Exponential Methods for Anisotropic Diffusion

(in the context of Cosmic Rays)

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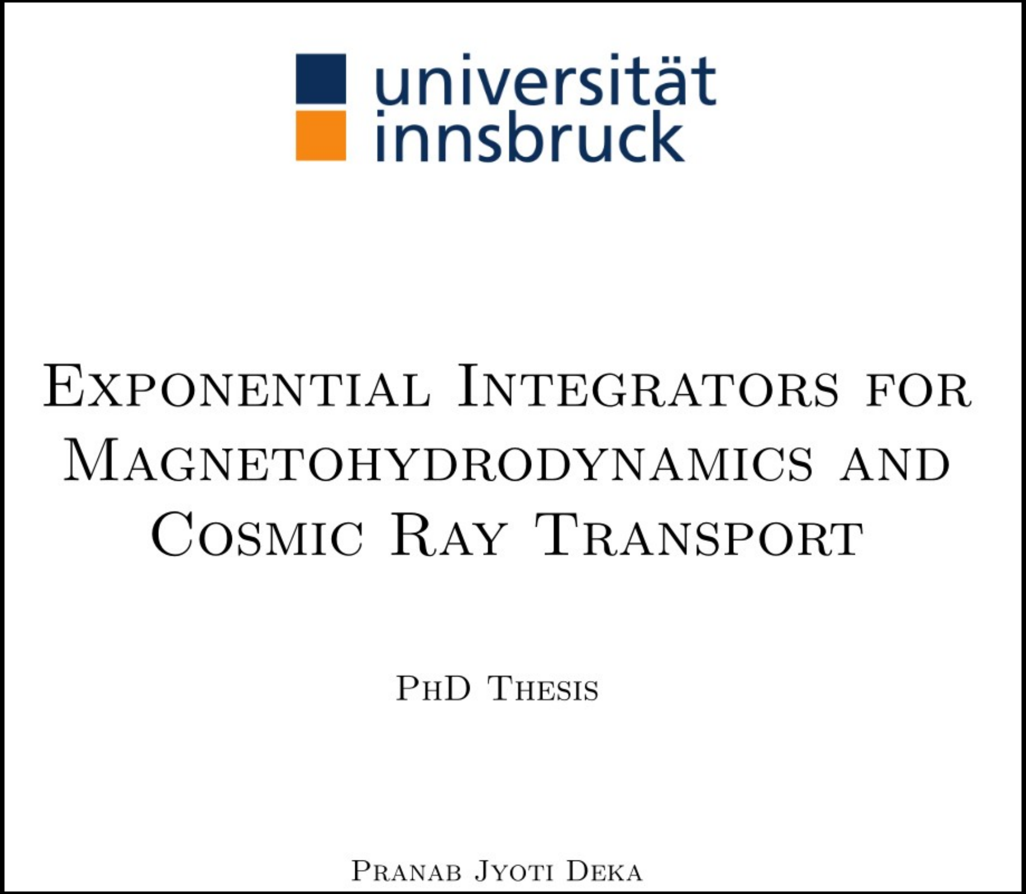
My Background



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EXPONENTIAL INTEGRATORS FOR MAGNETOHYDRODYNAMICS AND COSMIC RAY TRANSPORT

PHD THESIS

PRANAB JYOTI DEKA

- solve time-dependent ODEs/PDEs using computers

- develop mathematical and computational algorithms for HPC systems

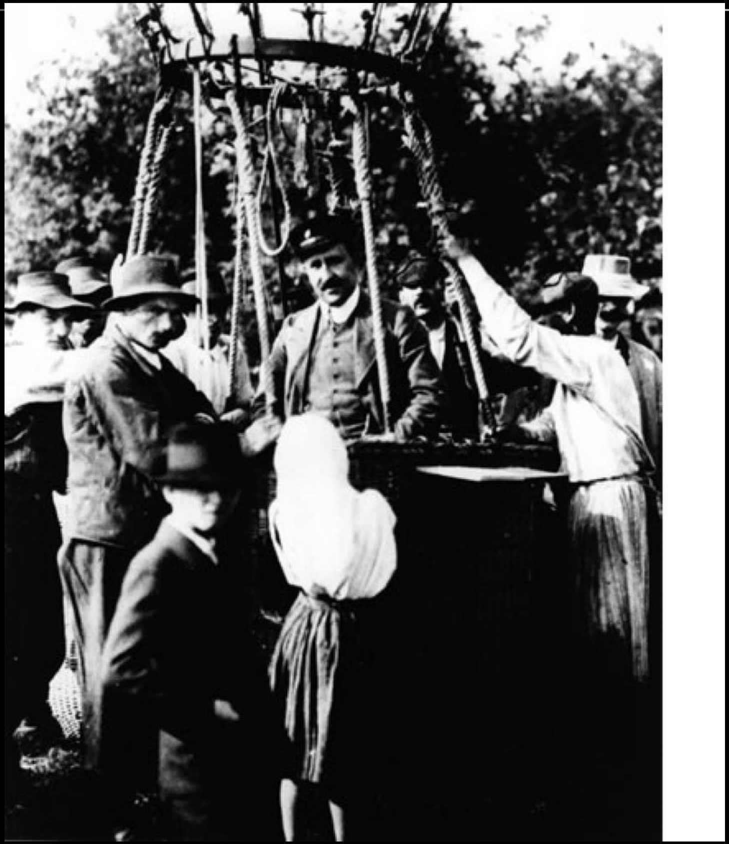
- applications: MHD, anisotropic diffusion (in the context of cosmic ray diffusion)

Cosmic Rays

Discovered by ***Victor Francis Hess***
(24 June 1883 – 17 December 1964)

Nobel Prize in **Physics** (1936)

Professor – University of Innsbruck (1931)



August 1912

Cosmic Rays

- Highly-energetic charged particles

- (Galactic) CRs are transported mainly via ***diffusion***

- CR diffusion − morphology and strength of the (stronger) large-scale ordered Galactic magnetic field and the (weaker) small-scale turbulent fields

- CRs are diffused along the ordered field lines − CR diffusion is ***anisotropic***

Galactic Cosmic Rays

$$\frac{\partial \psi(\vec{r}, p, t)}{\partial t} = S(\vec{r}, p, t) - \nabla \cdot (\vec{v}\psi) + \nabla \cdot (\mathcal{D}\nabla\psi)$$
$$- \frac{\partial}{\partial p}\left(p^2 D_{pp} \frac{\partial}{\partial p}\left(\frac{\psi}{p^2}\right)\right) + \frac{\partial}{\partial p}\left(\dot{p}\psi - \frac{p}{3}(\nabla \cdot \vec{v})\psi\right)$$
$$- \frac{1}{\tau_f}\psi - \frac{1}{\tau_r}\psi$$

- advect through the Galaxy
- diffuse, in space, upon scattering
- reaccelerate (diffusion in momentum space)
- momentum losses (synchrotron, IC, bremstrahlung)
- fragment and decay (create new elements/isotopes)

Galactic Cosmic Rays

$$\frac{\partial \psi(\vec{r}, p, t)}{\partial t} = S(\vec{r}, p, t) - \nabla \cdot (\vec{v}\psi) + \nabla \cdot (\mathcal{D}\nabla\psi) - \frac{\partial}{\partial p}\left(p^2 D_{pp} \frac{\partial}{\partial p}\left(\frac{\psi}{p^2}\right)\right) + \frac{\partial}{\partial p}\left(\dot{p}\psi - \frac{p}{3}(\nabla \cdot \vec{v})\psi\right) - \frac{1}{\tau_f}\psi - \frac{1}{\tau_r}\psi$$

GOAL

We want to develop algorithms suitable for solving this equation over long periods of time on modern HPC systems (such as GPUs)

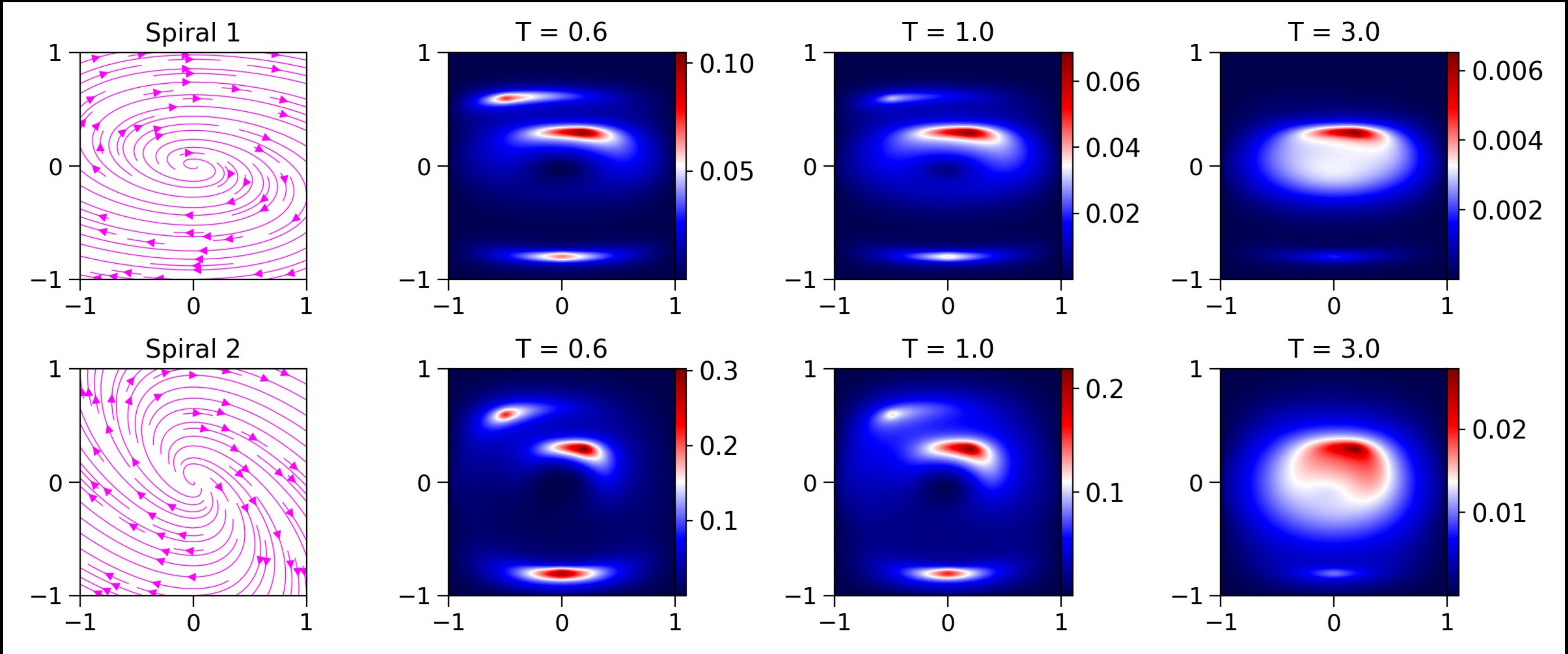
Anisotropic Diffusion

Anisotropic diffusion + advection + time-dependent sources

$$\frac{\partial u}{\partial t} = \nabla \cdot (\mathcal{D} \nabla u) + \nabla \cdot (\vec{a} u) + S(x, y, t);$$

Anisotropic diffusion $\mathcal{D} = \begin{bmatrix} D_{xx}(x, y) & D_{xy}(x, y) \\ D_{yx}(x, y) & D_{yy}(x, y) \end{bmatrix}$

Anisotropic Diffusion



Exponential methods

$$\frac{\partial u}{\partial t} = f(u)$$

$$u^{n+1} = u^n + f(u^n)\Delta t$$

$$u^{n+1} = u^n + \text{exponential-like function}(f(u^n))\Delta t$$

Exponential methods

Exponential midpoint (2nd order):

$$u^{n+1} = u^n + \varphi_1(\mathcal{A}\Delta t)\left(\mathcal{A}u^n + S\left(t^n + \frac{1}{2}\Delta t\right)\right)\Delta t$$

$$\frac{\partial u}{\partial t} = \nabla \cdot (\mathcal{D}\nabla u) + \nabla \cdot (\vec{a}u) + S(x, y, t);$$

$$\mathcal{A} = \nabla(\mathcal{D}\nabla(\cdot)) + \nabla(\vec{a})$$

Exponential methods

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$$\varphi_1(z) = \frac{\exp(z) - 1}{z}$$

Approximate the action of the exponential-like functions on vectors using polynomial interpolation

Exponential methods for Anisotropic Diffusion

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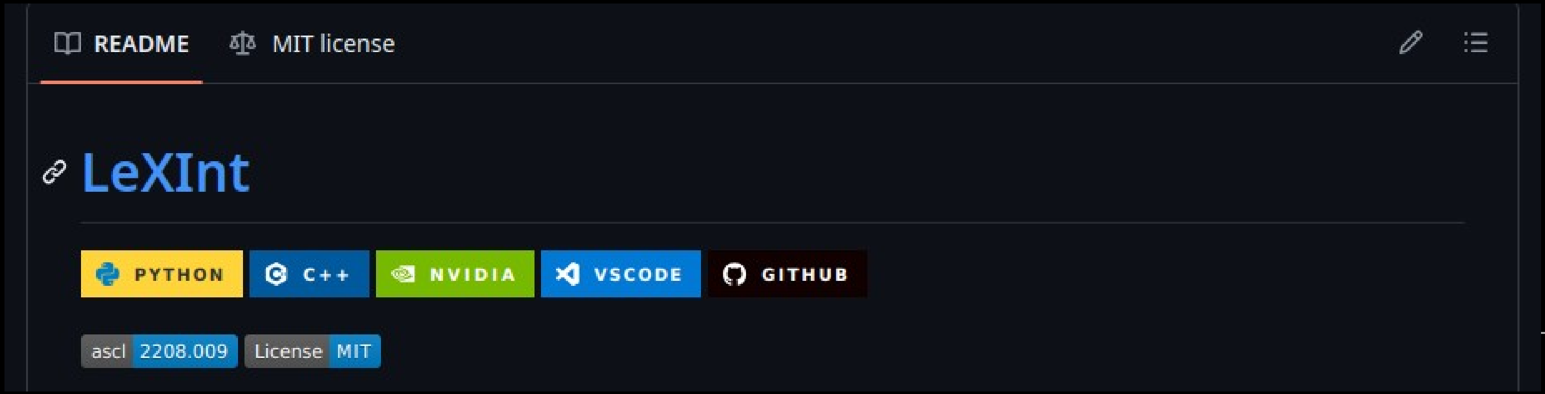
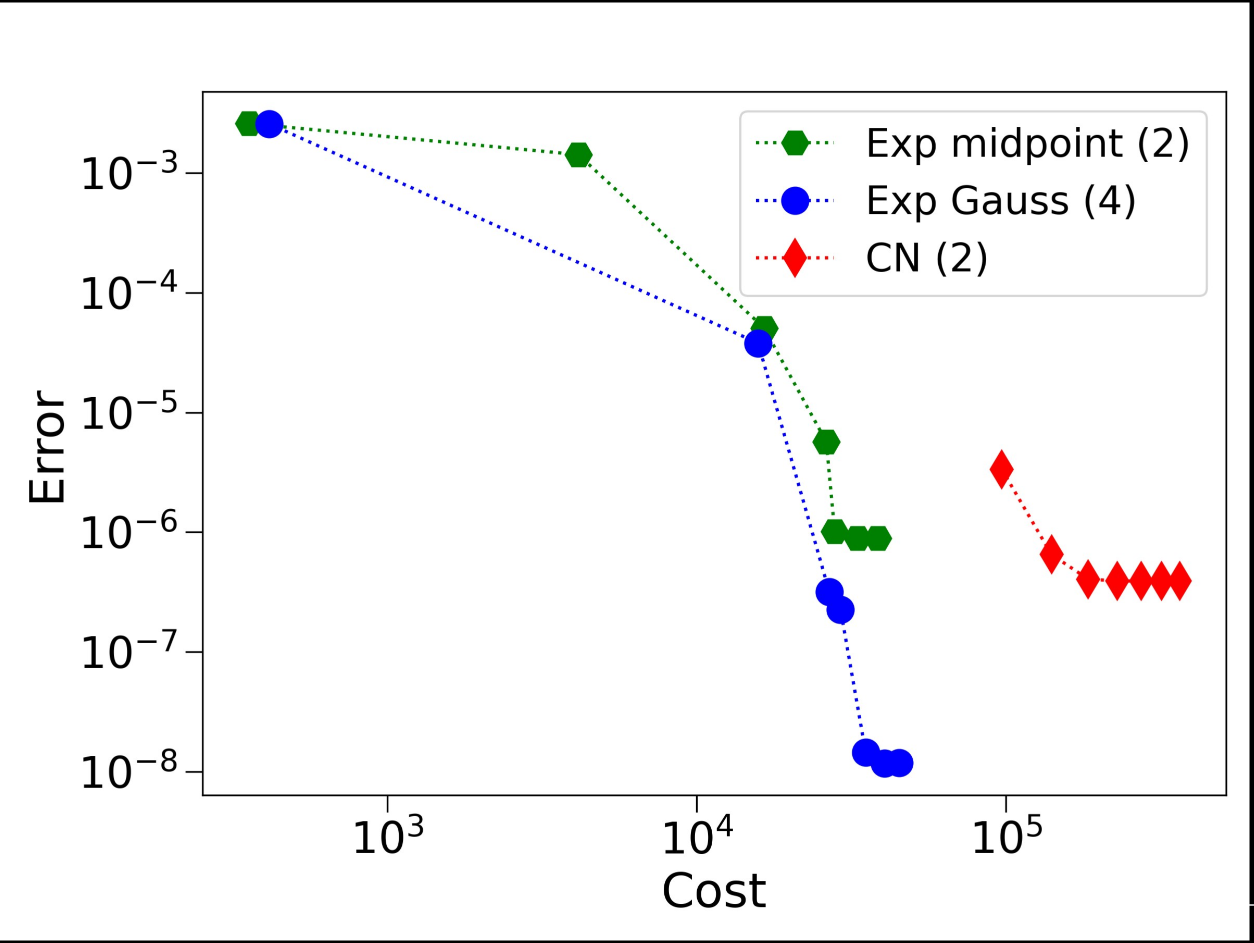


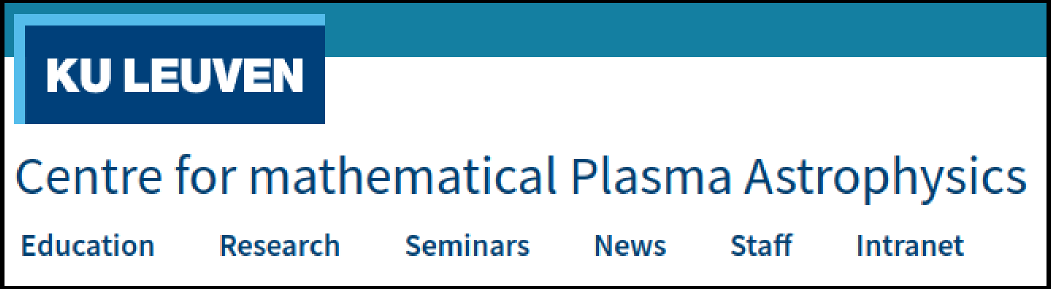
Python
C++ (OpenMP)
CUDA

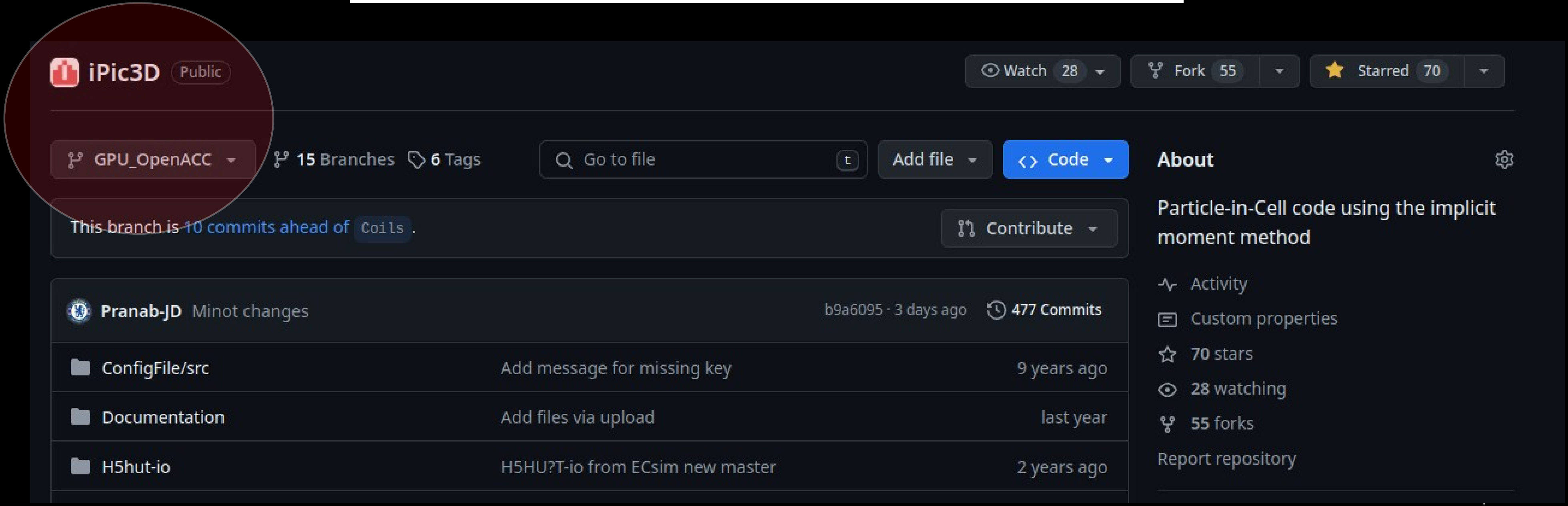
MPI (coming soon ...)

Exponential methods for Anisotropic Diffusion



What am I doing now?





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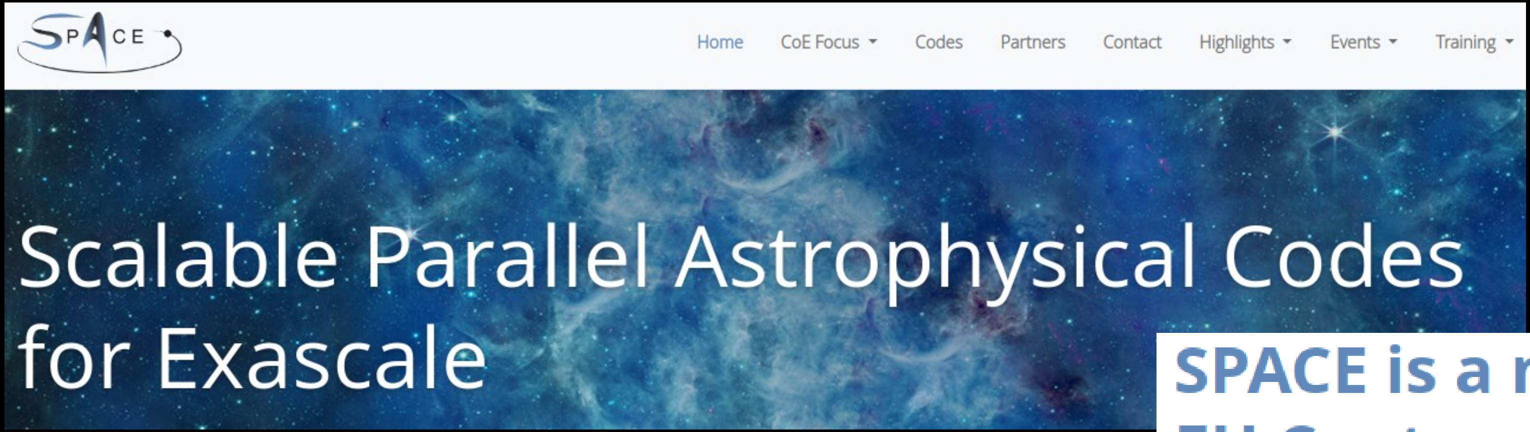

SPACE is a newly-funded EU Centre of Excellence focused on astrophysical and cosmological applications

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Budget of nearly 8 million euros

What am I doing now?

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