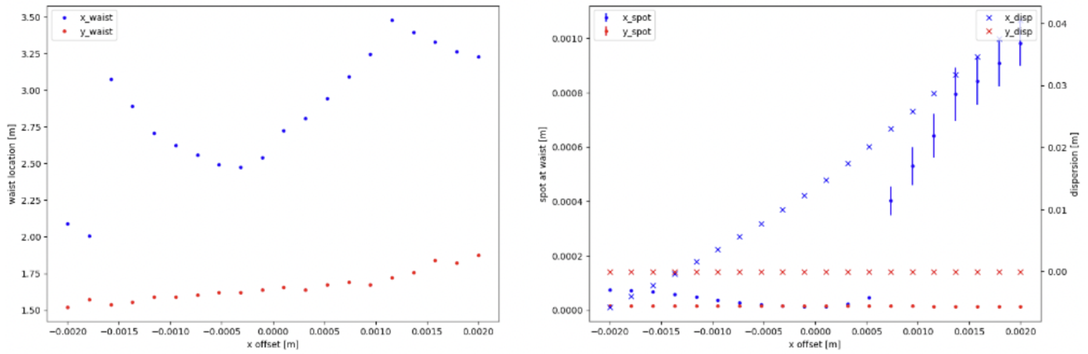

Figure 6: The effects on the electron bunch from alignment component 2 (x-dispersion) in terms of sextupole offset in meters. Dispersion was calculated at the plasma entrance.

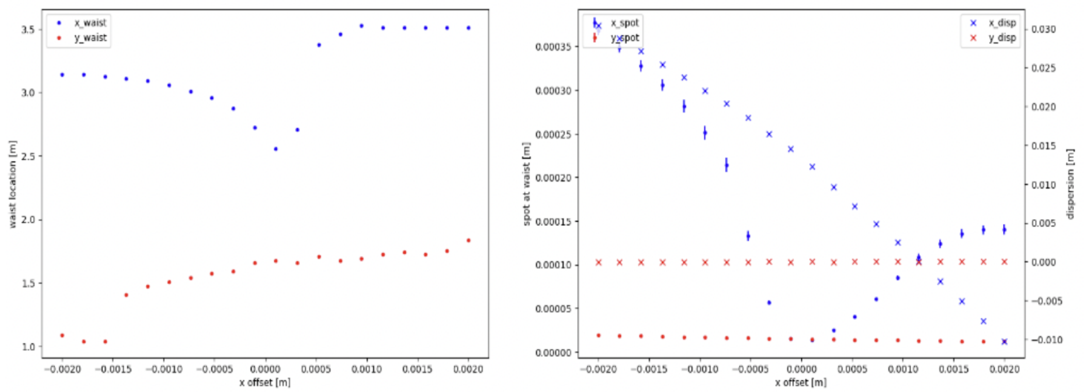

Figure 7: The effects on the electron bunch from alignment component 3 (y-waist) in terms of sextupole offset in meters. Dispersion was calculated at the plasma entrance.

beam. Because of this, these alignment components alone cannot be used to change only one of the beam properties at a time. However, it can be seen that it is possible to affect only the bunch properties in the x-direction, namely the beam waist and dispersion, without a significant change in the y-dependant properties. Similarly, component 6 is able to manipulate the y-dispersion of the beam without too significant of a change in the x-dependant properties.

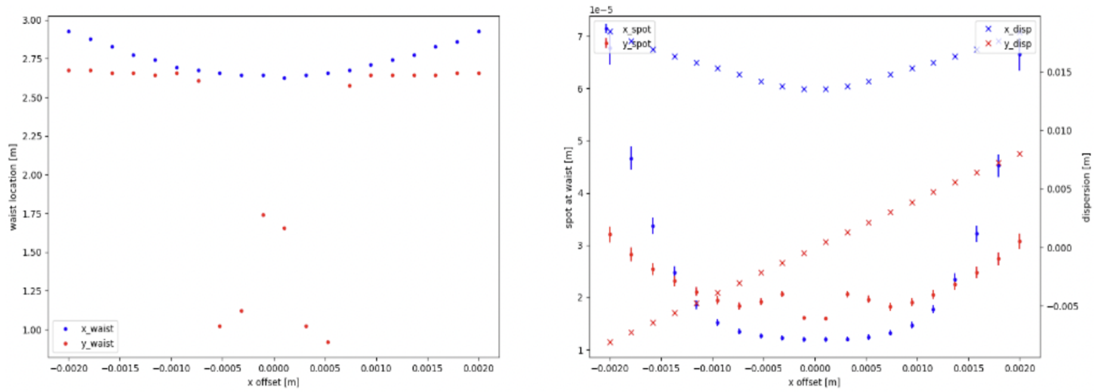

Figure 8: The effects on the electron bunch from alignment component 6 (y-dispersion) in terms of sextupole offset in meters. Dispersion was calculated at the plasma entrance.

3.2 Waist Optimization

The next method utilized to manipulate the sextupoles offsets was Multiobjective Bayesian Optimization [5]. In general, multi-objective optimization seeks to determine what is known as a Pareto front, a set of points that optimally balances the tradeoffs between competing optimization objectives with different optimization algorithms achieving this goal in different manners. In the case of MOBO, the concept of hypervolume improvment is employed. The hypervolume is defined as the volume bounded by the current Pareto front and a reference point defined when the algorithm is first initialized. The MOBO algorithm creates a fast approximation function for the target function that is used to compute the expected improvement to the hypervolume for different studied points in the parameter space. The parameters which are expected to generate a point which improves the hypervolume to the greatest extent are then evaluated in the objective function. It is in this way that the algorithm iteratively and efficiently improves the hypervolume until a good enough approximation of the true Pareto front is obtained. These concepts can be visualized in Figure 9.

For this project, the two quantities that were minimized were the absolute difference between the x-waist and the model waist position and the absolute difference between the y-waist and the model waist position. The model waist position occurs at the plasma entrance (PENT) as the PWFA process is improved by a smaller spot size at this interaction point. Figure 10 illustrates the waist differences as a function of the algorithm iteration.

As is illustrated in Figure 10, the algorithm does not converge to or begin to approach a point where the waist differences are simultaneously minimized. This demonstrates that adjustments of only the sextupole offsets are not enough to move the beam's waist to the desired location for an arbitrary beam. Instead, other accelerator components must be

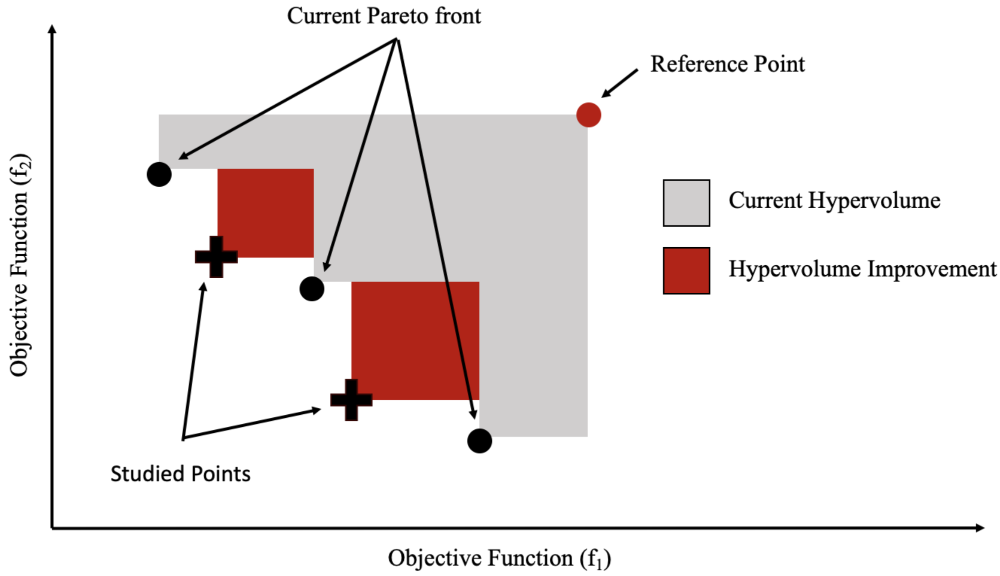

Figure 9: A multi-objective optimization problem with the goal of minimizing both objective functions.

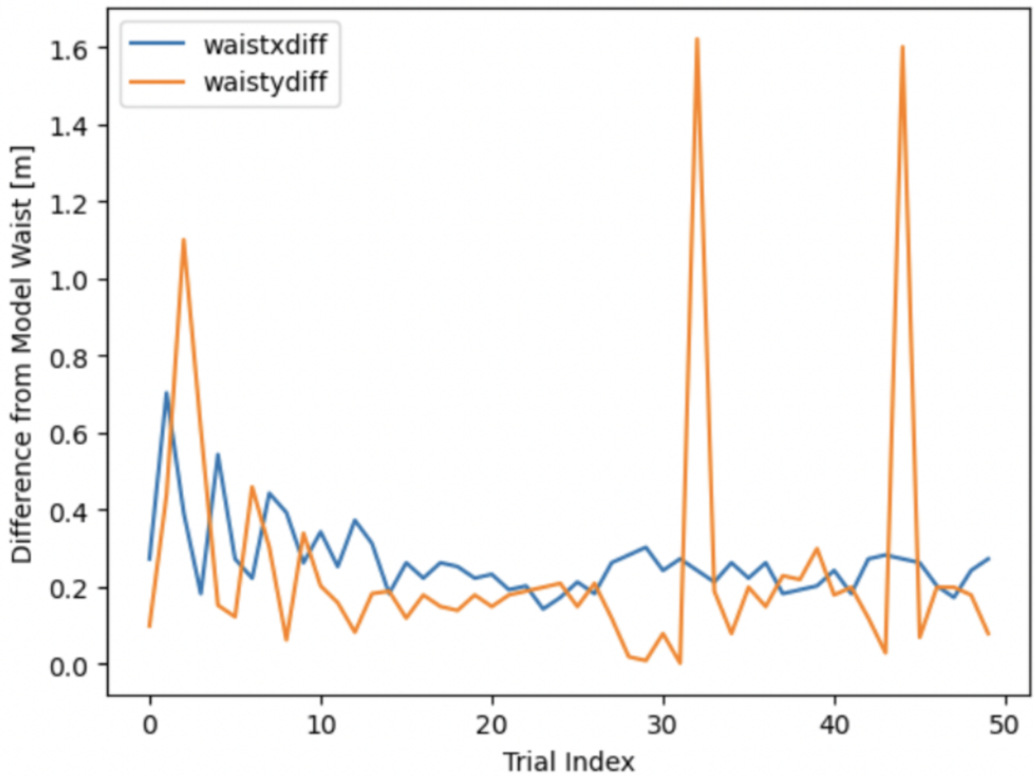

Figure 10: The waist differences from the model waist through 50 iterations of MOBO optimization.

adjusted for this to be achieved. However, this optimization was effective in illustrating the trade-off between minimizing either the x or y waist difference from the model waist. This can be seen in the determined Pareto front of the optimization, shown in Figure 11.

Graphs such as these are important in determining the range of possible beam properties that can be achieved by altering properties of the accelerator and their relationships with other properties of the beam.

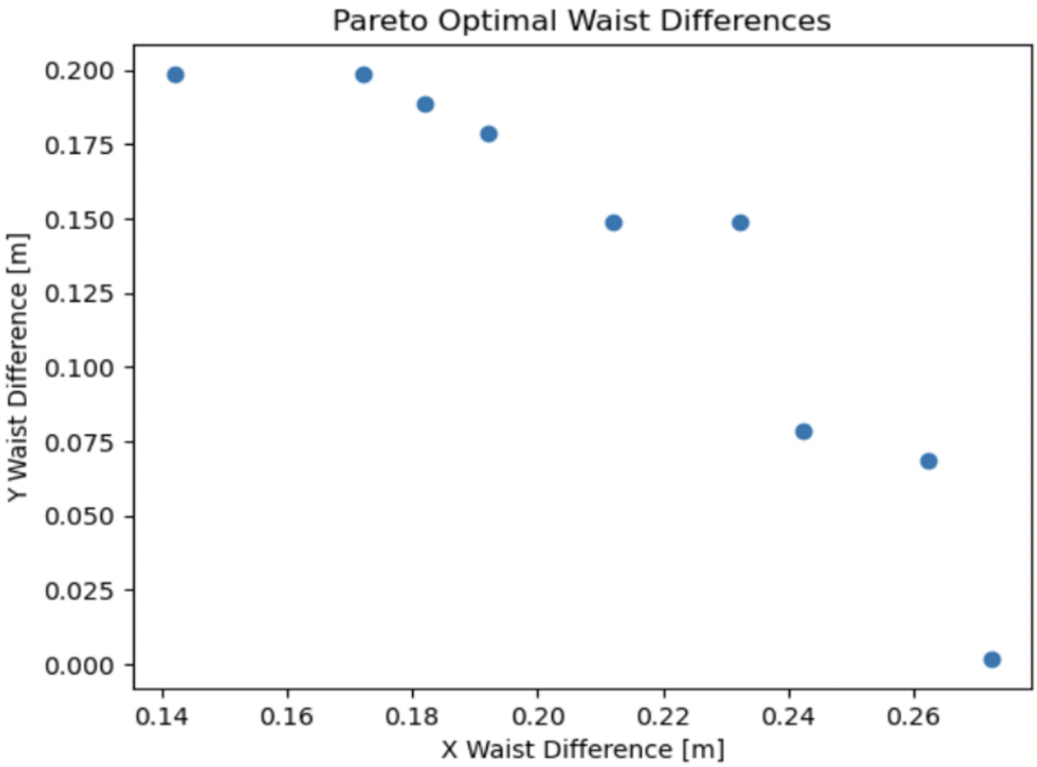

Figure 11: The established Pareto front of the optimization, showcasing the tradeoff between the x-waist and y-waist difference from the model waist.

3.3 Current Work

Current progress is being made in transferring the beam output by the BMAD simulations to a program that can simulate the plasma interaction, WAKE-T [6]. This will allow for simulations to be performed to determine the sextupole and other accelerator setting that will create the most efficient PWFA interaction.

4 Conclusion

This study demonstrates the critical role of sextupole magnets in optimizing electron beam properties for Plasma Wakefield Acceleration (PWFA) at the FACET-II facility. Through detailed simulations, the effects of sextupole alignment on beam characteristics such as waist location and chromaticity were explored, showing that these magnets can significantly influence the transverse dimensions of the beam. However, the study also reveals that adjustments to sextupole offsets alone are insufficient to achieve the desired beam waist position, indicating that additional accelerator components must be fine-tuned for optimal performance.

The use of Multi-Objective Bayesian Optimization (MOBO) provided valuable insights into the trade-offs between minimizing beam waist offsets in either transverse dimension. Although the algorithm did not converge to a single optimal solution, it highlighted the complex indemerminacies between beam properties and the need to take this into account when adjusting accelerator components.

Future work will focus on integrating the simulated beam output into the WAKE-T program to model the plasma interaction. This will enable further optimization of sextupole and other accelerator settings to enhance the efficiency and effectiveness of the PWFA process at FACET-II.

5 Acknowledgements

I would first like to acknowledge my mentor Doug Storey for all the support and wisdom he provided throughout the internship. I would also like to thank Nathan Majernik for all the helpful code and indispensable advice he provided and Alex Knetsch for his help with WAKE-T. Finally, I would like to thank the whole of the FACET-II team and the positive environment they have created in building 52.

6 Code Documentation

6.1 UTILITY_Waist.py

6.1.1 waist

Definition:

```
def waist(inDict):
```

Description: This function calculates the waist (the position along the beam where the beam size is the smallest) for a given beam configuration based on the input dictionary `inDict`. It initializes the Tao simulation environment, sets sextupole offsets, tracks the beam, and evaluates the waist in the x and y directions. The function also compares the calculated waist positions with model values provided in `inDict`.

Parameters:

- `inDict`: A dictionary containing beam parameters and model values for waist calculation.

Returns:

- `outDict`: A dictionary containing the calculated waist positions (`waistx`, `waisty`), the differences between the calculated and model waist positions (`waistxdiff`, `waistydiff`), and the beam parameters at the x and y waist positions (`Px`, `Py`, `Ppent`).

6.1.2 optimizeWaist

Definition:

```
def optimizeWaist(iterations, modxwaist, modywaist, beamStart='BEGBC20',
                  inFile='beams/BEGBC20from2024-02-16_2bunch_1e5Downsample_nudgeWeigh
```

Description: This function optimizes the beam waist positions using the Ax optimization library. It performs a specified number of iterations to minimize the difference between the calculated and model waist positions in the x and y directions.

Parameters:

- `iterations`: The number of iterations for the optimization process.
- `modxwaist`: The model x waist position.
- `modywaist`: The model y waist position.
- `beamStart`: The starting point of the beam (default is `'BEGBC20'`).
- `inFile`: The input file containing the beam data (default is a specific file path).

Returns:

- `best_parameters`: The parameters that result in the optimal waist positions.
- `axObj`: The Ax optimization object used for the experiment.

6.1.3 getBestWaists

Definition:

```
def getBestWaists(best_params, plotB=False):
```

Description: This function analyzes the results of the waist optimization process. It identifies and returns the best waist offsets in terms of x, y, and total offset.

Parameters:

- `best_params`: The parameters obtained from the waist optimization process.
- `plotB`: A boolean indicating whether to plot the waist offsets (default is `False`).

Returns:

- A dictionary containing the indices and parameters for the best x, y, and total waist offsets.

6.1.4 calculateChromaticity

Definition:

```
def calculateChromaticity(inDict):
```

Description: This function calculates the chromaticity of the beam in both the x and y directions by analyzing how the waist position changes with respect to energy. It fits a linear model to the waist positions as a function of energy slices and returns the chromaticity values.

Parameters:

- `inDict`: A dictionary containing parameters for the chromaticity calculation.

Returns:

- `xchrom`: The chromaticity in the x direction.
- `ychrom`: The chromaticity in the y direction.

6.2 componentScans.py

6.2.1 sexScan

Definition:

```
def sexScan(t, scan_range, drift, waist_res, sext1, sext2,
            x1_rng, x2_rng, y1_rng, y2_rng):
```

Description: This function simulates the beam waist positions in the x and y directions as a function of varying sextupole offsets. It initializes the Tao simulation environment, applies specific sextupole offsets, tracks the beam, and calculates the resulting beam waist positions at specified points along the beamline.

Parameters:

- `t`: The time variable for tracking the beam.
- `scan_range`: A list or array of values representing the range over which the sextupole offsets are varied.
- `drift`: The distance over which the beam drifts after the sextupole magnet.
- `waist_res`: The resolution of the waist position measurement.
- `sext1`: The identifier or label for the first sextupole magnet.
- `sext2`: The identifier or label for the second sextupole magnet.
- `x1_rng`: A list or array representing the range of x-axis offsets for the first sextupole magnet.
- `x2_rng`: A list or array representing the range of x-axis offsets for the second sextupole magnet.
- `y1_rng`: A list or array representing the range of y-axis offsets for the first sextupole magnet.
- `y2_rng`: A list or array representing the range of y-axis offsets for the second sextupole magnet.

Returns:

- `waistxs`: An array containing the calculated waist positions in the x direction for each value in `scan_range`.
- `waistys`: An array containing the calculated waist positions in the y direction for each value in `scan_range`.
- `P_pent`: A list of beam properties at the plasma entrance point for each scan.

6.2.2 sexScans

Definition:

```
def sexScans(tao, scan_range, drift, waist_res):
```

Description: This function performs multiple scans of sextupole offsets and records the resulting waist positions in both the x and y directions. It is a higher-level wrapper that calls the `sexScan` function for different sextupole configurations, facilitating comprehensive beam alignment analysis.

Parameters:

- `tao`: The Tao object used for initializing and tracking the beam.
- `scan_range`: A list or array of values representing the range over which the sextupole offsets are varied.
- `drift`: The distance over which the beam drifts after the sextupole magnet.
- `waist_res`: The resolution of the waist position measurement.

Returns:

- `x_waistss`: A 2D array containing the calculated waist positions in the x direction for each value in `scan_range` across multiple scans.
- `y_waistss`: A 2D array containing the calculated waist positions in the y direction for each value in `scan_range` across multiple scans.

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