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A Theoretical Fourier-Transformation Model for the Formation of X-ray Time Lags from Black Hole Accretion Disks

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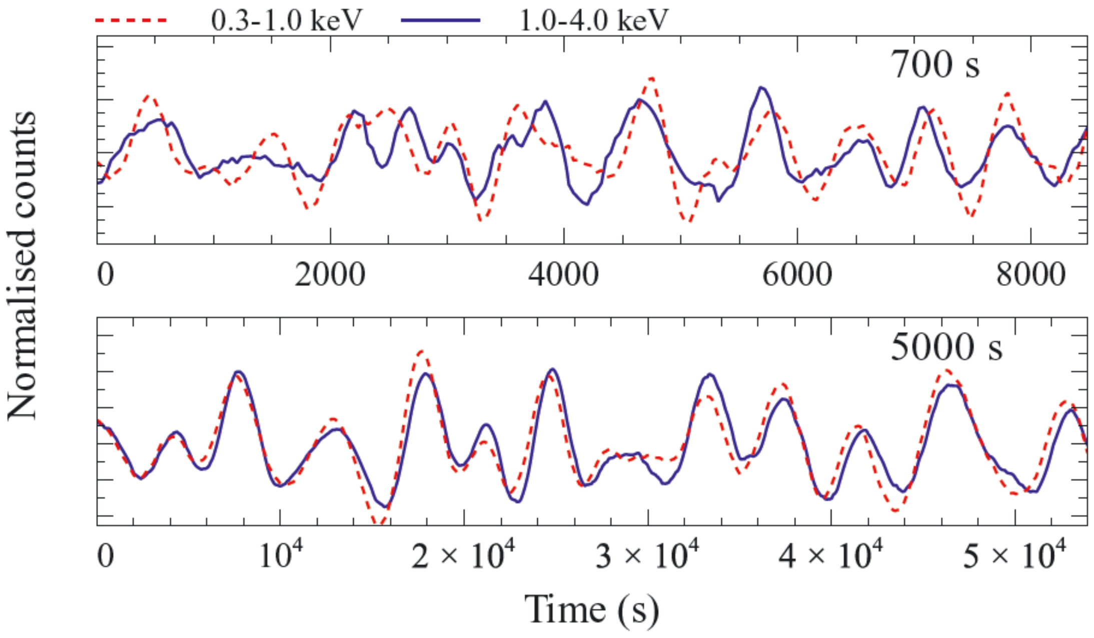
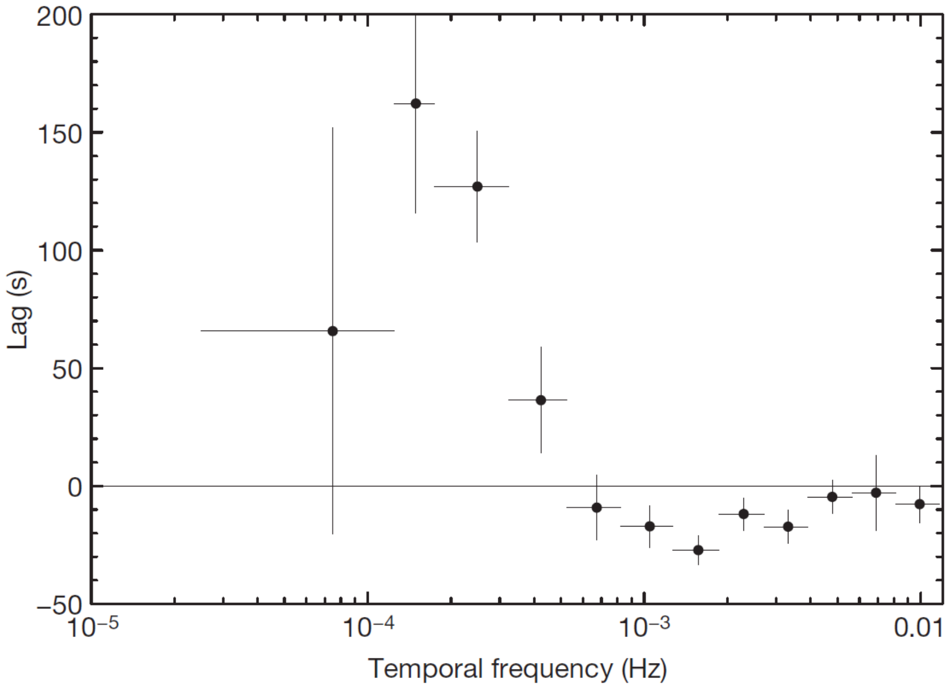
Definition of Time Lags



Source: Zoghbi et al. 2010



Source: Fabian et al. 2009

Definition of Time Lags

$$\mathfrak{s}(t) = \mathscr{F}(\epsilon_{\rm s}, t)$$
$$\mathfrak{h}(t) = \mathscr{F}(\epsilon_{\rm h}, t)$$

$$\mathfrak{S}(\omega) = \tilde{\mathscr{F}}(\epsilon_{\rm s}, t) = \int_{-\infty}^{+\infty} \mathfrak{s}(t)\, e^{i\omega t} dt$$

$$\mathfrak{H}(\omega) = \tilde{\mathscr{F}}(\epsilon_{\rm h}, t) = \int_{-\infty}^{+\infty} \mathfrak{h}(t)\, e^{i\omega t} dt$$

$$\mathfrak{C}(\omega) = \mathfrak{S}^*(\omega)\mathfrak{H}(\omega)$$

$$\phi(\omega) = \mathrm{Arg}[\mathfrak{S}^*(\omega)\mathfrak{H}(\omega)]$$

$$\delta t(\omega) = \frac{\phi(\omega)}{\omega}$$

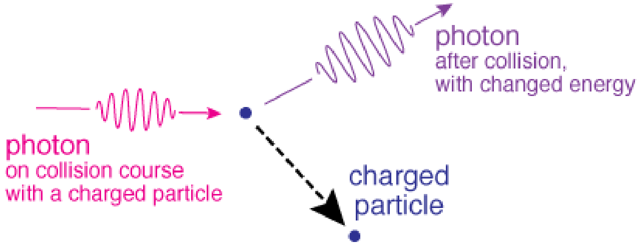
$$\mathfrak{h}(t + \Delta t) = \mathfrak{s}(t)$$
$$\mathfrak{H}(\omega) = \int_{-\infty}^{+\infty} \mathfrak{h}(t)\, e^{i\omega t} dt$$
$$\mathfrak{S}(\omega) = \int_{-\infty}^{+\infty} \mathfrak{h}(t + \Delta t)\, e^{i\omega t} dt$$
$$\mathfrak{S}(\omega) = e^{-i\omega\Delta t} \int_{-\infty}^{+\infty} \mathfrak{h}(t + \Delta t)\, e^{i\omega(t+\Delta t)} d(t + \Delta t)$$
$$= e^{-i\omega\Delta t}\mathfrak{H}(\omega)$$
$$= e^{i\omega\Delta t}|\mathfrak{H}(\omega)|^2$$
$$\phi(\omega) = \mathrm{Arg}[e^{i\omega\Delta t}|\mathfrak{H}(\omega)|^2] = \omega\Delta t$$
$$\delta t(\omega) = \frac{\omega\Delta t}{\omega} = \Delta t$$

X-ray Emission

XMM-Newton

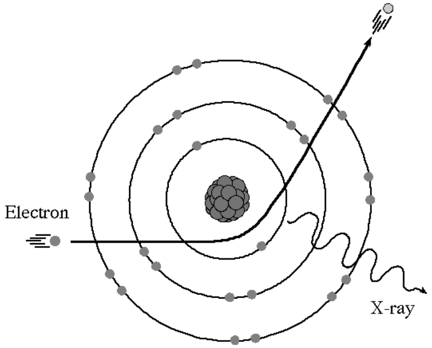
Source: ESA - D. Ducros

Comptonization & Inverse Comptonization



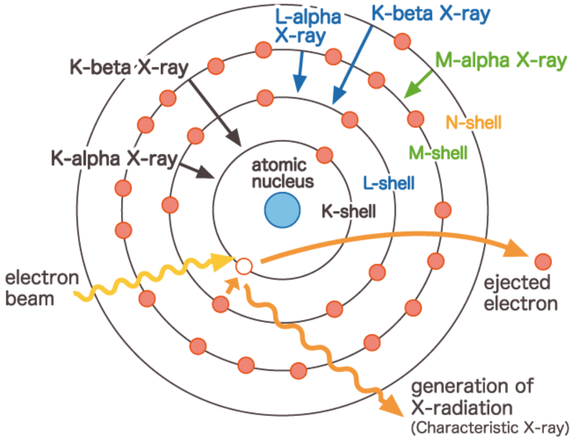
Source: seramarkoff.com

Bremsstrahlung



Source: teledyneicm.com

X-ray Fluorescence (Fe Kα & Lα lines)

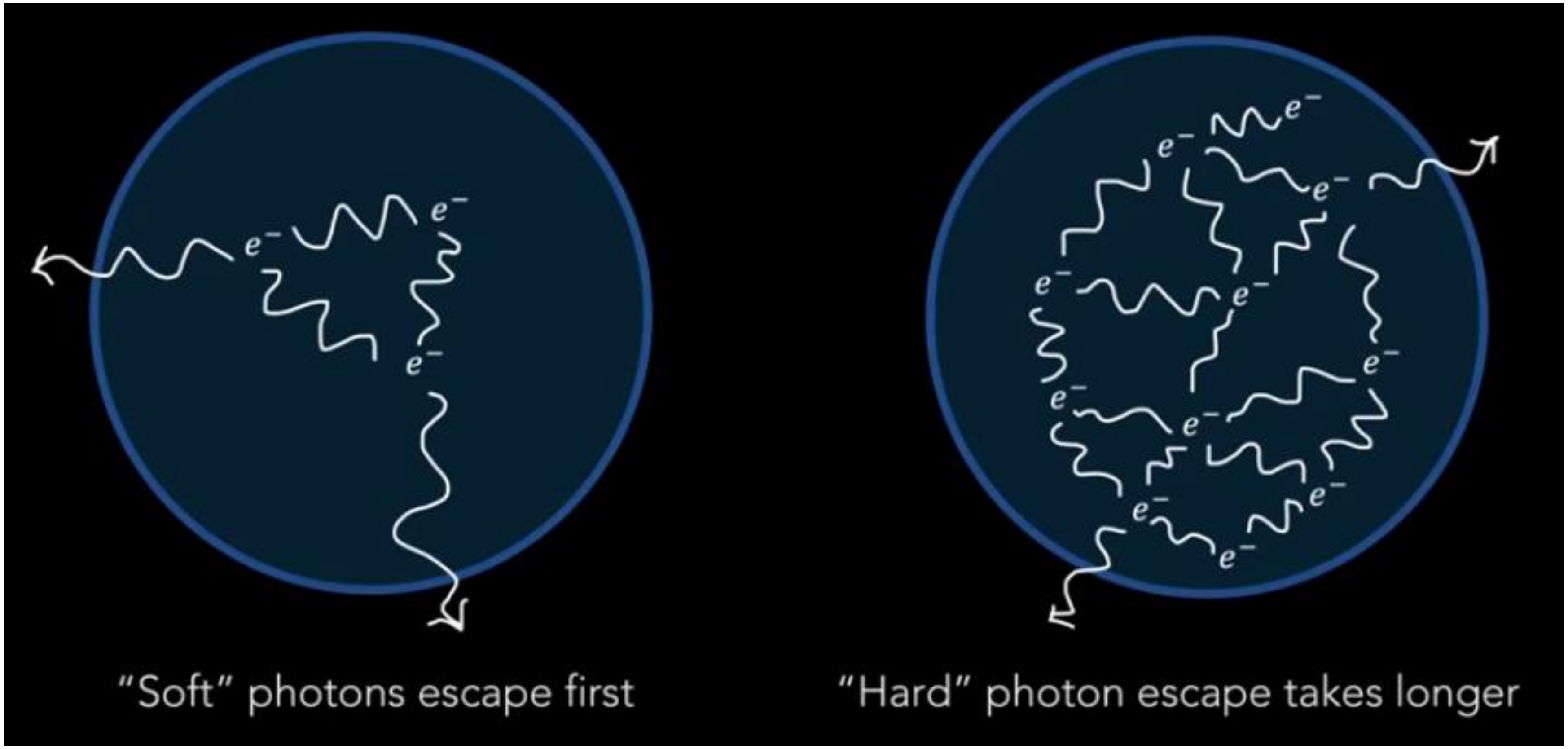


Source: matsusada.com/column/sem-tech3.html

Blackbody radiation

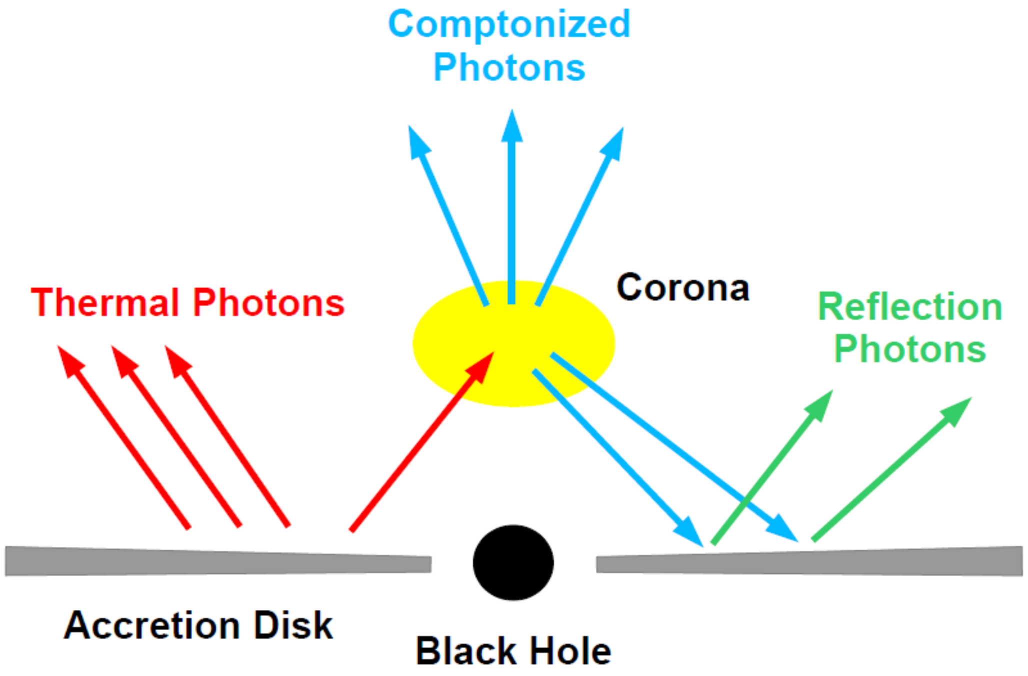
Source: blacksmithu.com

Previous Model

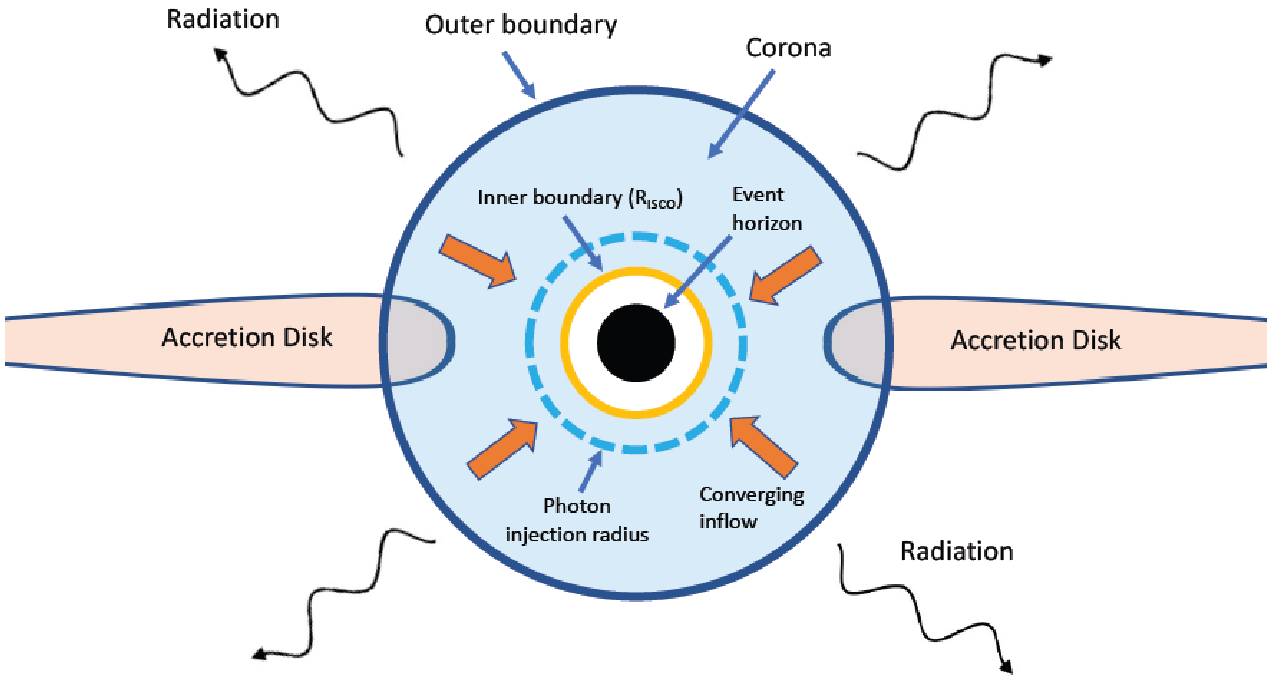


Source: Baughman & Becker (2022)

Previous Model

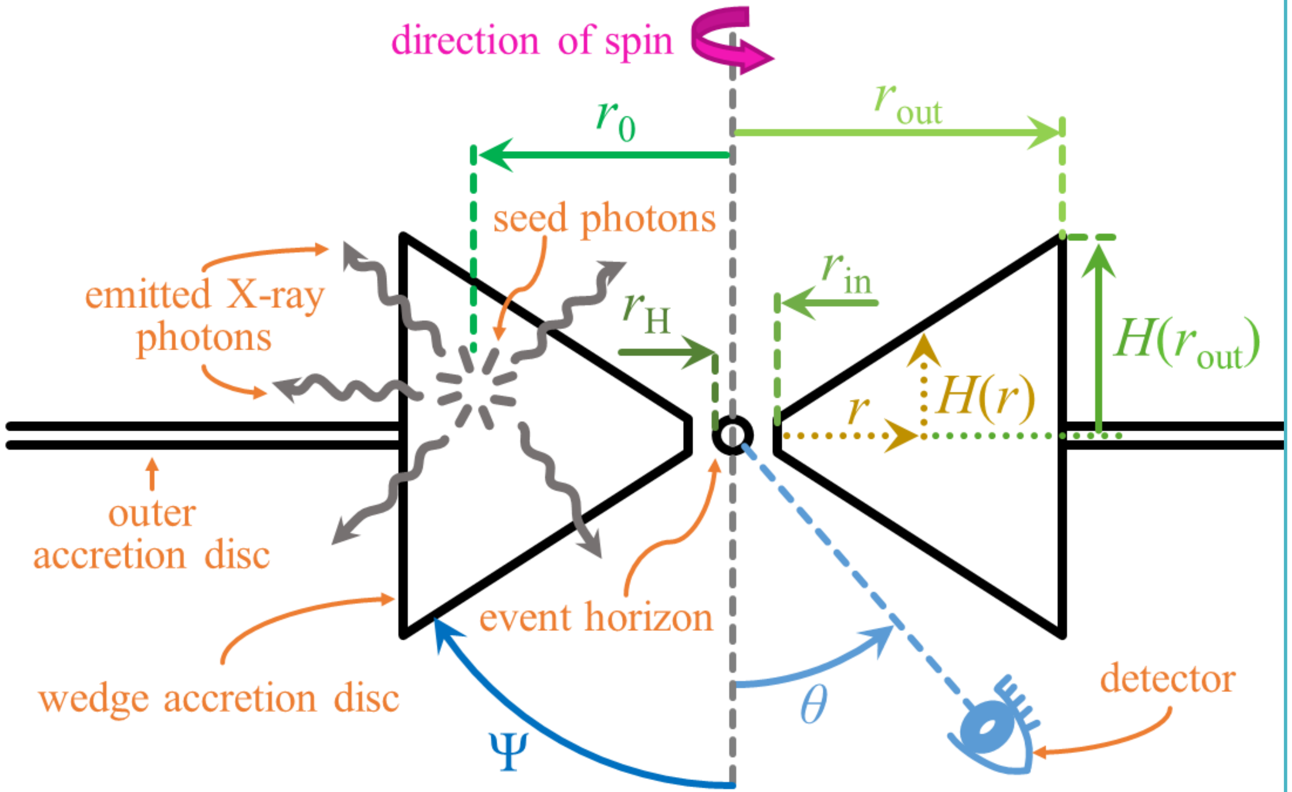


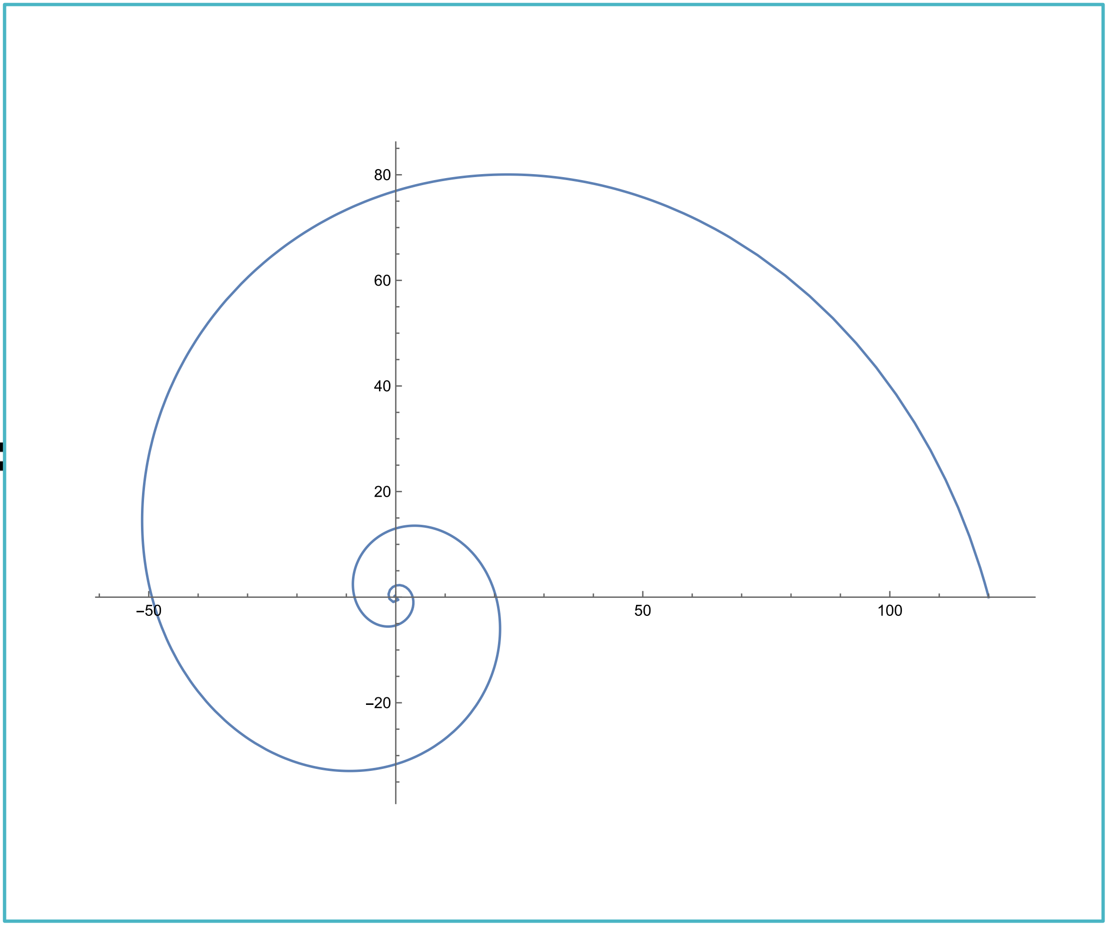
Source: Bambi (2021)



Source: Baughman & Becker (2022)

Model Review: Diagram





Model Derivation

Radiation distribution function $f(\vec{r}, \epsilon, t)$

Vertical average
+ introduce Green's function
+ Fourier transform
+ introduce dimensionless parameters

Fourier-Transform of Green's function $F_{\mathrm{G}}(\tau, \chi, q)$

$$-i\tilde{\omega}\frac{\hat{u}\hat{b}\tilde{r}^{3/2}}{\dot{m}}F_{\mathrm{G}} - \hat{u}\tilde{r}^{-1/2}\frac{\partial F_{\mathrm{G}}}{\partial \tau} = -\frac{\hat{u}^2\hat{b}}{2\dot{m}}\chi\frac{\partial F_{\mathrm{G}}}{\partial \chi} + \frac{2\hat{u}\hat{b}\tilde{r}^{1/2}}{3\dot{m}}\frac{\partial F_{\mathrm{G}}}{\partial \tau} + \frac{1}{3}\frac{\partial^2 F_{\mathrm{G}}}{\partial \tau^2} - \frac{\hat{u}\tilde{r}^{1/2}}{\dot{m}}\mathrm{Min}\left(\frac{\hat{u}\tilde{r}^{1/2}}{\dot{m}}, 1\right)F_{\mathrm{G}} + \frac{\Theta}{\chi^2}\frac{\partial}{\partial \chi}\left[\chi^4\left(F_{\mathrm{G}} + \frac{\partial F_{\mathrm{G}}}{\partial \chi}\right)\right] + \frac{N_0\delta(\tau-\tau_0)\delta(\chi-\chi_0)e^{i\tilde{\omega}q_0}}{4\pi c R_g^2\tilde{r}_0^2\hat{b}\Theta^3(m_e c^2)^3\chi_0^2}$$

Apply separation of variables

$$F_{\mathrm{G}}(\tau, \tau_0, \chi, \chi_0, \tilde{\omega}) = \sum_{n=0}^{\infty} C_n(\tau_0, \chi_0)\,\mathcal{G}_n(\tau, \tilde{\omega})\,\mathcal{H}_n(\chi, \tilde{\omega})$$

- Spatial-dependent equation

$$\frac{d^2\mathcal{G}}{d\tau^2} + \left[2\frac{\hat{u}\hat{b}}{\dot{m}}\tilde{r}^{1/2} + 3\hat{u}\tilde{r}^{-1/2}\right]\frac{d\mathcal{G}}{d\tau} + \left[i\frac{\hat{u}\hat{b}\tilde{\omega}}{\dot{m}}\tilde{r}^{3/2} - \frac{\hat{u}\tilde{r}^{1/2}}{\dot{m}}\mathrm{Min}\left(\frac{\hat{u}\tilde{r}^{1/2}}{\dot{m}}, 1\right) + \frac{\hat{u}^2\hat{b}}{2\dot{m}}\lambda\right]3\mathcal{G} = 0$$

- Energy-dependent equation

$$\frac{1}{\chi^2}\frac{d}{d\chi}\left[\chi^4\left(\mathcal{H} + \frac{d\mathcal{H}}{d\chi}\right)\right] - \frac{\hat{u}^2\hat{b}}{2\dot{m}\Theta}\chi\frac{d\mathcal{H}}{d\chi} - \frac{\hat{u}^2\hat{b}}{2\dot{m}\Theta}\lambda\mathcal{H} = 0$$

Solve for $C_n(\tau_0, \chi_0)$, $\mathcal{G}_n(\tau, \tilde{\omega})$, and $\mathcal{H}_n(\chi, \tilde{\omega})$ to obtain the Green's function F_{G}

Model Derivation

Fourier transform of X-ray light curve is given by

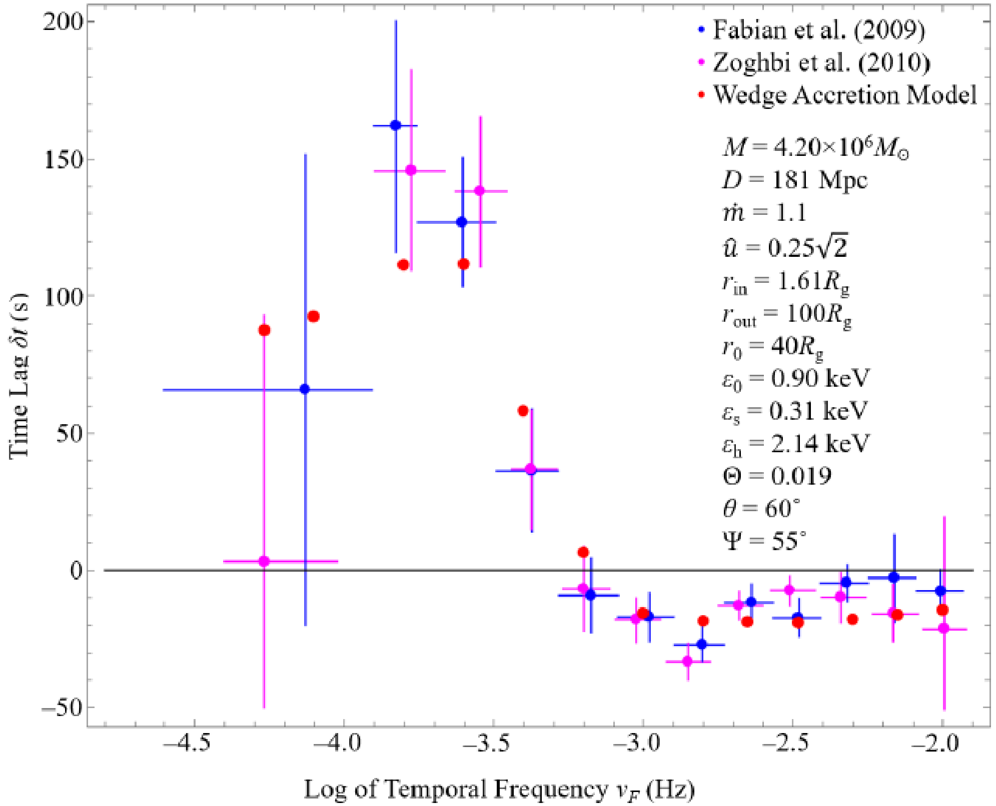
- $\tilde{\mathcal{F}}_{\rm side}(\chi,\tilde{\omega}) = R_g \hat{b} \left(\frac{\chi k_{\rm B} T_e R_g \tilde{r}_{\rm out}}{D} \right)^2 F_{\rm G}(\chi, \tau_{\rm out}, \tilde{\omega})$
- $\tilde{\mathcal{F}}_{\rm face}(\chi,\tilde{\omega}) = \frac{\chi^2 \Theta^2 (m_e c^2)^2 \hat{b} R_g^3}{D^2} \int_0^{\tau_{\rm out}} \frac{\hat{u}\tilde{r}^{5/2}}{\dot{m}} {\rm Min}\left(\frac{\hat{u}\tilde{r}^{1/2}}{\dot{m}}, 1 \right) F_{\rm G}(\chi, \tau, \tilde{\omega})\, d\tau$

$$\tilde{\mathcal{F}}(\chi,\tilde{\omega}) = \tilde{\mathcal{F}}_{\rm side}(\chi,\tilde{\omega}) \sin\theta + \tilde{\mathcal{F}}_{\rm face}(\chi,\tilde{\omega}) \cos\theta \;= \tilde{\mathcal{F}}(\epsilon,\omega)$$

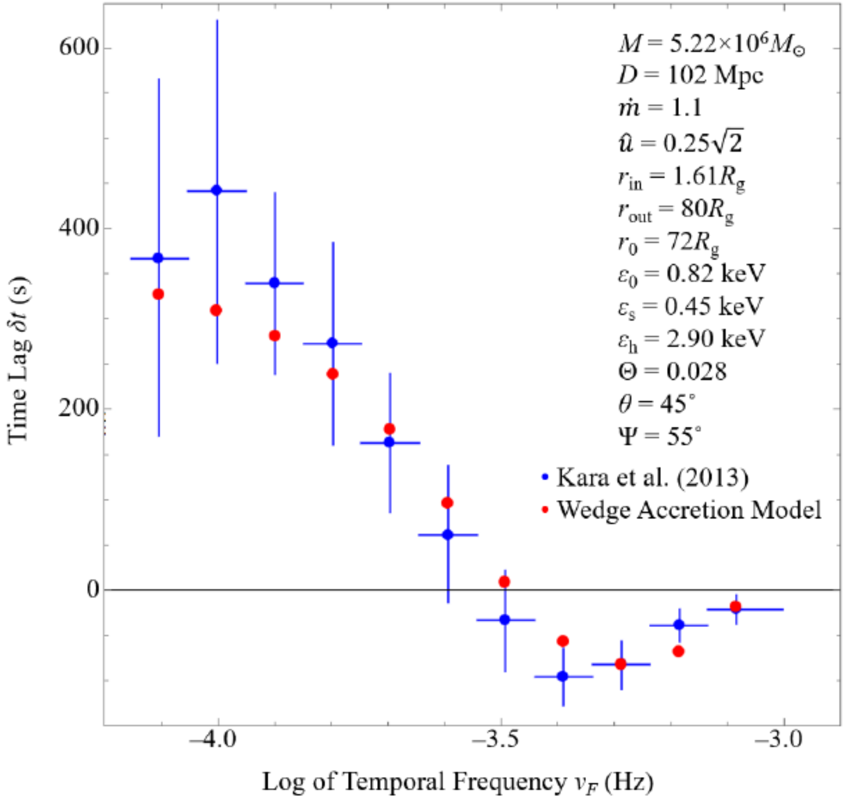
- Phase lag $\phi(\omega) = {\rm Arg}[\mathcal{S}^*(\omega)\mathcal{H}(\omega)]$
- Time lag $\delta t(\omega) = \frac{\phi(\omega)}{\omega}$

Applications

- 1H 0707-495



- Ark 564



Discussion

- Local thermal instability → bremsstrahlung emission flash → Fe L-line and K-line photons
- $k_\mathrm{B}T_e$ = 9.71 keV in 1H 0707-495 and $k_\mathrm{B}T_e$ = 14.31 keV in Ark 564
- $\epsilon \sim$ 0.7 keV for Fe L-line photons and $\epsilon \sim$ 6.4 keV for Fe K-line photons
- Fe L-line photons produce time lags in the observational window ($\sim$ 0.3 – 4 keV)
- Fe K-line photons are outside the observational window but may illuminate the outer cooler disk, creating the reverberation feature noted by Kara et al. (2013)

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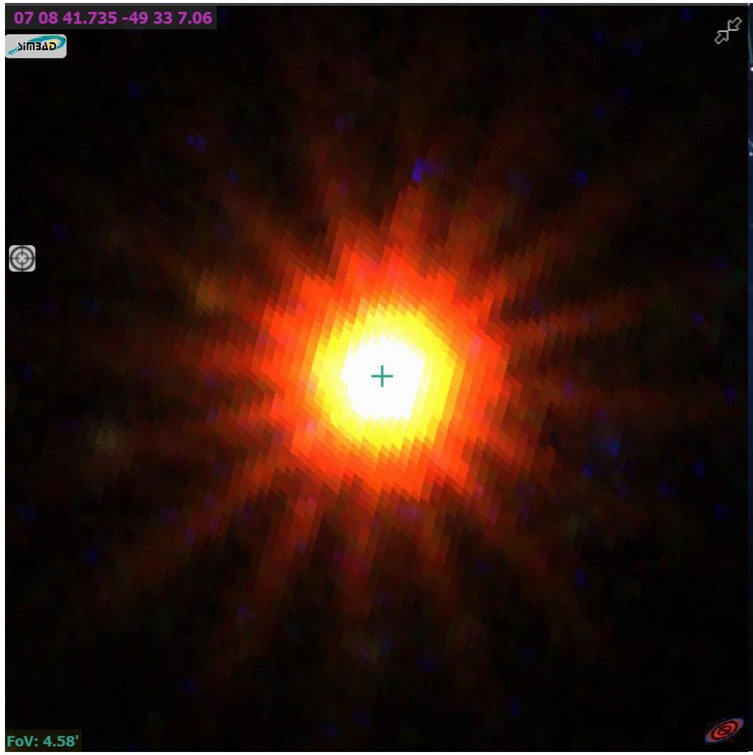
Questions?

THANK YOU

Application to 1H 0707-495



Source: simbad.u-strasbg.fr XMM-Newton Catalog

Source: stellarium-web.org

Also known as PGC 88588

type I Seyfert galaxy

RA: 07h 08m 41.488661470s

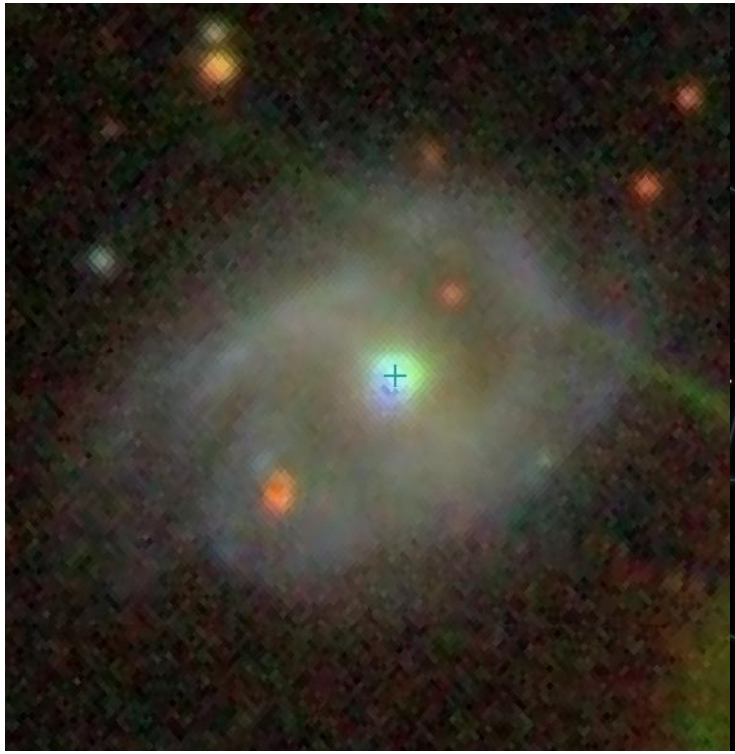
Dec: -49° 33' 06.308921700"

Magnitude: 15.70

The Black hole mass: $2\times10^6 \lesssim M/M_\odot \lesssim 6.45\times10^6$

Distance: 181 Mpc

Application to Ark 564


Source: simbad.u-strasbg.fr XMM-Newton Catalog

Source: stellarium-web.org

Also known as PGC 69553

type I Seyfert galaxy

RA: 22h 42m 39.3363009144s

Dec: +29° 43' 31.302092640 “

Magnitude: 14.16

The Black hole mass: $0.59\times10^6 \lesssim M/M_\odot \lesssim 5.88 \times 10^6$

Distance: 102 Mpc