

Exponential Methods for Anisotropic Diffusion

(in the context of Cosmic Rays)

Pranab J Deka

Lukas Einkemmer

Ralf Kissmann



My Background



EXPONENTIAL INTEGRATORS FOR MAGNETOHYDRODYNAMICS AND COSMIC RAY TRANSPORT

PHD THESIS

PRANAB JYOTI DEKA

- solve **time-dependent** ODEs/PDEs using computers
- develop mathematical and computational algorithms for HPC systems
- applications: MHD, anisotropic diffusion (in the context of cosmic ray diffusion)

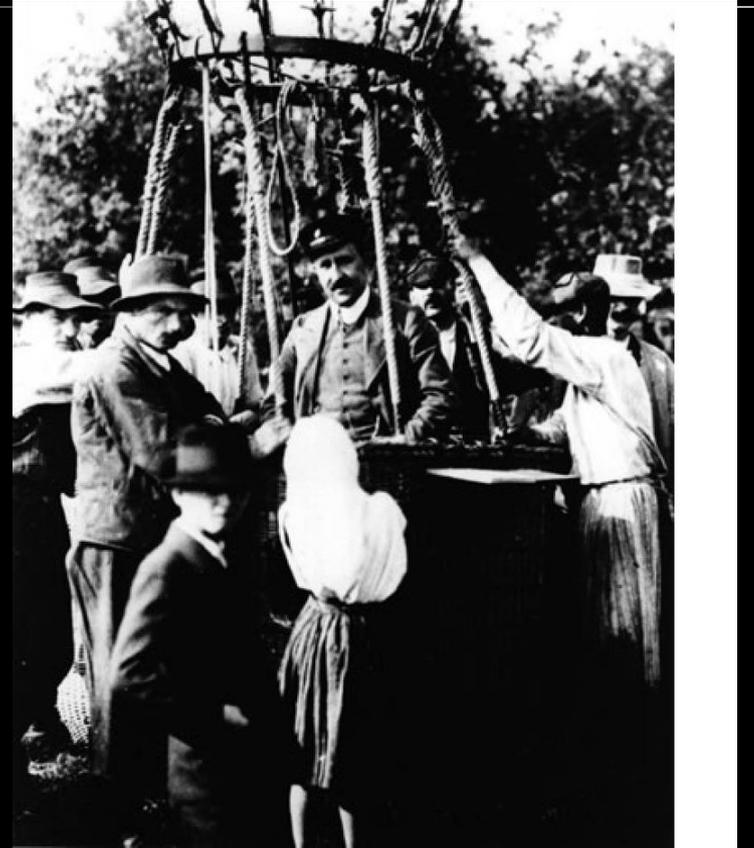
Cosmic Rays



Discovered by **Victor Francis Hess**
(24 June 1883 – 17 December 1964)

Nobel Prize in Physics (1936)

Professor – University of Innsbruck (1931)



August 1912

Cosmic Rays

- Highly-energetic charged particles
- (Galactic) CRs are transported mainly via *diffusion*
- CR diffusion – morphology and strength of the (stronger) large-scale ordered Galactic magnetic field and the (weaker) small-scale turbulent fields
- CRs are diffused along the ordered field lines – CR diffusion is *anisotropic*

Galactic Cosmic Rays

$$\begin{aligned} \frac{\partial \psi(\vec{r}, p, t)}{\partial t} &= S(\vec{r}, p, t) - \nabla \cdot (\vec{v}\psi) + \nabla \cdot (\mathcal{D}\nabla\psi) \\ &\quad - \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial}{\partial p} \left(\frac{\psi}{p^2} \right) \right) + \frac{\partial}{\partial p} \left(\dot{p}\psi - \frac{p}{3} (\nabla \cdot \vec{v})\psi \right) \\ &\quad - \frac{1}{\tau_f} \psi - \frac{1}{\tau_r} \psi \end{aligned}$$

- **advect** through the Galaxy
- **diffuse**, in space, upon scattering
- **reaccelerate** (diffusion in momentum space)
- **momentum losses** (synchrotron, IC, bremsstrahlung)
- **fragment** and **decay** (create new elements/isotopes)

Galactic Cosmic Rays

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GOAL

We want to develop algorithms suitable for solving this equation over long periods of time on modern HPC systems (such as GPUs)

Anisotropic Diffusion

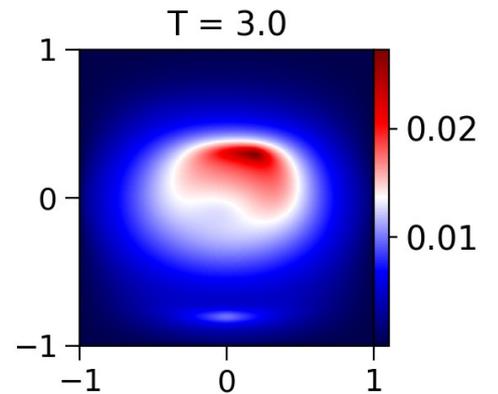
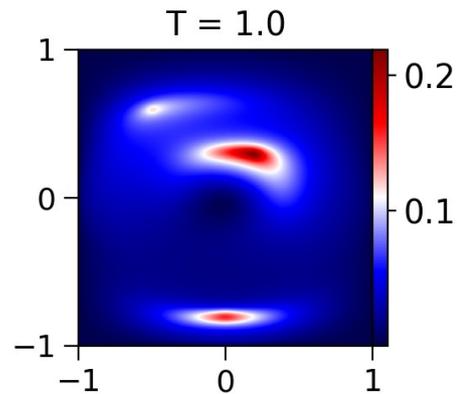
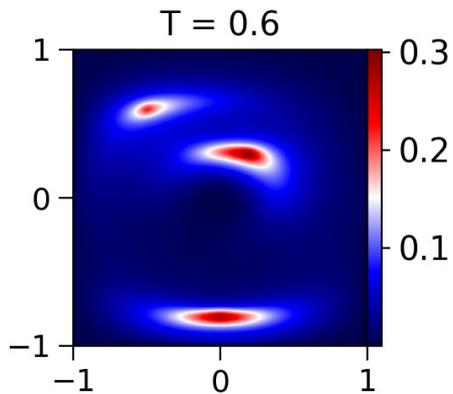
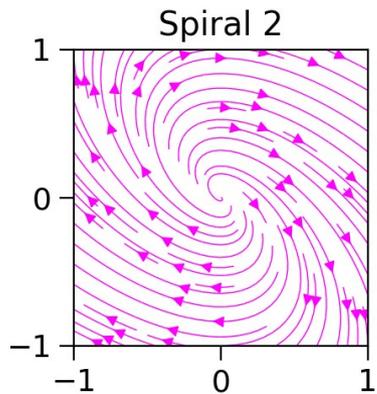
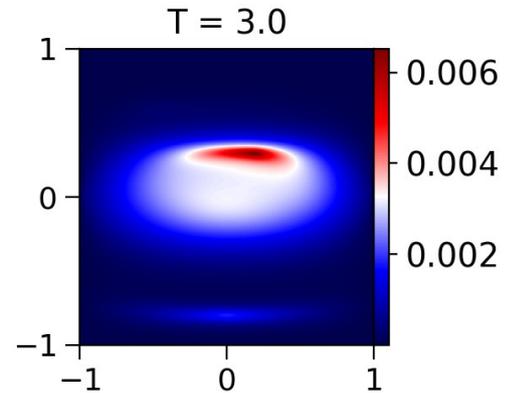
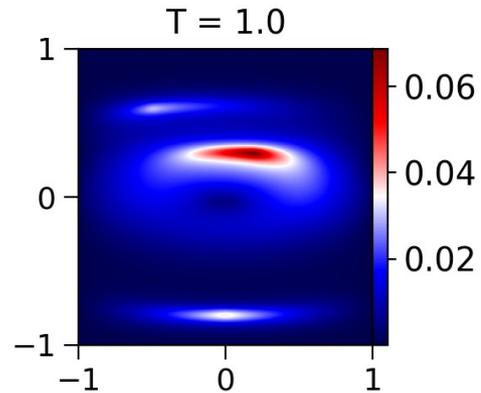
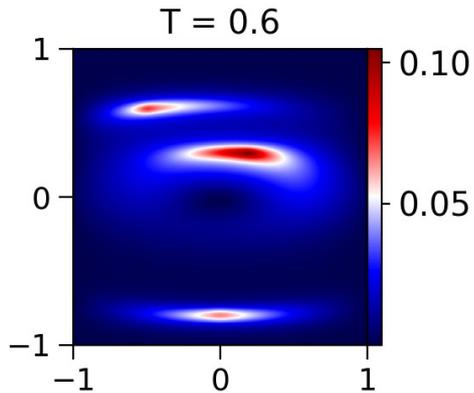
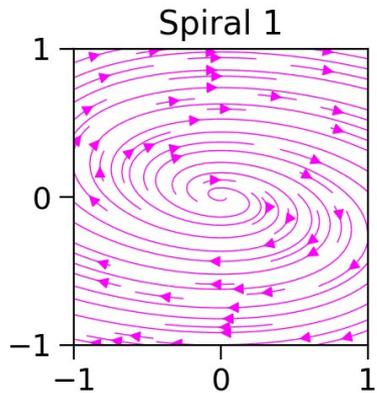
Anisotropic diffusion + advection + time-dependent sources

$$\frac{\partial u}{\partial t} = \nabla \cdot (\mathcal{D} \nabla u) + \nabla \cdot (\vec{a}u) + S(x, y, t);$$

Anisotropic diffusion

$$\mathcal{D} = \begin{bmatrix} D_{xx}(x, y) & D_{xy}(x, y) \\ D_{yx}(x, y) & D_{yy}(x, y) \end{bmatrix}$$

Anisotropic Diffusion



Exponential methods

$$\frac{\partial u}{\partial t} = f(u)$$

$$u^{n+1} = u^n + f(u^n)\Delta t$$

$$u^{n+1} = u^n + \text{exponential-like function}(f(u^n))\Delta t$$

Exponential methods

Exponential midpoint (2nd order):

$$u^{n+1} = u^n + \varphi_1(\mathcal{A}\Delta t) \left(\mathcal{A}u^n + S \left(t^n + \frac{1}{2}\Delta t \right) \right) \Delta t$$

$$\frac{\partial u}{\partial t} = \nabla \cdot (\mathcal{D} \nabla u) + \nabla \cdot (\vec{a}u) + S(x, y, t),$$

$$\mathcal{A} = \nabla(\mathcal{D} \nabla(\cdot)) + \nabla(\vec{a})$$

Exponential methods

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$$\varphi_1(z) = \frac{\exp(z) - 1}{z}$$

Approximate the action of the exponential-like functions on vectors using polynomial interpolation

Exponential methods for Anisotropic Diffusion

Exponential midpoint (2nd order):

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$$\mathcal{A} = \nabla(\mathcal{D}\nabla(\cdot)) + \nabla(\vec{a})$$

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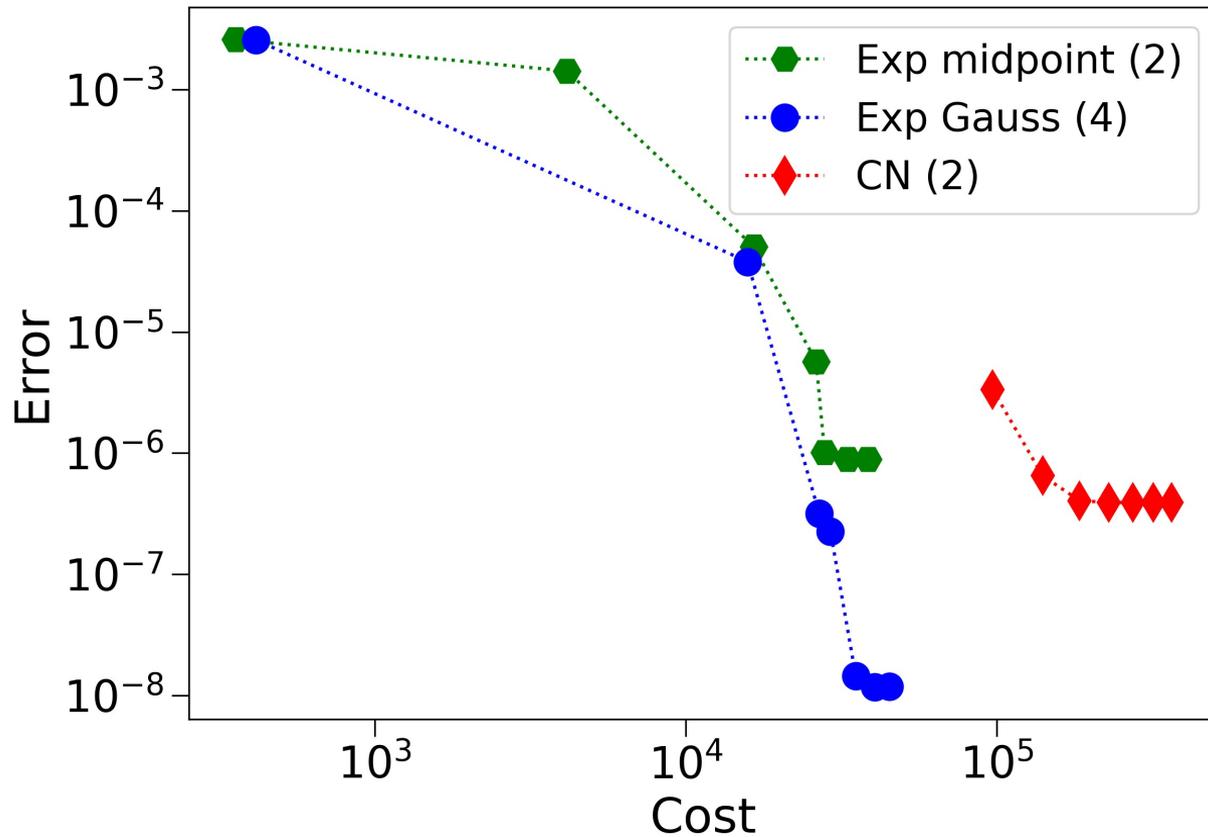
Approximate the action of the exponential-like functions on vectors using polynomial interpolation

The screenshot shows a GitHub repository page for 'LeXInt'. At the top, there are links for 'README' and 'MIT license'. The repository name 'LeXInt' is prominently displayed. Below the name, there are several colored buttons representing different technologies: 'PYTHON' (yellow), 'C++' (blue), 'NVIDIA' (green), 'VSCODE' (blue), and 'GITHUB' (black). At the bottom left, there are two small buttons: 'ascl 2208.009' and 'License MIT'.

Python
C++ (OpenMP)
CUDA

MPI (coming soon ...)

Exponential methods for Anisotropic Diffusion



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This branch is 10 commits ahead of Coils .

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 **Pranab-JD** Minor changes

b9a6095 · 3 days ago 477 Commits

ConfigFile/src

Add message for missing key

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Documentation

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H5HU?T-io from ECsim new master

2 years ago

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