Galactic supernova remnants

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Contents

Disclaimer

Those are my personal notes, everything here is wrong: almost no references, the style is not academic, and there are tons of typos, but still, it sums up what I will talk about today.

1 Why supernova remnants?

1. They are perfect to study particle acceleration mechanisms. 2. They're cool. 3. In Science, everytime something remains unexplained for long enough, a "supernova" related hypothesis shows up (Epidemic outbreaks? Dinosaurs mass extinction? Cosmic rays up to the knee?). Here, we will mainly focus on the latest, because it is almost impossible to talk about supernova remnants (SNRs) without talking about cosmic rays (CRs).

2 Cosmic rays

CRs are charged particles, mainly protons ($\approx 90\%$), but also alpha particles ($\approx 9\%$) and heavier nuclei, arriving to us from all directions. Because they are charged, they do not point back to their source(s), and therefore, it is hard to answer to natural question: where do they come from?

In the exact words of T. Gaisser (Gaisser, 1990): *"Cosmic rays particles hit the Earth?s atmosphere at the rate of about 1000 per square meter per second. They are ionized nuclei - about 90% protons, 9% alpha particles and the rest heavy nuclei - and they are distinguished by their high energies. Most cosmic rays are relativistic, having energies comparable or somewhat greater than their masses. A very few of them have ultrarelativistic energies extending up to* 10^{20} *eV (about 20 Joules), eleven order of magnitudes greater than the equivalent rest mass energy of a proton. The fundamental question of cosmic ray physics is: Where do they come from? and in particular: How are they accelerated to such high energies?"*

Also, electrons are present in the cosmic radiation $\sim 1\%$.

2.1 Cosmic rays at Earth

Let's look at the *local* CR spectrum in Fig. 1 compiling measurements from many experiments. There are a few remarkable features on the spectrum. First, it is very well fitted by a power–law $\mathrm{d}N/\mathrm{d}t \propto E^{-\Gamma}$ over about 10 orders of magnitude in energy and over about 30 orders of magnitude in flux. But looking closely, several deviations are observed. At about 10^{10} eV, the propagation of CRs is affected by the heliosphere. Just above, $\Gamma = 2.7$, till a characteristic energy of about $\approx 1-4\times10^{15}$ eV, called the *knee*, where the spectrum steepens and $\Gamma = 3.1$. A second weak steepening of the CR spectrum is observed att about $4-8\times10^{17}$eV, called the *second knee*. At $\approx 4\times10^{18}$eV, the *ankle*, where the spectrum recovers the low energy slope until a cut–off at the energy of $\approx 5\times10^{19}$eV. Additional fun facts:

1. the CR energy density is $w_{\rm CR} \approx 1\mathrm{eV/cm3}$. This number is typically of the same order of magnetic field energy density $w_{\rm B}$ or the thermal gas energy density $w_{\rm T}$.
2. from geological measurements, meteorites, lunar rocks, etc., smart people have estimates than the CR flux at Earth was constant over at least $\sim 10^9$yr. Thus the CR flux should be constant long the orbit of the Sun (many revolution in a Gyr).
3. stability in time is a hint for spatial homogeneity.

Let's focus here on the part of spectrum below the *knee*. Since it is such a beautiful power–law, intuitively, the community started looking for one type/one class of sources, and the following questions arose:

1. What sources can produce such a beautiful spectrum at the Earth?
2. How can they do it?

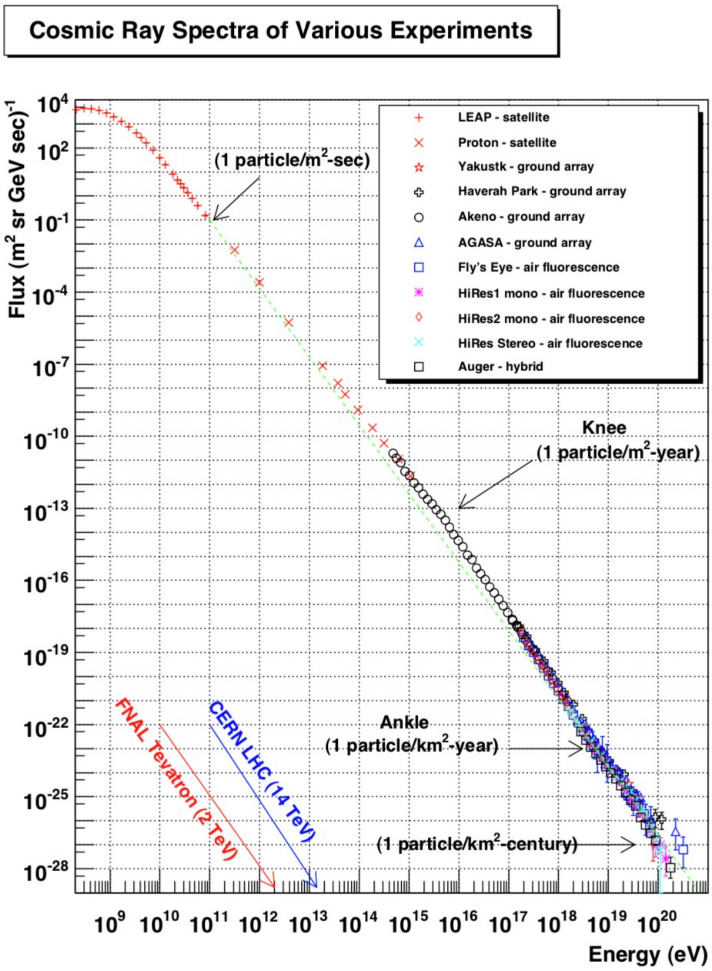

Figure 1: Local cosmic ray spectra of various experiment (Hanlon, 2008)

3. Can they go up to PeV energies?

Just like for the Dinosaur mass extinction, the supernova hypothesis was formulated, suggesting that if a reasonable fraction $\approx 10\%$ of the total explosion energy of supernovae was converted into CRs, the observed level of CRs could be explained. But then a bit later, the community realized that more than supernovae, the supernova remnants expanding in the interstellar medium could do. Because of a mechanism *diffusive shock acceleration* (DSA), and the refined modern versions called *non–linear diffusive shock acceleration* (NLDSA), the spectrum of accelerated particles could be compatible with measurements, and the PeV energies of CRs also, thanks to a *magnetic field amplification* allowed in NLDSA.

Why do we stop at the *knee*? Well, first, this is a Fermi school, so I thought it would make sense to talk about Fermi, and the range covered by Fermi. In the 1950's, good people had already realized that the proton CRs should undergo hadronic interactions with protons of the Galactic disk, and that the Galaxy should shine in gamma rays! In the MeV and up to the multi–GeV domain, see Fig. 2,

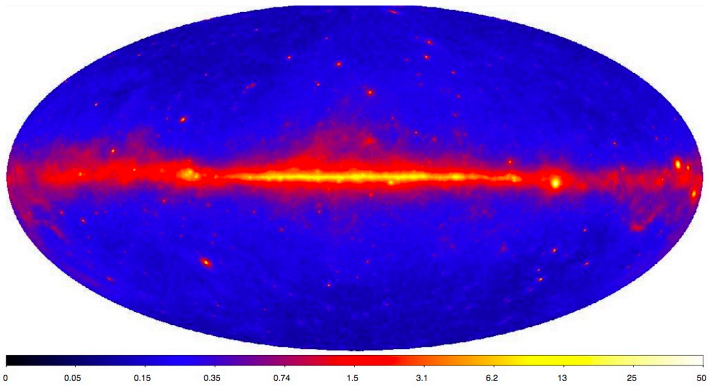

Figure 2: Sky map of the energy flux derived from 24 months of observations. The image shows γ–ray energy flux for energies between 100 MeV and 10 GeV n units of 10^7 erg cm^{-1} s^{-1} sr^{-1} (Morselli, 2011)

showing how the CRs fill the whole Galactic disk.

$$\begin{aligned} & p + p \to p + p + \pi^0 \\ & \pi^0 \to \gamma + \gamma \end{aligned} \tag{1}$$

From the Fig. 2 picture, it seems that CRs are homogeneously distributed in the Galactic disk, but could it be that they are homogeneously distributed in the entire Universe and not only the Galaxy? A simple argument can be made just looking at the Large Magellanic Cloud, located outside the disk. From this cloud, less gamma rays than expected are observed, therefore giving an argument in favor of CRs from the Galaxy. It is often stated than in fact, CRs below the knee correspond to the Galactic origin, and above the knee, extra–Galactic origin, but is this a clear cut? Let's estimate very simply the confinement of CRs, and consider that the CRs are confined when their Larmor radius $R_{\rm L}$ is smaller than the Galactic halo thickness h.

$$\begin{aligned} & \tfrac{E(\mathrm{eV})}{300\, B(\mathrm{G})} = R_{\rm L} \lesssim h \\ & E \lesssim 10^{18} \left(\tfrac{h}{\mathrm{kpc}}\right) \left(\tfrac{B}{\mu\mathrm{G}}\right) eV = 10^{17} - 10^{20} eV \end{aligned} \tag{2}$$

In other words, the confinement depends on the magnetic field and the halo thickness, both poorly constrained, so that the transition between Galactic and extra–Galactic is not *that* obvious.

2.2 Power from CR sources

Let's call $\mathcal{E}_{\rm CR}$ the total CR energy in the Galactic disk. We'll call $V_{\rm disk}$ the volume of the disk, and we have $\mathcal{E}_{\rm CR} = w_{\rm CR} \times V_{\rm disk}$, assuming a spatial homogeneity. If $P_{\rm CR}$ is the total CR power from CR sources, then we have:

$$\frac{\mathrm{d}\mathcal{E}_{\rm CR}}{\mathrm{d}t} = P_{\rm CR} - \frac{\mathcal{E}_{\rm CR}}{t_{\rm disk}} \tag{3}$$

where $t_{\rm disk}$ is the typical confinement time of CRs in the Galactic disk.

How can we estimate t_{disk}? Well, there is one method, using *spallation* measurements. In the solar system, the abundance of light elements (Li, B, Be) is low (several orders of magnitude lower) compared to other elements (e.g. C,N,O), due to the fact that they are not produced in stellar nucleosynthesis, but only the primordial nucleosynthesis. But in the CRs, the abundance of these light elements is typically of the same order than the level observed for most elements. The abundance of light elements in the CRs can be explain by spallation: interactions of high–energy particle interact with matter and produce the light elements. To put numbers, at $\sim$ 1GeV, the anomaly is explained if CRs go through $\lambda \approx 5$ g/cm^2. Assuming propagation in the Galactic disk, the typical diffusion length is $l_{\text{s}} = \frac{\lambda}{\rho_{\text{ISM}}} \approx 1$ Mpc, leading to an estimated $t_{\text{disk}} = l_{\text{s}}/c \approx 3 \times 10^6$yr. By the way, in comparison, the typical confinement time in the halo (also for GeV particles) is $t_{\text{halo}} \approx 10 \times 10^6$yr.

We've talked about stability in time, in this case, in Eq. 3, the term $\frac{\mathrm{d}\mathcal{E}_{\text{CR}}}{\mathrm{d}t}$ is null. Plugging numbers, we find $P_{\text{CR}} = \frac{\mathcal{E}_{\text{CR}}}{t_{\text{disk}}} \approx 10^{41}$erg/s.

Remark (Energy loss term due to p–p interactions)**.** *Should we take it into account and add a term "$-\dot{\mathcal{E}}_{pp}$" in Eq. 3? Let's look at the energy loss rate for p–p interactions.* $t_{pp} = (n_{\text{gas}}\sigma_{pp}ck)^{-1}$. *With* $\sigma_{pp} = 4 \times 10^{-26}$ cm^{-2} *and* $k = 0.45$, *we get* $t_{pp} \approx 60\frac{n_{\text{gas}}}{cm^{-3}}Myr >> t_{\text{disk}}$. *This means we can safely neglect CR energy losses.*

A supernova (SN) releases typically $\approx 10^{51}$erg in the form of kinetic energy. In the Galaxy, the observed SN rate is about 1/30yr^{-1}, meaning the total Galactic SN power is $P_{\text{SN}} = 10^{42}$erg/s. If SN could maintain the CR population provided that 10% of their kinetic energy is somehow converted in CRs, then bingo.

2.3 Diffusion of CRs and consequences

CRs follow tortuous path before escaping the Galaxy. Let's take a look at a very simplified picture, and consider small cloudlets with a magnetic field B inside, and B outside. The particle energy from cloudlet to cloudlet is unchanged (Lorentz force). Let's define a few quantities.

$$\begin{aligned} &\lambda \equiv \text{mean free path} \\ &\tau_{\text{c}} = \tfrac{\lambda}{c} \equiv \text{collision time} \\ &N = \tfrac{t}{\tau_{\text{c}}} \equiv \text{number of collision after time t} \end{aligned} \tag{4}$$

The diffusion length l_{d} is:

$$l_{\text{d}} = \lambda\sqrt{N} = \lambda\sqrt{\frac{t}{\tau_{\text{c}}}} = \lambda\sqrt{\frac{tc}{\lambda}} = \sqrt{\lambda ct} = \sqrt{Dt} \tag{5}$$

with D the diffusion coefficient. For a straight line propagation, we would have had $l_{\text{straightline}} = ct \propto t$, while $l_{\text{d}} \propto \sqrt{t}$. For the Galactic disk, we can then estimate $D = \frac{l^2_{\text{disk}}}{t_{\text{disk}}}$. At 1 GeV, a typical value is $D \approx 10^{28}$ cm^2 s^{-1}.

Spallation measurements at different energies showed $t \propto E^{-0.3}$, which corresponds to $D \propto E^{0.3}$.

And this has consequences on the requirement for the CR sources! The quantity of CRs N_{CR} in the Galaxy is described by:

$$\frac{\mathrm{d}N_{\text{CR}}(E)}{\mathrm{d}t} = Q_{\text{CR}}(E) - \frac{N_{\text{CR}}(E)}{t_{\text{disk}}} \tag{6}$$

with Q_{CR} the injection from the sources, and $-\frac{N_{\text{CR}}(E)}{t_{\text{disk}}}$ the escape rate from the disk. Assuming stability in time, we get $Q_{\text{CR}}(E) = \frac{N_{\text{CR}}(E)}{t_{\text{disk}}} \propto N_{\text{CR}}(E)D(E) \propto E^{-2.4}$.

$\propto \mathbf{E^{-2.4}}$ let's keep this in mind, this what we want the sources to provide!

2.4 Gamma rays from SNRs

We've talked about energy/power from sources, the required spectra, but what about the fact that we see gamma rays from SNRs, isn't this enough for direct evidence that SNR are sources of CRs? And this answer is: it's not that clear. As we've learned gamma rays can be produce by protons or electrons. Accelerated protons can interact with protons of the ISM as shown in Eq. 1. Accelerated electrons can produce gamma rays via inverse Compton scattering on soft photons or Bremsstrahlung.

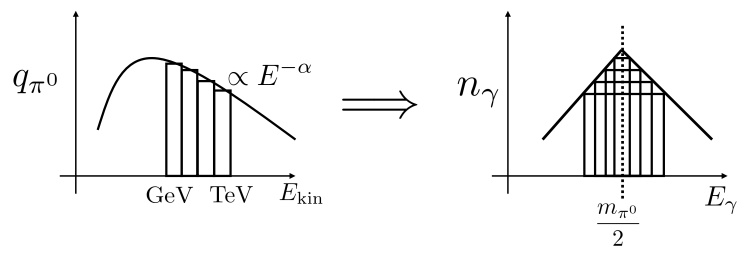

Figure 3: Shape of the gamma–ray spectrum from the pions spectrum.

2.5 Hadronic interactions

From two protons, we form two protons and one π^0. So we can intuitively guess that their is a threshold energy $E_{\rm threshold}$ for this production, we need enough energy to form the mass of the π^0. After the collision, we want at least two protons at rest and one π^0. We use the conservation of the interval $E^2 - p^2$, after and before the collision. $m^2_{\rm after} = (2m_pc^2 + m_{\pi^0}c^2)^2$ Before collision is $m^2_{\rm before} = (E_p + m_pc^2)^2 - (p_pc)^2$ Imposing that $m^2_{\rm before}$ is at least $m^2_{\rm after}$, one gets:

$$E_p - m_pc^2 \geq 2m_{\pi^0}c^2 + \left(\frac{m_{\pi^0}}{2m_p}\right) m_{\pi^0}c^2 \approx 280\text{MeV} \tag{7}$$

It is possible to derive the expected shape of the spectrum in γ rays, starting from the proton distribution.

Let's assume a power–law for the protons: $N_{\rm p} \propto E_p^{-\alpha}$. The rate of production of the π^0 is given by:

$$Q_{\pi^0} = \int {\rm d}E_p N_{\rm p}(E_p)\delta(E_{\pi^0} - f_{\pi^0}E_{p,{\rm kin}})\sigma_{pp}(E_p)n_{\rm gas}c \tag{8}$$

with f_{π^0} the fraction of proton kinetic energy transferred to π^0, obtained from data $f_{\pi^0} \approx 0.17$. σ_{pp} is constant for energies greater than $\sim$GeV, and cutting–off towards low energy at $E_{\rm threshold}$. From the GeV range on, Q_{π^0} as a function of $E_{\rm kin}$ is therefore a power–law of index $-\alpha$.

We must now derive the γ spectrum. Since the photon spectrum is the result of a one body decay, the photon spectrum must exhibit a feature at an energy related to the pion mass. In the π^0 rest frame, the two γ emitted get half of the π^0 mass. In the lab rest frame, Lorentz transform give:

$$\begin{aligned} E^\star_\gamma &= \tfrac{m_{\pi^0}}{2} \rightarrow \pi^0\text{rest frame} \\ E_\gamma &= \gamma(E^\star_\gamma + vp^\star_\gamma cos\theta^\star) \rightarrow \text{lab rest frame} \end{aligned} \tag{9}$$

From Eq. 9, the minimum and maximum photon energy are found:

$$E^{\rm min}_\gamma = \frac{m_{\pi^0}}{2}\sqrt{\frac{1-\beta}{1+\beta}} \leq E_\gamma \leq \frac{m_{\pi^0}}{2}\sqrt{\frac{1+\beta}{1-\beta}} = E^{\rm max}_\gamma \tag{10}$$

Let's plot the gamma–ray spectrum, noticing two things:

1. In log-scale, the centre of the interval defined by Eq. 10 is half of the π^0 mass. $\frac{\log E^{\rm min}_\gamma + \log E^{\rm max}_\gamma}{2} = \log(\frac{\pi^0}{2})$
2. In the π^0 rest frame, the photon distribution is isotropic : $\frac{{\rm d}n_\gamma}{{\rm d}\Omega^\star} = \frac{1}{4\pi}$. Since ${\rm d}\Omega^\star \propto {\rm d}(cos\theta^\star)$, we have ${\rm d}E_\gamma \propto {\rm d}(cos\theta^\star)$ and $\frac{{\rm d}n_\gamma}{{\rm d}E_\gamma} = $ constant.

See Fig. 3.

Looking at Fig. 3, few remarks:

1. the gamma ray spectrum is symmetric (in log–log) with respect to $\frac{m_{\pi^0}}{2} \approx 70$MeV.

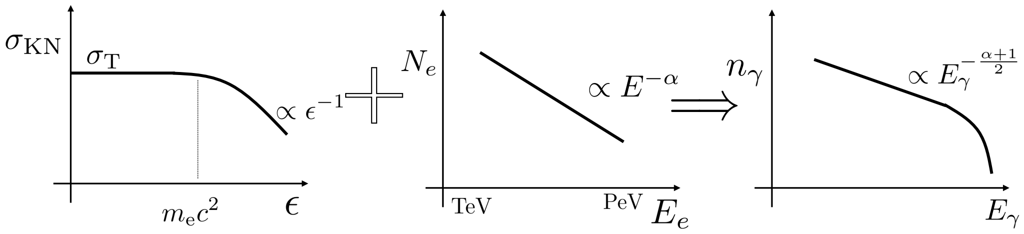

Figure 4: Gamma ray spectrum from Inverse Compton scattering of electrons on soft photons.

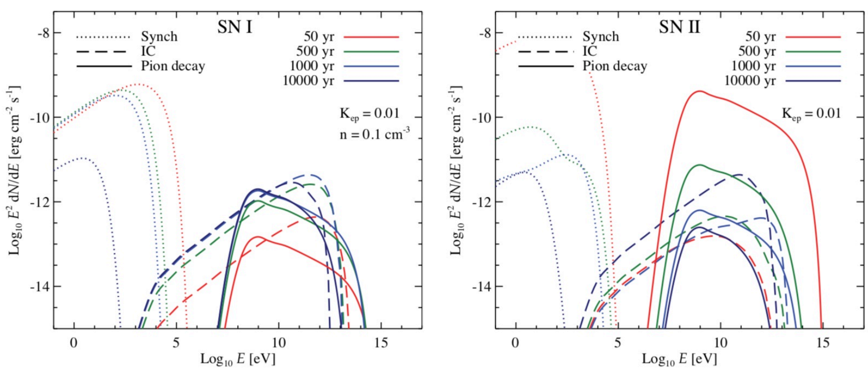

Figure 5: Gamma rays from SNRs taken from Gaggero et al. (2018)

2. At high energy, the spectrum mimics the CR spectrum with roughly $E_\gamma = \frac{E_{\rm CR}}{10}$
3. the CR and γ power–law are the same

Remark (Why are the CR and γ power–law are the same?)**.** $E_\gamma^{\rm max} = \frac{m_{\pi^0}}{2}\sqrt{\frac{1+\beta}{1-\beta}} = \frac{m_{\pi^0}}{2}(1+\beta)\gamma \overset{\gamma\to\infty}{\longrightarrow} m_{\pi^0}\gamma$ $E_\gamma^{\rm min} = \frac{m_{\pi^0}}{2}\sqrt{\frac{1-\beta}{1+\beta}} = \frac{m_{\pi^0}}{2}[(1+\beta)\gamma]^{-1} \overset{\gamma\to\infty}{\longrightarrow} \frac{m_{\pi^0}}{4}\gamma^{-1}$ *To produce a photon of energy E_γ, a proton of energy $E_p \gtrsim \frac{m_p}{m_{\pi^0}}E_\gamma$ is needed.*

$$N_\gamma(E_\gamma) \propto \int_{\frac{m_p}{m_{\pi^0}}E_\gamma}^{+\infty} dE_p \frac{N_p(E_p)}{E_\gamma^{\rm max} - E_\gamma^{\rm min}} \propto \int_{\frac{m_p}{m_{\pi^0}}E_\gamma}^{+\infty} dE_p \frac{E_p^{-\alpha}}{E_p} \propto E_\gamma^{-\alpha} \tag{11}$$

On Eq. 11, we see that we get the same slope for CRs and gammas!

2.6 Leptonic interactions

Relativistic electrons can interact with soft background photons (such as CMB, IR, and optical background) to produce gamma rays. For an incident photon of energy $\epsilon_{\rm i}$, in the electron rest frame the photon energy is:

$$\epsilon'_{\rm i} = \epsilon_{\rm i}\gamma(1-\beta\cos\theta) \tag{12}$$

with $\beta = v/c$. In the electron rest frame (Thomson scattering), $\epsilon'_{\rm f} = \epsilon'_{\rm i}$

As an illustration, a plot taken from Gaggero et al. (2018) computation of gamma rays from SNRs Type Ia and Type II as different ages. The bremsstrahlung of electrons is not shown as it is dominated by inverse Compton scattering.

3 Supernova remnants

3.1 Supernovae

Lists of sources can be boring. But still. If we take a look at known Galactic SN, we realize we don't know that much of them. Why? Of course, a simple reason is that in visible light, we do not see them for long, making them harder to catch. In gamma rays, we can easily stay bright for $10^4 - 10^5$ yr.

3.2 Shocks

So far, we have several good arguments in favor of SNRs being the sources of Galactic CRs: the power input from supernovae, the fact that we detect SNRs in gamma rays with gamma–ray spectral indices not too far from the CR spectral indices. But one of the strongest support for the SNR hypothesis, is a mechanism: the famous *diffusive shock acceleration* (DSA), or the modern refined version *non–linear diffusive shock acceleration* (NLDSA).

Let's just try to understand the basics of those, by simply understanding how shocks form.

Small perturbations in a a fluid can form sound waves. When studying a fluid element, we remember from high–school that we can follow a fluid element along a field line (Lagrangian description, see Eq. 13 with changes in time and space taken into account), or look at a fixed position and see what comes in and goes out (Eulerian description).

$$\frac{\mathrm{d}}{\mathrm{d}t} \longrightarrow \frac{\partial}{\partial t} + \vec{u} \cdot \vec{\nabla} \tag{13}$$

Let's consider whatever random fluid. For this random fluid, the mass and momentum conversation read:

$$\begin{array}{l} \frac{\partial \rho}{\partial t} + \nabla(\rho \vec{v}) = 0 \\ \rho \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\vec{\nabla} P \longrightarrow \frac{\partial v}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} = -\frac{\vec{\nabla} P}{\rho} \end{array} \tag{14}$$

This boring fluid has, when in equilibrium $\vec{v} = \vec{0}$, $\rho = \rho_0$ and $P = P_0$. Let's perturbe it $\vec{\delta v}$, δP, $\delta\rho$ and rewrite Eq. 14.

$$\begin{array}{l} \frac{\partial \delta\rho}{\partial t} + \rho_0 \nabla \cdot \vec{\delta v} = -\nabla \cdot \left(\delta\rho \vec{\delta v}\right) \\ \frac{\partial \vec{\delta v}}{\partial t} = -\vec{\delta v} \cdot \vec{\nabla} \vec{\delta v} - \frac{1}{\rho_0 + \delta\rho} \nabla \delta P \end{array} \tag{15}$$

We can simplify Eq. 15, getting rid of second order δ's and considering $\delta\rho << \rho$, getting:

$$\begin{array}{l} \frac{\partial \delta\rho}{\partial t} + \rho_0 \nabla \cdot \vec{\delta v} = 0 \\ \rho_0 \frac{\partial \vec{\delta v}}{\partial t} = -\nabla \delta P \end{array} \tag{16}$$

Let's do the classic trick and derive $\frac{\partial}{\partial t}$ the mass equation and $\nabla \cdot$ the momentum equation.

$$\begin{array}{l} \frac{\partial^2 \delta\rho}{\partial t^2} + \rho_0 \frac{\partial \nabla \cdot \vec{\delta v}}{\partial t} = 0 \\ \rho_0 \nabla \cdot \frac{\partial \vec{\delta v}}{\partial t} = -\nabla^2 \delta P \end{array} \tag{17}$$

Subtracting the mass and the momentum equation, we obtain:

$$\frac{\partial^2 \delta\rho}{\partial t^2} - \nabla^2 \delta P = 0 \tag{18}$$

Hum, this almost look like a wave equation, if only we could connect P and ρ.. Wait, the adiabatic equation of state gives:

$$\begin{array}{l} P \propto \rho^\gamma \\ \delta P = \left(\frac{\partial P}{\partial \rho}\right)_{\mathrm{s}} \delta\rho = \frac{\gamma P}{\rho} \delta\rho = c_{\mathrm{s}}^2 \delta\rho \end{array} \tag{19}$$

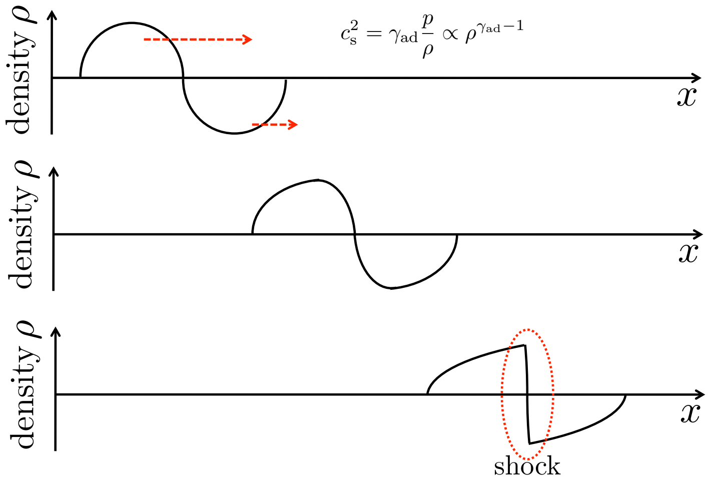

Figure 6: Formation of a shock waves.

The tiny s means "for constant entropy". For a monoatomic gas $\gamma = 5/3$. For small perturbation, the sound speed is $c_s = c_{s,0} = \frac{\gamma P_0}{\rho_0}$. Tadam, Eq. 20 becomes:

$$\frac{\partial^2 \delta\rho}{\partial t^2} - c_s^2 \nabla^2 \delta\rho = 0 \tag{20}$$

We know how to solve this. Typical solution is:

$$\begin{aligned} &\frac{\delta\rho}{\rho_0} = A \ \exp(i(\vec{k}\cdot\vec{x}) \pm wt) \\ &A << 1 \quad \text{and} \quad w^2 = k^2 c_s^2 \end{aligned} \tag{21}$$

Now we are going to do something not fair - but still. We are going to relax the assumption of small perturbations, and consider finite amplitude perturbations. Let's plot ρ as a function of x, direction of propagation of the wave. The propagation velocity is a function of the density $c_s \propto \rho^{\gamma-1}$. This will create a steeping of the wave, and a formation of a discontinuity: this is the formation of a shock wave, as shown on Fig. 6.

3.3 Strong shocks

Let's write the jump condition at the shock discontinuity for a strong shock. We assign the index 1 and 2 to the regions upstream and downstream of the discontinuity respectively, and work in the shock rest frame. The mass, momentum and energy conservation have to be respected:

$$\begin{aligned} &\rho_1 u_1 = \rho_2 u_2 \\ &\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2 \\ &\frac{1}{2}\rho_1 u_1^3 + \rho_1 u_1 \left(\epsilon_1 + \frac{P_1}{\rho_1}\right) = \frac{1}{2}\rho_2 u_2^3 + \rho_2 u_2 \left(\epsilon_2 + \frac{P_2}{\rho_2}\right) \end{aligned} \tag{22}$$

with ϵ the internal energy per unit mass and $\epsilon + \frac{P}{\rho}$ the specific enthalpy. Combining the mass and momentum equations, we get:

$$1 + \frac{P_1}{\rho_1 u_1^2} = \frac{u_2}{u_1} + \frac{P_2}{\rho_1 u_1^2} \tag{23}$$

It's a good time to introduce the Mach number $\mathcal{M} = \frac{u_1}{c_s}$

$$\mathcal{M} \gg 1 \longrightarrow \frac{\rho_1 u_1^2}{\gamma P_1} \gg 1 \tag{24}$$

It is usual to make the *cold medium* assumption. $P_1 \approx 0$, $\rho_1 \approx 0$ and $\epsilon_1 \approx 0$. We introduce the compression factor $r = \frac{\rho_2}{\rho_1}$. We also remember (I never remember it, but people who pay attention in

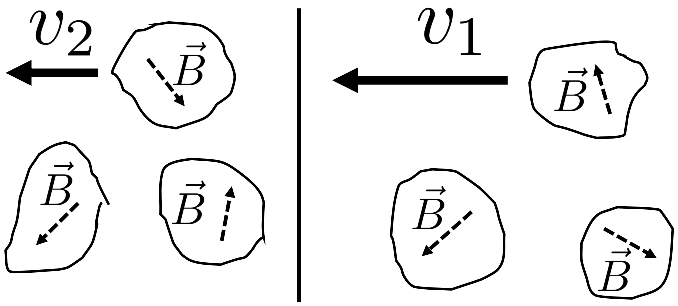

Figure 7: Clouds upstream (1) and downstream (2) of the shock in the shock rest frame.

thermodynamics 101 might remember) : $P_2 = (\gamma - 1)\epsilon_2\rho_2$. With this, Eq. 23 leads to:

$$\begin{aligned} &\tfrac{P_2}{\rho_1 u_1^2} = 1 - \tfrac{1}{r} \\ &\tfrac{1}{2}\rho_1 u_1^3 = \tfrac{1}{2}\rho_2 u_2^3 + \rho_2 u_2 \left(\epsilon_2 + \tfrac{P_2}{\rho_2}\right) \to \rho_1 u_1^3 = \rho_2 u_2^3 + \tfrac{2\gamma}{\gamma-1} P_2 \rho_2 \\ &\longrightarrow \tfrac{u_1^2}{u_2^2} = 1 + \tfrac{2\gamma}{\gamma-1}\left(\tfrac{P_2}{\rho_1 u_1^2}\right)\tfrac{u_1}{u_2} \longrightarrow \boxed{r^2 - 1 = \frac{2\gamma}{\gamma - 1}(r-1)} \end{aligned} \tag{25}$$

To this equation, two solutions : $r = 1$ (boring and unphysical) $r = \frac{2\gamma}{\gamma-1} - 1 = \frac{\gamma+1}{\gamma-1} \overset{\gamma=5/3}{\longrightarrow} 4$.

Remark (Name dropping)**.** *These jump conditions are also sometimes known as the Rankine - Hugoniot conditions (a Scottish and a french, guess who is who).*

Few comments on this "4" and in general, on strong shocks. A strong shocks is something that:

1. compresses moderately the gas. $r = \frac{\rho_2}{\rho_1} \overset{\gamma=5/3}{\longrightarrow} 4$.

2. makes the downstream gas subsonic: $\mathcal{M}_2^2 = \frac{u_2^2}{c_{s,2}^2} = \frac{\rho_2 u_2^2}{\gamma P_2} = \frac{\gamma-1}{2\gamma} \overset{\gamma=5/3}{\longrightarrow} \frac{1}{5} < 1$

3. heats significantly the gas: $P_2 = \frac{2}{\gamma+1}\rho_1 u_1^2 = \frac{\rho_2}{m_p} kT_2$. In other words, if we want to estimate the thermal energy $kT_2 = \frac{2(\gamma-1)}{(\gamma+1)^2} m_p u_1^2 \overset{\gamma=5/3}{\longrightarrow} \frac{3}{16} m_p u_1^2 \sim \left(\frac{u_1}{1000\mathrm{km/s}}.\right)^2$ keV. For a typical value of SNR shock speed, this is a HOT plasma!

At this point, we could calculate the typical Thermal Bremsstrahlung and (quite) easily realize that this goes up to the keV, multi–keV range, but not much more, and therefore not interesting for Fermi.

3.4 Particle acceleration at strong shocks: diffusive shock acceleration

A magnetized plasma is a good environment for non–thermal particles. Of course, you can have thermal particles and it can be difficult to discriminate between thermal and non–thermal, but let's not get into this.

Let's consider a medium made of cloudlets magnetized in random directions. We look at the two regions *downstream* (shocked) and *upstream* (unshocked) of the shock, in the shock rest frame, and we consider only one particle of mass $m \ll M_{\text{could}}$ and energy E. It's all going to be a game of going from one rest frame to another.

1. In the shock rest frame, the upstream cloudlets are coming at v_1, and the downstream cloudlets are going away at v_2, as illustrated in Fig. 7.

2. In the upstream rest frame, the upstream cloudlets are at rest, and the shock is coming at v_1 (with a minus sign if we pay attention to sign right). The cloudlets of downstream are also coming but less fast, they come at $\boxed{v_1 - v_2}$.

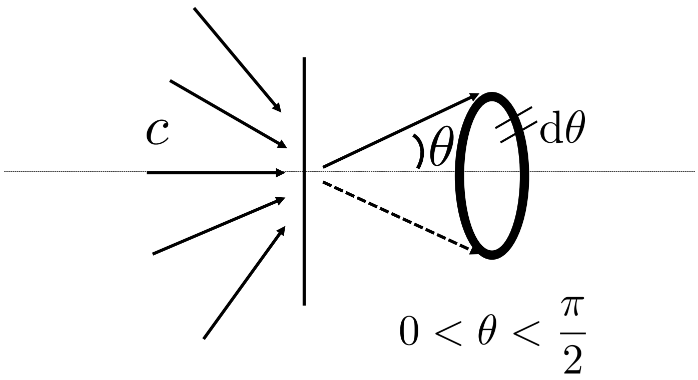

Figure 8: Particles arriving on plane shock.

3. In the downstream rest frame, the downstream cloudlets are rest. The shock is going away at v_2 and the upstream is coming at $\boxed{v_1 - v_2}$.

This was the trickiest part. We can know formulate an argument of symmetry: for a particle downstream, the upstream is coming at $v_1 - v_2$. For a particle upstream, the downstream is coming at exactly the same speed! In this, there is *symmetry*. On the other hand, in this infinite plane shock problem, there is also some *asymmetry*. Upstream particles always come back to the shock, while downstream particles may be advected and not return. Let's call E, p the initial energy and momentum of the particle upstream. This particle sees the downstream fluid at $v = v_1 - v_2$, and a Lorentz factor γ_v. In the downstream rest frame, this particle has an energy:

$$E' = \gamma_v (E + p\cos(\theta) v) \tag{26}$$

Assuming the shock is non–relativistic, we get $\gamma_v = 1$, assuming the particle is relativistic $E = pc$. Eq. 26 becomes:

$$E' = E + \frac{E}{c} v\cos(\theta) \longrightarrow \boxed{\frac{\Delta E}{E} = \frac{v}{c}\cos(\theta)} \tag{27}$$

This represents the gain going from upstream to downstream (half a cycle). What happens next? The particles are rapidly isotropized by magnetic field irregularities. We work under the assumption than particles up and downstream of the shock are rapidly isotropized by magnetic field irregularities. Particles arrive on the plane shock at c, with angle θ to normal, as in Fig. 8. The number of particles between θ and $\theta + \mathrm{d}\theta$ is $\propto \sin(\theta)\mathrm{d}\theta$, and the rate at which particles cross the shock is $\propto c\cos(\theta)$. Therefore, the probability for a particle to cross the shock is $p(\theta) \ \propto \sin(\theta)\cos(\theta)\ \mathrm{d}\theta$. In fact, taking θ between 0 and $\pi/2$, the probability is:

$$p(\theta) \ = 2\sin(\theta)\cos(\theta)\ \mathrm{d}\theta \tag{28}$$

Remark (probability of crossing the shock)**.** *To obtain the result, integrate and change variable $t = sin(\theta)$*

Combining Eq. 27 and Eq. 28, we can express the average gain per half–cycle:

$$< \frac{\Delta E}{E} >= 2\left(\frac{v}{c}\right)\int_0^{\frac{\pi}{2}} \mathrm{d}\theta\cos^2(\theta)\sin(\theta) \tag{29}$$

Calculating the integral (changing variable using $\cos(\theta) = t$), we obtain:

$$< \frac{\Delta E}{E} >= \frac{2}{3}\left(\frac{v}{c}\right) \tag{30}$$

Going from upstream to downstream is equivalent to going from downstream to upstream, thus, for a full cycle:

$$< \frac{\Delta E}{E} >= \frac{4}{3}\left(\frac{u_1 - u_2}{c}\right) \tag{31}$$

This is a typical *first order* Fermi mechanism, because first order in $\frac{v}{c}$. So, what happens after n cycles of acceleration?

$$\begin{aligned} \frac{E_{\rm i+1}-E_{\rm i}}{E_{\rm i}} &= \frac{<E>}{E} = \frac{4}{3}\frac{v}{c} \\ E_{\rm i+1} &= \left(1+\frac{4}{3}\frac{v}{c}\right)E_{\rm i} \\ E_{\rm i+1} &= \beta E_{\rm i} \quad \text{with} \quad \boxed{\beta = 1+\frac{4}{3}\frac{u_1-u_2}{c} = 1+\frac{u_1}{c}} \end{aligned} \tag{32}$$

So if P is the probability for a particle to remain within the accelerator after each cycle, after n cycle, there are $N = N_0 P^{\rm n}$ particles with energy above $E = E_0\beta^{\rm n}$. Taking the log, we obtain the integral spectrum:

$$N(>E) = N_0\left(\frac{E}{E_0}\right)^{\frac{\log P}{\log\beta}} \tag{33}$$

and therefore a differential spectrum:

$$n(E) \propto E^{-1+\frac{\log P}{\log\beta}} \tag{34}$$

Now we need P the probability that the particle remains within the accelerator after each cycle. Let's estimate $1-P$, the probability that the particle leaves. If we call $R_{\rm in}$ the rate of particle coming in the system and $R_{\rm out}$ the rate of particles leaving the system: $\frac{R_{\rm out}}{R_{\rm in}} = 1-P$. n being the density of particles accelerated close to the shock, and supposed to be isotropic: $\mathrm{d}n = \frac{n}{4\pi}\mathrm{d}\Omega$, and the velocity of the particles crossing the shock is $c\cos(\theta)$.

$$R_{\rm in} = \int_{\rm up\to down} \mathrm{d}n c\cos(\theta) = \frac{nc}{4\pi}\int_0^{\frac{\pi}{2}}\cos(\theta)\sin(\theta)\mathrm{d}\theta\int_0^{2\pi}\mathrm{d}\psi = \frac{1}{4}nc \tag{35}$$

and $R_{\rm out} = nu_2$. In other words:

$$1-P = \frac{R_{\rm out}}{R_{\rm in}} = \frac{nu_2}{\frac{1}{4}nc} = \frac{u_1}{c} \ll 1 \quad \text{thus} \quad \boxed{P = 1-\frac{u_1}{c}} \tag{36}$$

$\log P = \log\left(1-\frac{u_1}{c}\right) \sim -\frac{u_1}{c}$ and $\log\beta = \log\left(1+\frac{u_1}{c}\right) \sim \frac{u_1}{c}$, so that the differential spectrum of Eq. 34 can be rewritten as a *universal spectrum*:

$$\boxed{n(E) \propto E^{-2}} \tag{37}$$

Remarkably, It does not depend on (hence the "universal"):

1. shock velocity and Mach number
2. gas density/ pressure
3. magnetic field intensity and structure
4. diffusion coefficient

It is power–law ever reaching a maximum? We can estimate the maximum energy looking at the typical parameters used before. Let's say the accelerated particles diffuse of a typical diffusion length $l_{\rm d} \approx \sqrt{Dt_{\rm d}}$, while the particle diffuses the shock moves by $u_1 t_{\rm d}$, this leads to $t_{\rm d} \approx \frac{D}{u_1^2}$, and $l_{\rm d} \approx \frac{D}{u_1}$. (Downstream calculation leads to same result.) $t_{\rm d}$ gives us the order–of–magnitude estimate for the time of a cycle. Because D increases with E, and E is increasing at each cycle, the last cycle is the longest. An argument can then be made for the maximum energy reached! Taking $t_{\rm d}$ as the acceleration time equals to the age of the system (age of the SNR), we obtain $E_{\rm max}$:

$$t_{\rm d} \approx \frac{D}{u_1^2} = t_{\rm age} \quad \text{and} \quad D = D_0 E^{\alpha} \implies E_{\rm max} = \left(\frac{u_1^2 t_{\rm age}}{D_0}\right)^{\frac{1}{\alpha}} \tag{38}$$

$E_{\rm max}$ does:

1. increase with time
2. depends on age, shock speed
3. depends on magnetic field intensity and structure (D)

It is very much NOT universal.

3.5 Non–linear diffusive shock acceleration

Remark (What if the shock is not strong?)**.** *Short answer:* $n(E) \propto E^{-\alpha}$ *with* $\alpha = \frac{r+2}{r-1}$*, making acceleration less efficient at weak shocks.*

The very remarkable par of the universal spectrum in Eq. 37 is that only a few assumptions were needed to obtain it:

1. strong shock
2. isotropy both up and downstream of the shock
3. plane shock
4. test particle (CR pressure negligible)

In the three main hypothesis, the two first are easily satisfied by SNRs. The third one is more problematic: mainly because for the CR hypothesis we need a somewhat efficient acceleration of particles and tend to violate the hypothesis. Because of the intrinsically efficient process, the cosmic–ray pressure is slowing down the upstream flow, and leading to the formation of a precursor. At low energy, the slope is greater than 2 (see Rem. 3.5), and at high energy, the slope is smaller than 2. In other very simple words, the CR pressure is modifying the shock, hence the *non–linear* terminology. A variety of subtleties can be found under this NLDSA label, but we'll keep it this simple here. There are many papers on the topic, with different approaches and names (see e.g. Bell, 1978, 2004; Giacalone & Jokipii, 2007; Amato & Blasi, 2009; Caprioli, 2012, and references).

Conclusion on the universal spectrum

The universal spectrum is close to $\propto E^{-2}$ (more or less, taking DSA or NLDSA) and we have seen that we need that the sources of CRs inject in the Galaxy a spectrum $\propto E^{-2}$, so this seem like a good ending for the story. BUT, as always, it's slightly more subtle. $\propto E^{-2}$ produced at strong shocks is the *accelerated* spectrum, and not the one *released* in the ISM. Thus this is NOT the end of the story here..

3.6 Dynamical expansion of SNRs

It is quite usual to classify SN into two types: thermonuclear, and core–collapse SN. Thermonuclear SN result from the super–critical accretion onto a white dwarf star and the subsequent thermonuclear explosion of the system, leaving a SNR without a neutron star. The mass of the ejecta is $M_{\rm ej} \approx 1.4$ $\rm M_\odot$.

Core–collapse SN result from the gravitational collapse of the star on its core and a subsequent explosion outwards, forming the SNR shock. Typical masses are > 1.4 $\rm M_\odot$. The typical total explosion energy of the SN is $\approx 10^{51}$erg and we'll keep this value in mind.

The problem is quite simple. An amount of matter $M_{\rm ej}$ is ejected with velocity v_0 and kinetic energy $E_{\rm SN}$. The goal of this tiny section is to simply derive estimates of what happens in the different phase of the evolution of the SNR shock.

3.6.1 The free expansion phase

$$E_{\rm SN} = \frac{1}{2}M_{\rm ej}v_0^2 \longrightarrow v_0 = 10^4\left(\frac{E_{\rm SN}}{10^{51}{\rm erg}}\right)^{1/2}\left(\frac{M_{\rm ej}}{{\rm M}_\odot}\right)^{-1/2}\ {\rm km/s} \tag{39}$$

The sound speed in the ISM is:

$$c_{\rm s} = \left(\gamma\frac{kT}{m_p}\right)^{1/2} \approx 10\left(\frac{T}{10^4{\rm K}}\right)^{1/2}\ {\rm km/s} \longrightarrow \boxed{\mathcal{M}\approx 100} \rightarrow \text{This means shock wave!} \tag{40}$$

So $v_{\rm sh} = u_0$ and $R_{\rm sh} = v_0 t$, but till when? Let's define $M_{\rm sw} = \frac{4\pi}{3}R_{\rm sh}^3\rho_0$ the mass of swept–up gas. In the *free expansion phase* (FE), the phase we just described right above, $M_{\rm sw} \ll M_{\rm ej}$.

$$\begin{aligned} M_{\rm ej} &\approx M_{\rm sw} \longrightarrow R_{\rm ej} \approx 2\left(\tfrac{M_{\rm ej}}{{\rm M}_\odot}\right)^{1/3}\left(\tfrac{n_{\rm ISM}}{{\rm cm}^{-3}}\right)^{-1/3}{\rm pc} \\ t_{\rm ej} &\approx \tfrac{R_{\rm ej}}{v_0} \approx 2\times10^2\left(\tfrac{M_{\rm ej}}{{\rm M}_\odot}\right)^{5/6}\left(\tfrac{E_{\rm SN}}{10^{51}{\rm erg}}\right)^{-1/2}\left(\tfrac{n_{\rm ISM}}{{\rm cm}^{-3}}\right)^{-1/3}{\rm yr} \end{aligned} \tag{41}$$

So when $M_{\rm sw} \sim M_{\rm ej}$, this ends.

3.6.2 The adiabatic phase

Let us consider the case $M_{\rm sw} \gg M_{\rm ej}$. We describe the shell as a boring sphere (circle in 2D). The mass inside the shell is $M = M_{\rm ej} + 4\pi\int_0^{R_{\rm sh}} {\rm d}R\ R^2\rho_0 \approx \frac{4\pi}{3}R_{\rm sh}^3\ \rho_0$. Is is usual to use the *thin shell approxmation*, considering that most of the matter is located in a thin shell at $R_{\rm sh}$ from the center. In this case, if ΔR is the thickness of the thin shell, we can write:

$$\frac{4\pi}{3}R_{\rm sh}^3\rho_0 = 4\pi R_{\rm sh}^2\Delta R \longrightarrow \frac{\Delta R}{R_{\rm sh}} = \frac{1}{12} \approx 0.08 \tag{42}$$

At the shock, the shell has a very high density, inside, a very low hot density. We can again write the energy and momentum conservation:

$$\begin{aligned} E_{\rm SN} &= E_{\rm k} + E_{\rm th} = \tfrac{1}{2}Mv_{\rm sh}^2 + \tfrac{P_{\rm in}}{\gamma-1}\tfrac{4\pi}{3}R_{\rm sh}^3[\text{energy}] \\ \tfrac{{\rm d}Mv_{\rm sh}}{{\rm d}t} &= 4\pi R_{\rm sh}^2(P_{\rm in}-P_{\rm out})[\text{momentum}] \end{aligned} \tag{43}$$

We want to find $R_{\rm sh}$ and $v_{\rm sh}$. Let's look for power–law solutions in the form $R = At^\beta$ (with A, *beta* constants, and thus $v_{\rm sh} = \frac{{\rm d}R_{\rm sh}}{{\rm d}t} \propto t^{\beta-1}$). In the shock rest frame : the velocity of the shock upstream is $v_{\rm sh}$ and downstream $v_{\rm sh}/4$. In the lab rest frame, the shock is moving forward at $v_{\rm sh}$ and the shell speed is compressed moving at $3/4v_{\rm sh}$. In the strong shock hypothesis, $P_{\rm out} = 0$

Let's rewrite Eq. 43:

$$\begin{aligned} &\tfrac{\rho_0}{4}\tfrac{{\rm d}R_{\rm sh}^3 v_{\rm sh}}{{\rm d}t} = R_{\rm sh}^2 P_{\rm in}[\text{momentum}] \\ &\longrightarrow \boxed{P_{\rm in} = \frac{(4\alpha-1)}{4\alpha}\rho_0 v_{\rm sh}^2} \\ &E_{\rm SN} = \tfrac{3\pi}{8}\rho_0 R_{\rm sh}^3 v_{\rm sh}^2 + 2\pi R_{\rm sh}^3 P_{\rm in} \\ &\longrightarrow E_{\rm SN} = \tfrac{\pi}{8}\rho_0 A^5\alpha(19\alpha-4)t^{5\alpha-2}[\text{energy}] \end{aligned} \tag{44}$$

Now, in this second phase of expansion we are discussing, the most important hypothesis is the *adiabatic* one.

$$E_{\rm SN} = {\rm const} \rightarrow E_{\rm SN} \propto t^{5\alpha-2} \rightarrow \alpha = \tfrac{2}{5} \rightarrow \boxed{A = \left(\frac{50}{9\pi}\frac{E_{\rm SN}}{\rho_0}\right)^{1/5}} \tag{45}$$

During this adiabatic phase, also called Sedov phase (SP), or Sedov–Taylor phase - other usual names for the adiabatic phase, we have $E_{\rm k} = \frac{1}{3}E_{\rm SN}$ and $E_{\rm th} = \frac{2}{3}E_{\rm SN}$. Putting numbers into Eq. 45, we obtain:

$$\begin{aligned} R_{\rm sh} &\approx 5\left(\tfrac{E_{\rm SN}}{10^{51}{\rm erg}}\right)^{1/5}\left(\tfrac{n_{\rm ISM}}{{\rm cm}^{-3}}\right)^{-1/5}\left(\tfrac{t}{\rm kyr}\right)^{2/5}{\rm pc} \\ v_{\rm sh} &\approx 2\times10^3\left(\tfrac{E_{\rm SN}}{10^{51}{\rm erg}}\right)^{1/5}\left(\tfrac{n_{\rm ISM}}{{\rm cm}^{-3}}\right)^{-1/5}\left(\tfrac{t}{\rm kyr}\right)^{-3/5}{\rm km/s} \end{aligned} \tag{46}$$

Until when does this work?

3.6.3 The radiative phase

Let's look at the cooling function (bremmstrahlung + mainly lines emissons): $10^5 \lesssim T \lesssim 10^{7.5} \longrightarrow \Lambda \approx 2 \times 10^{-19} T^{-1/2} \text{erg cm}^3\text{s}^{-1}$.

The shock heating is given by:

$$T \approx 2 \times 10^7 \left(\frac{v_{\text{sh}}}{1000\text{km/s}}\right)^2 \text{K} \tag{47}$$

and consequently, the cooling time can be expressed as:

$$\tau_{\text{c}} \approx \frac{\epsilon_{\text{th}}}{n_{\text{i}} n_{\text{e}} \Lambda} = \frac{3nkT}{n^2\Lambda} \approx 10^6 \left(\frac{n_{\text{ISM}}}{\text{cm}^{-3}}\right)^{-1} \left(\frac{v_{\text{sh}}}{1000\text{km/s}}\right)^3 \text{yr} \tag{48}$$

When the age of the SNR becomes of the order of the cooling time $\tau_{\text{c}} \approx t_{\text{age}}$, the shell becomes *radiative.*

$$t_{\text{ad}} \approx 2 \times 10^4 \left(\frac{E}{10^{51}\text{erg}}\right)^{3/14} \left(\frac{n_{\text{ISM}}}{\text{cm}^{-3}}\right)^{-4/7} \text{yr} \tag{49}$$

The corresponding radius and shock velocity are:

$$\begin{cases} R_{\text{ad}} \approx 20 \left(\frac{E}{10^{51}\text{erg}}\right)^{2/7} \left(\frac{n_{\text{ISM}}}{\text{cm}^{-3}}\right)^{-3/7} \text{pc} \\ v_{\text{ad}} \approx 3 \times 10^2 \left(\frac{E}{10^{51}\text{erg}}\right)^{1/14} \left(\frac{n_{\text{ISM}}}{\text{cm}^{-3}}\right)^{1/7} \text{km/s} \end{cases} \tag{50}$$

Remark (Radiative isothermal shock: a particular case)**.** *What if the radiative losses are so effective that $T_2 = T_1$ (this is the isothermal approximation)? Then it means that at the shock transition, there must be a strong emission of photons (where the shocked gas is). In this isothermal context, the state equation is: $P \propto \rho$, i.e. $P = c_{\text{s}}^2 \rho$ where c_{s} is constant.*

$$\left.\begin{array}{ll} \textit{mass:} & \rho_1 u_1 = \rho_2 u_2 \\ \textit{momentum:} & \rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2 \end{array}\right\} = 1 + \frac{P_1}{\rho_1 u_1^2} = \frac{u_2}{u_1} + \frac{P_2}{\rho_1 u_1^2} \tag{51}$$

For a strong shock, the term $\frac{P_1}{\rho_1 u_1^2}$ can be neglected, leading to:

$$u_2^2 = u_2 u_1 + c_{\text{s}}^2 = 0 \implies u_2 = \frac{u_1}{2}\left[1 \pm \sqrt{1 - \frac{4c_{\text{s}}^2}{u_1^2}}\right] \tag{52}$$

Leading to two solutions $u_2 = u_1 - \frac{2c_{\text{s}}^2}{u_1} \sim u_1$ or $\boxed{u_2 = c_{\text{s}}^2/u_1}$.

$$\begin{array}{l} u_2 = \frac{c_{\text{s}}^2}{u_1} = \frac{u_1}{\mathcal{M}}^2 \overset{\mathcal{M}\to\infty}{\longrightarrow} 0 \\ r = \frac{\rho_1}{\rho_2} = \frac{u_1}{u_2} = \mathcal{M}^2 \overset{\mathcal{M}\to\infty}{\longrightarrow} \infty \end{array} \tag{53}$$

In other words, this means that for a strong shock a very thin and dense shell forms. The matter in the shell moves roughly at the same velocity that the shock. Thus, the thin shell approximation is even more justified for radiative shocks!

3.6.4 The end

The radiative phase, but until when? The fourth phase we are considering here is a pressure driven *snowplough* phase. At first, the dense shell cools, while the interior does not, because of the very low density. All the energy dissipated at the shock is radiated away and the SNR interior finally cools adiabatically. For P_{in} the pressure inside the shell:

$$P_{\text{in}} V^\gamma = \text{constant} \implies P_{\text{in}} \propto R_{\text{sh}}^{-3\gamma} \tag{54}$$

Using momentum conservation equation as written in Eq. 43, one obtains $R_{\text{s}} \propto t^\alpha$ with $\alpha = 2/7$. The density inside the SNR is largely smaller than in the shell $n_{\text{in}} = \epsilon n_{\text{sh}}$. The SNR interior cools

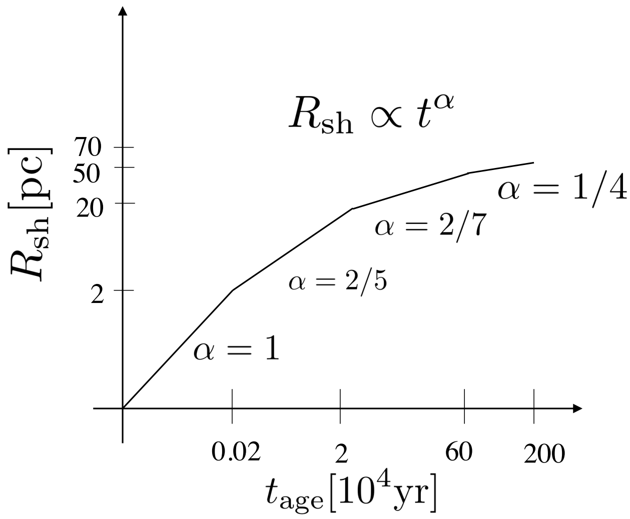

Figure 9: Typical radius evolution of a SNR. The breaks corresponds to $R_{\rm sh}$= (2,20,50,70) pc, $v_{\rm sh}$=(10,0.3,0.02,0.01) 10^3 km/s, and $M_{\rm sh}$= (1,10^3,10^4, 3 $\times 10^4$) $M_\odot$

much later than the SNR shell, and this snowplough pressure driven phase typically last a few tens of $t_{\rm ad}$ (given in Eq. 49). Once the interior cools, the motion of the shell is determined by momentum conservation:

$$Mv_{\rm sh} = \text{constant} \implies Mv_{\rm sh} = \tfrac{4\pi}{3}\rho_0 \alpha R_{\rm sh}^2 u_{\rm sh} = \tfrac{4\pi}{3}\rho_0 A^4 t^{4\alpha-1} \implies \boxed{\alpha = \frac{1}{4}} \tag{55}$$

$$\begin{cases} R_{\rm sh} \propto t^{1/4} \\ u_{\rm sh} \propto t^{-3/4} \end{cases} \tag{56}$$

The SNR will finally dissolve in the ISM when the shock Mach number becomes of the order of ~ 1, i.e. when $v_{\rm sh} \approx c_{\rm s} \approx 10$ km/s. Let us recap all this in Fig. 9.

The shock remains for a relatively *long* time, and efficient particle acceleration can happen until $10^4 - 10^5$ yr (Acero et al., 2016).

4 Few open questions on SNRs

1. The details of NLDSA: this includes, the maximum energy of accelerated particles, the magnetic field strength and structure, the structure of precursor, and in fact probably anything you can think of if you go into enough details
2. Are SNR the sources of Galactic cosmic rays? The ultimate "smoking gun" is thought to be the detection in the 100 TeV, proving the acceleration of protons up to PeV energies. But the acceleration of PeV particles could also be achieved at other sources.
3. Hadronic or Leptonic?
4. Diffusion of CRs away from their sources is still not well understood.

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