

Particle Acceleration Mechanisms (I)

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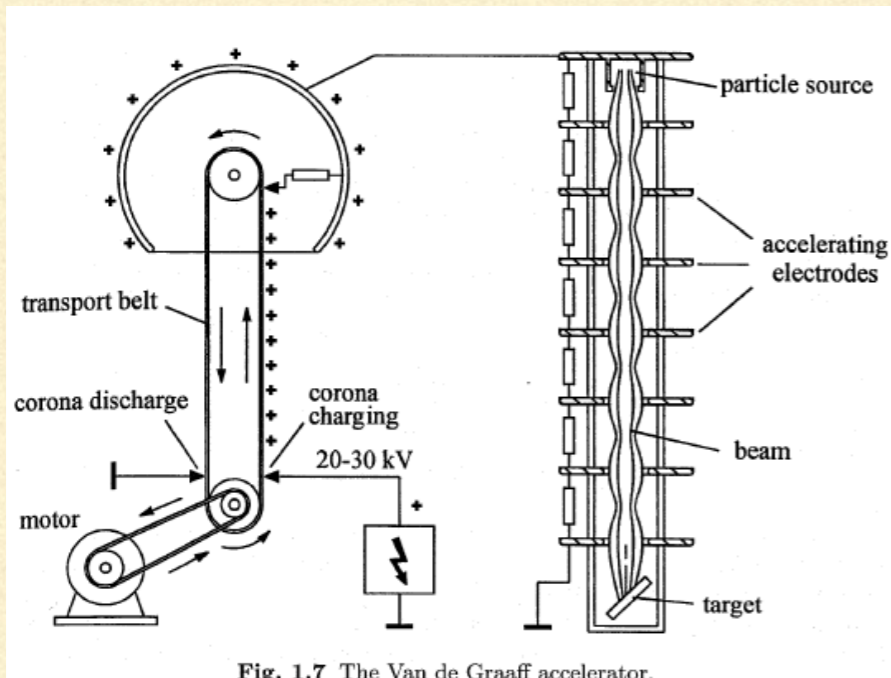
Collaborators: R. Blandford, W. East, K. Nalewajko, J. Zrake, A. Spitkovsky

Motion of charged particles

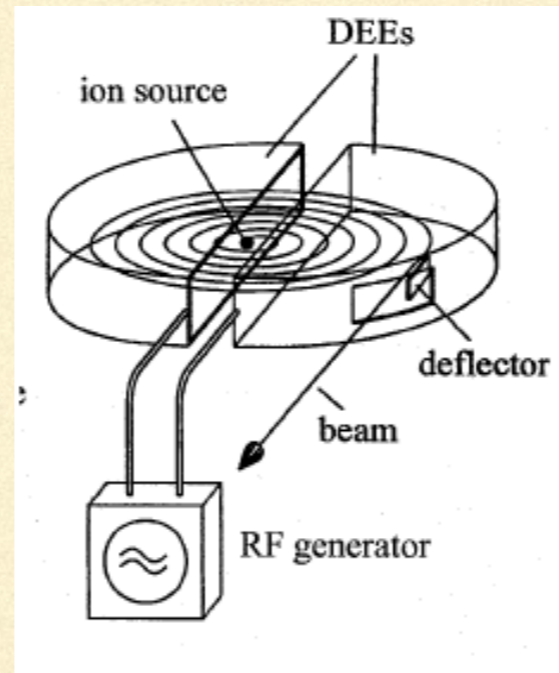
$$m \frac{d\mathbf{u}}{dt} = q(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B})$$

- Static magnetic field does not do work on particles.
- Need electric field!
 - Electrostatic field
 - Rapidly changing magnetic field

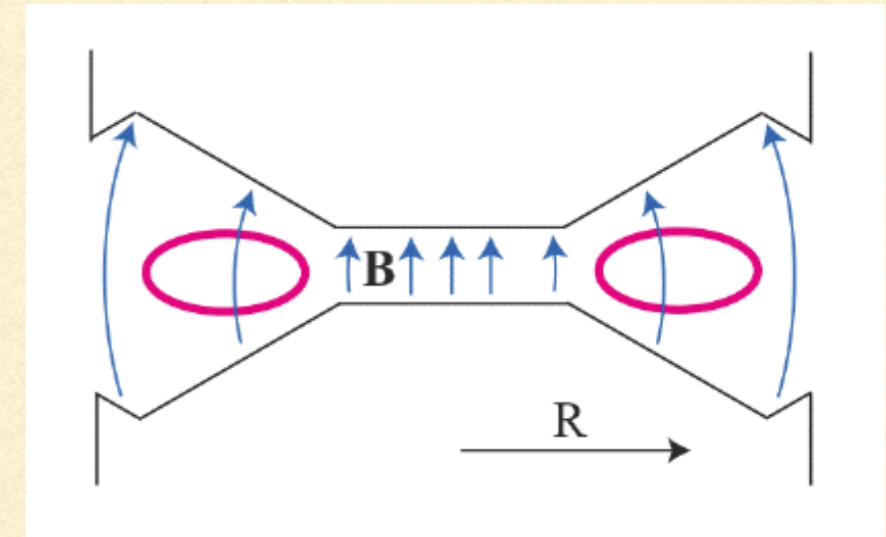
How we accelerate particles in laboratory



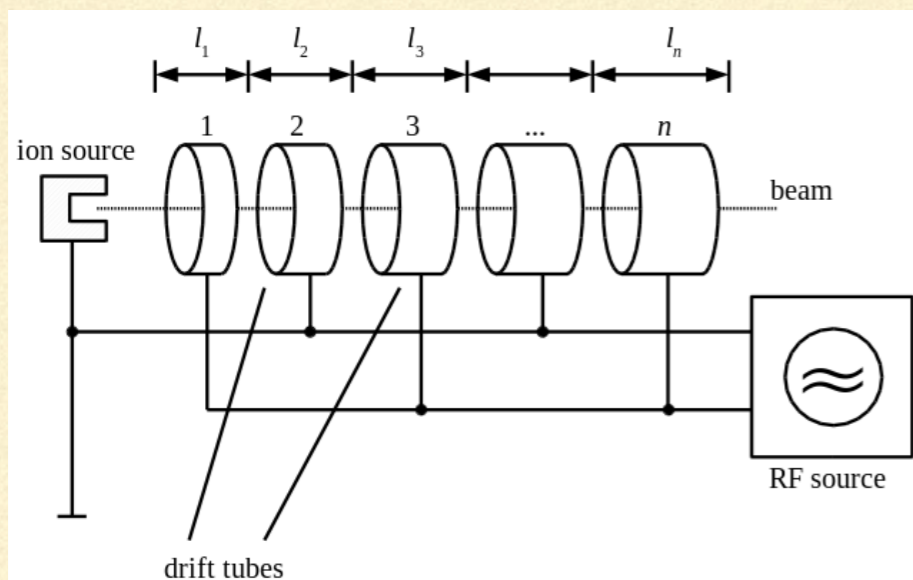
Direct electric field acceleration



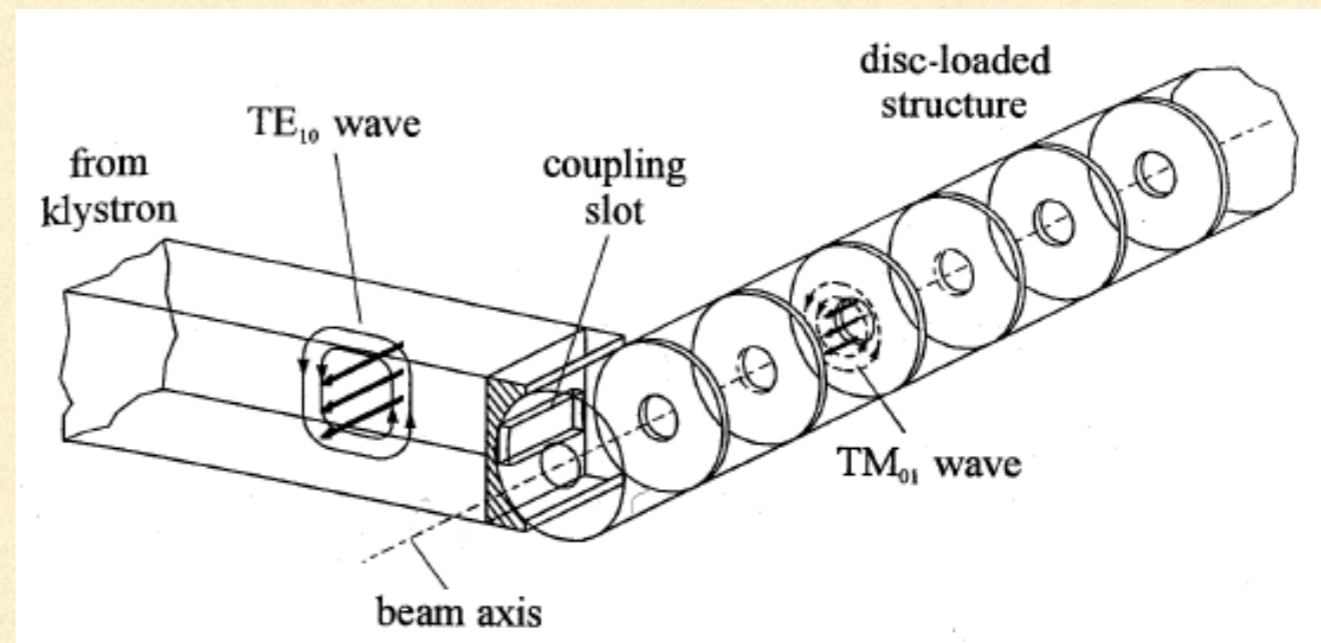
cyclotron



Betatron

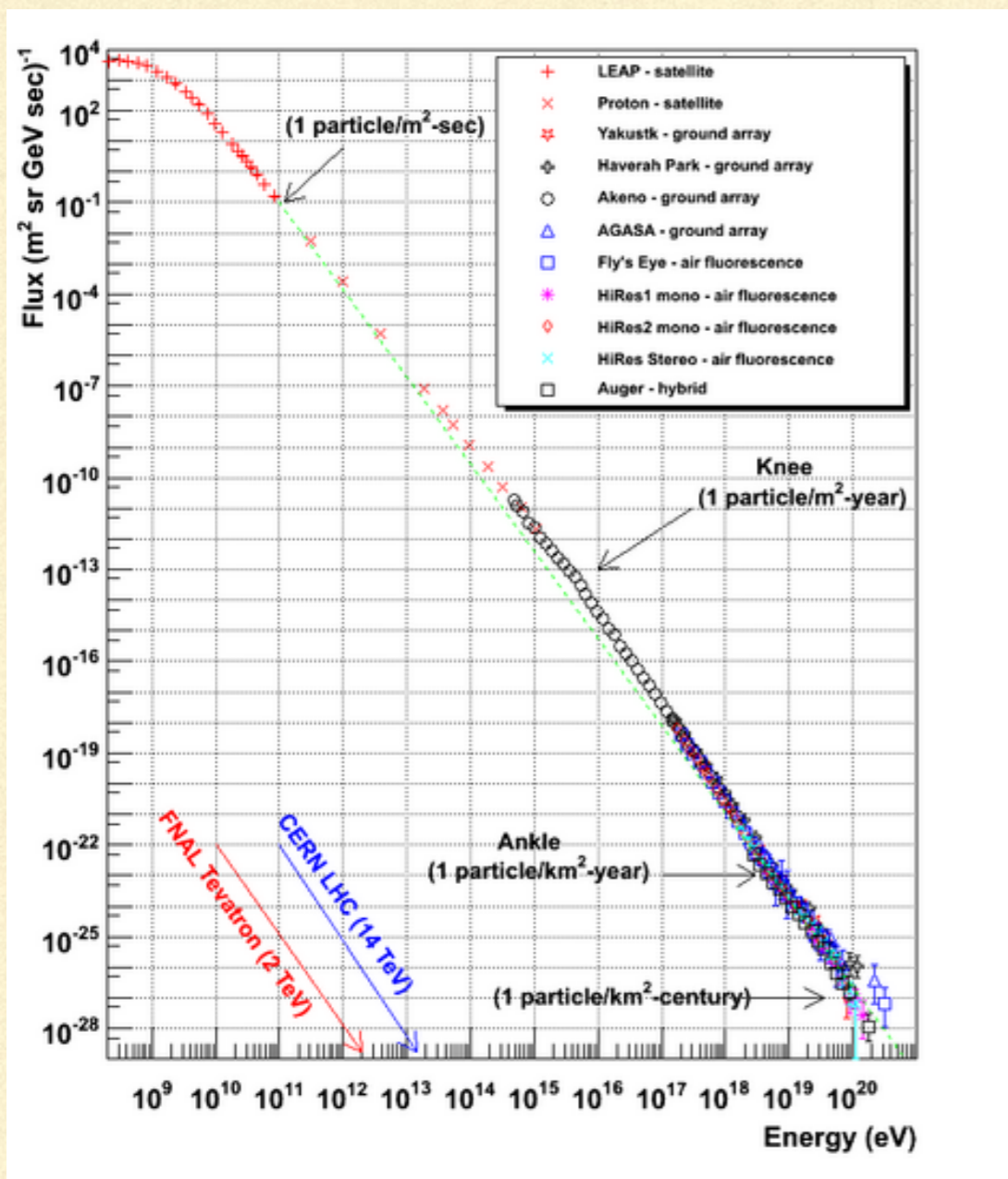


Linear accelerator



High energy particles from the cosmos

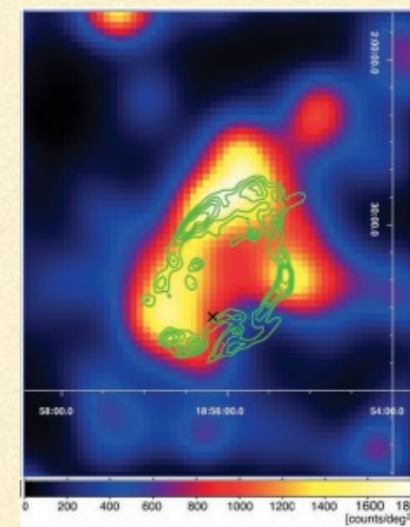
Cosmic rays



Gamma-ray sources



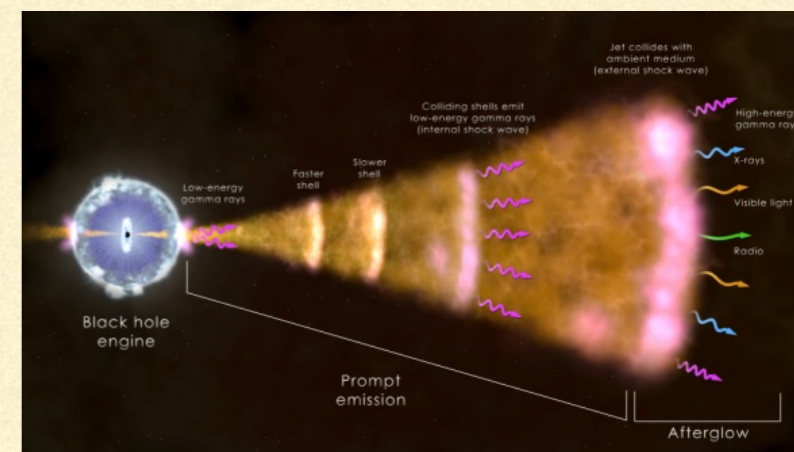
Pulsar/PWN



SNR



AGN



GRB

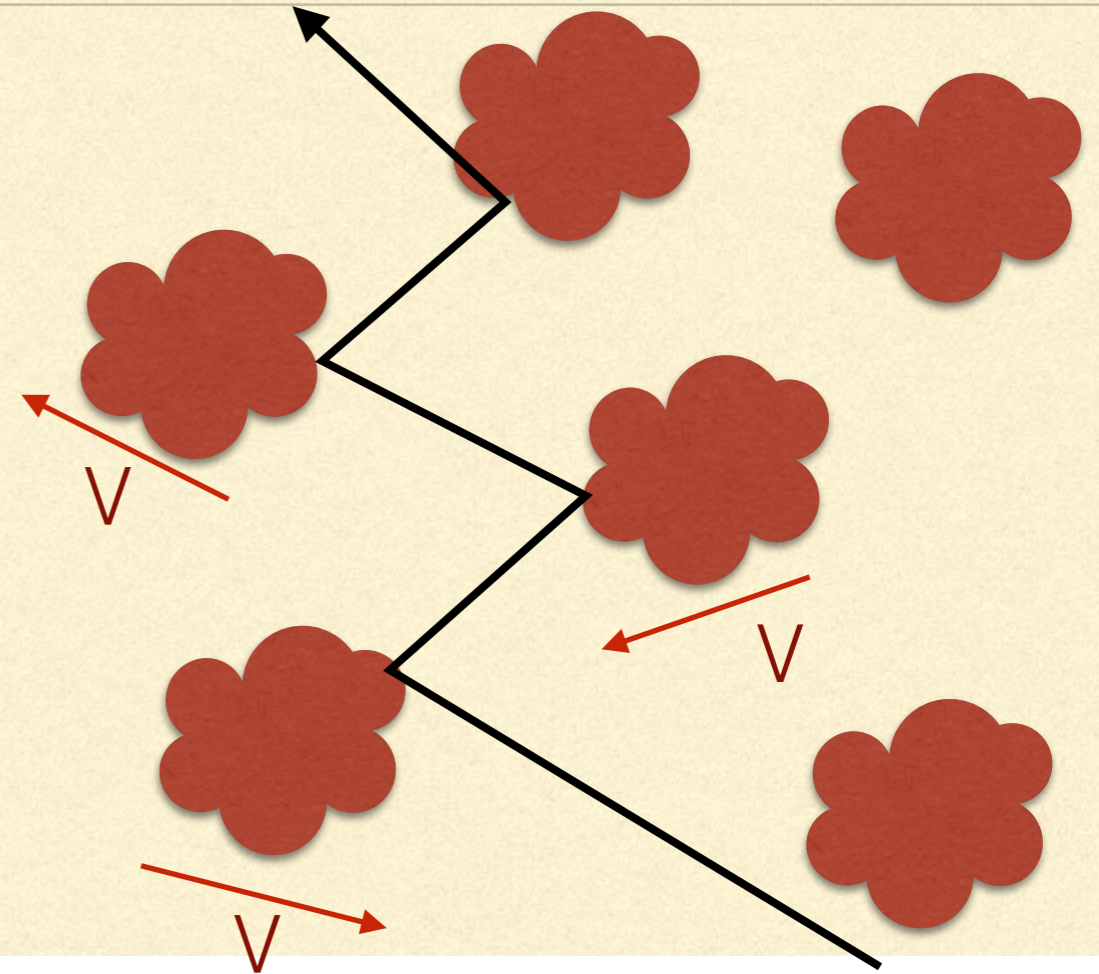
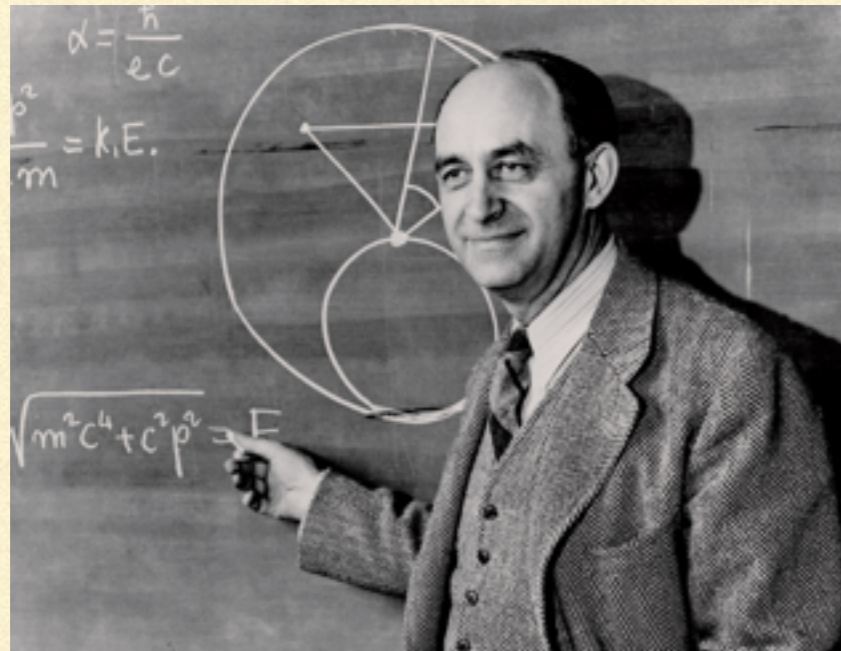
from <http://www.physics.utah.edu/~whanlon/spectrum.html>

...and X-ray/gamma-ray binaries, novae, etc...

Questions to answer

- How are particles accelerated?
- Why the universal power law?
- How to reach very high energies (PeV, EeV, ZeV)?

Fermi Acceleration



PHYSICAL REVIEW

VOLUME 75, NUMBER 8

APRIL 15, 1949

On the Origin of the Cosmic Radiation

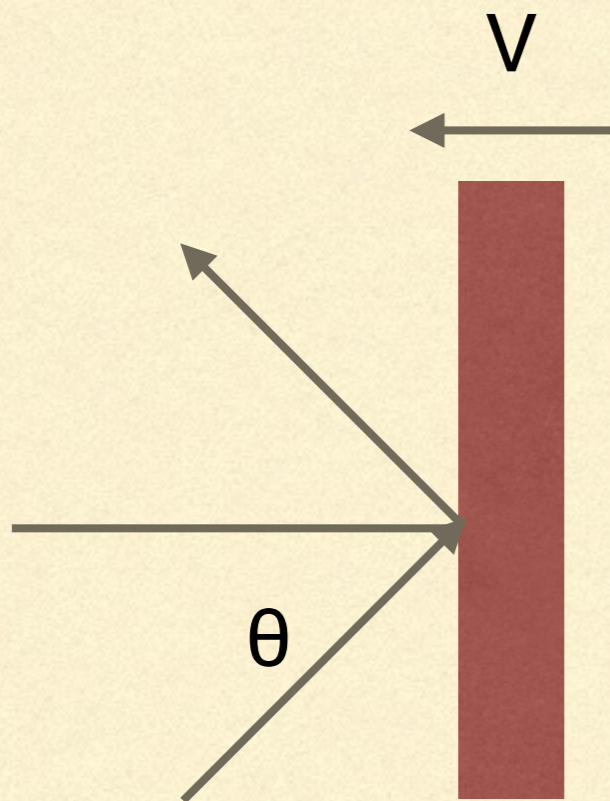
ENRICO FERMI

Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

Fermi acceleration—original idea



- Elastic collision between particle and magnetic cloud, each time the energy gain is

$$\Delta E = E \left(\frac{2V}{c} \cos \theta + \frac{2V^2}{c^2} \right)$$

- Head-on collision is slightly more frequent than overtaking collision
- Average over angle, energy gain is only second order in V/c , but proportional to initial energy E :

$$\langle \Delta E \rangle = \frac{8}{3} \left(\frac{V}{c} \right)^2 E$$

Why the power law?

Approach 1: diffusion-loss equation

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial E}(\dot{E}N(E)) = D\nabla^2 N - \frac{N}{\tau_{\text{esc}}} + Q(E) + \frac{1}{2} \frac{\partial^2}{\partial E^2}(d(E)N(E))$$

↑
↑
↑
↑
↑

convection in energy
diffusion in space
escape
injection
diffusion in energy

Consider a simple situation:

steady state, neglect diffusion in space and in energy, injection only at the low energy end, escape independent of energy

$$\frac{\partial}{\partial E}(\dot{E}N(E)) + \frac{N}{\tau_{\text{esc}}} = 0$$

$$\dot{E} = \alpha E \quad \longrightarrow \quad N = kE^{-x}, \quad x = 1 + \frac{1}{\alpha\tau_{\text{esc}}}$$

Approach 2: probabilistic view point

- For each collision, energy changes as $E=\beta E_0$
- The probability of remaining is P after each collision
- After k collisions, $N=N_0 P^k$ particle remaining, with energy $\geq E=\beta^k E_0$

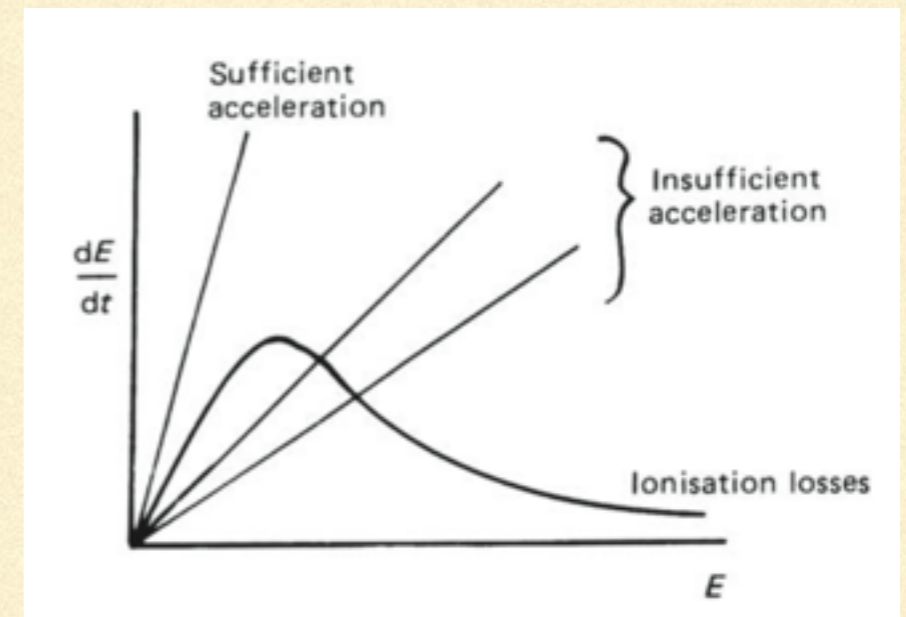
$$\frac{\ln(N/N_0)}{\ln(E/E_0)} = \frac{\ln P}{\ln \beta}, \quad \frac{N(\geq E)}{N_0} = \left(\frac{E}{E_0}\right)^{\frac{\ln P}{\ln \beta}}$$

$$N(E)dE \propto E^{-1+\ln P/\ln \beta} dE$$

$$\frac{\ln P}{\ln \beta} = -\frac{1}{\alpha \tau_{\text{esc}}}$$

A few problems

- Interstellar clouds $V/c \leq 10^{-4}$, mean free path 0.1 pc, very slow energy gain
- Injection problem
- Why should the power law index be a special value around 2.5?

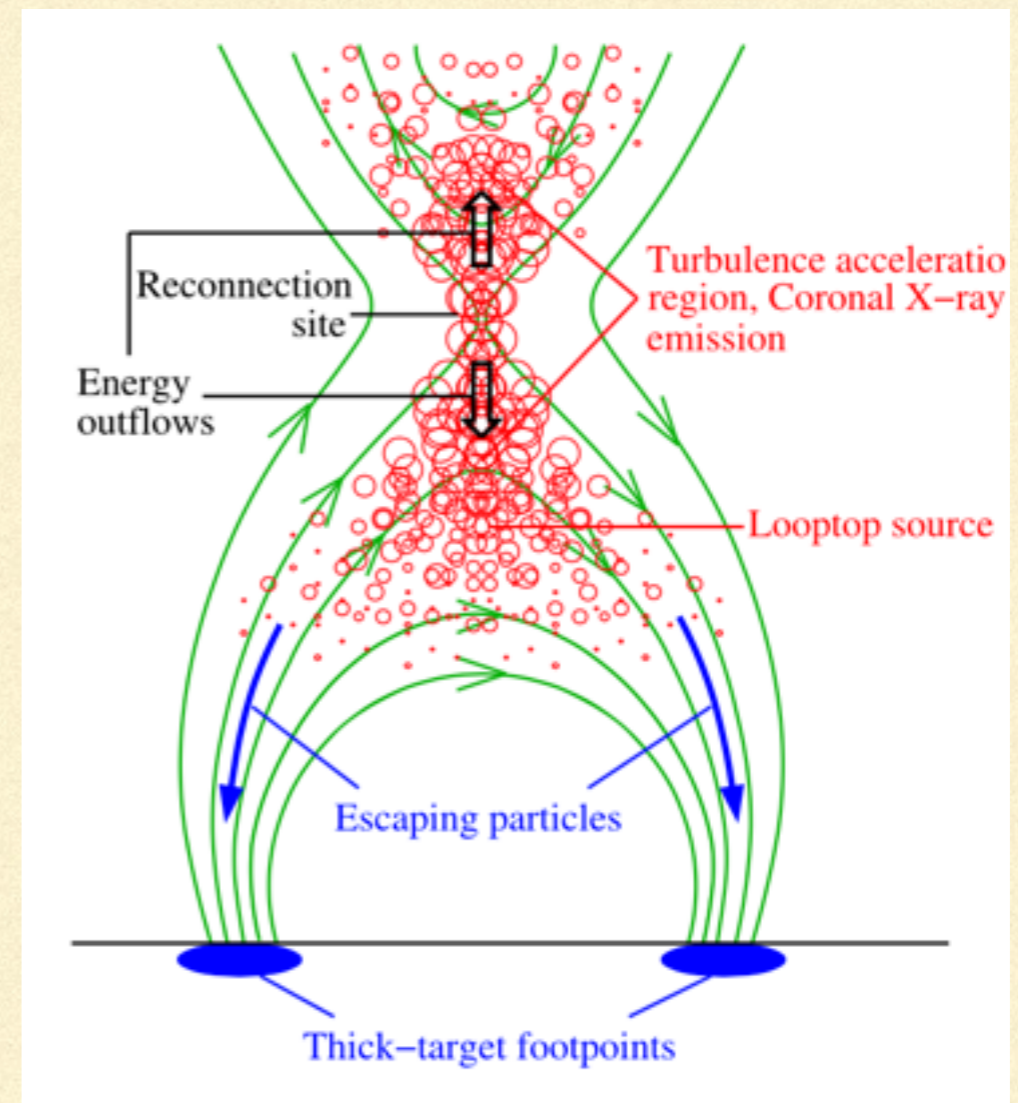


NB: including the diffusion in energy, the power law index changes into

$$x = \frac{3}{2} \left(1 + \frac{16}{9\alpha\tau_{\text{esc}}} \right)^{1/2} - \frac{1}{2}$$

Modern second order Fermi acceleration

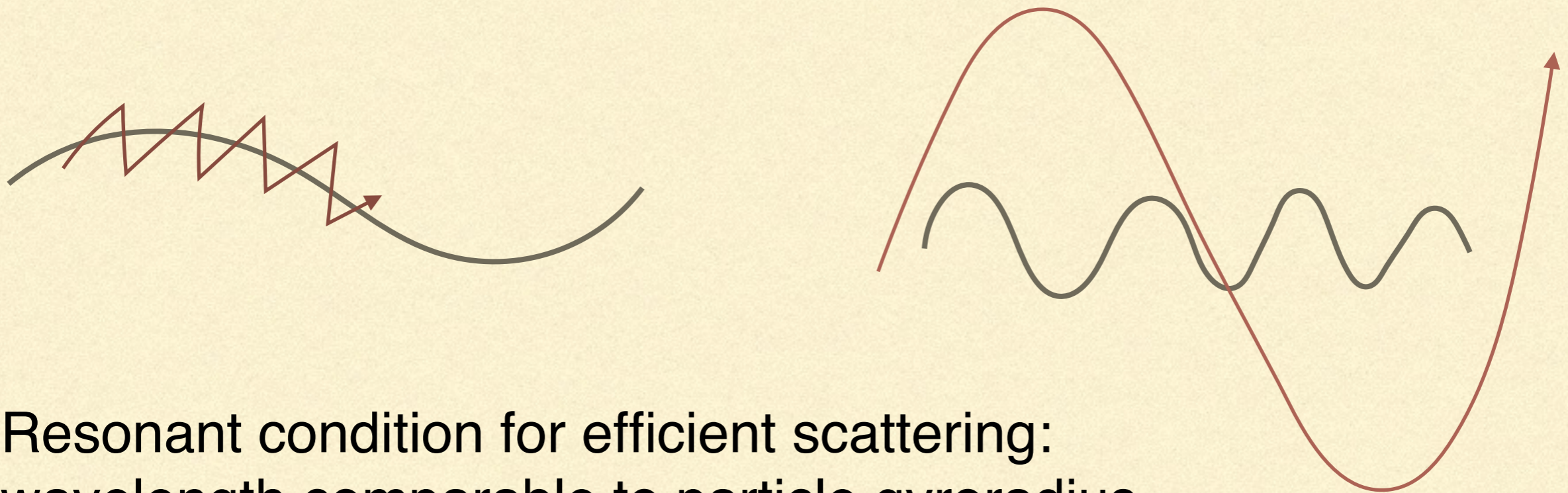
- Replace the magnetic clouds with plasma waves
- Turbulent medium



Petrosian 2012

Importance of scattering

Example: pitch angle scattering of cosmic rays by Alfvén waves
Wavelengths too small or too large are not efficient for scattering.



Resonant condition for efficient scattering:
wavelength comparable to particle gyroradius

$$k_z v_z - \omega = \pm \Omega, \quad \text{or} \quad \frac{\lambda}{2\pi} \approx r_L \quad \delta\theta \approx \pm \frac{\delta B}{B}$$

Can we get first order acceleration?

If only there's just head-on collisions...

Consider a shock!

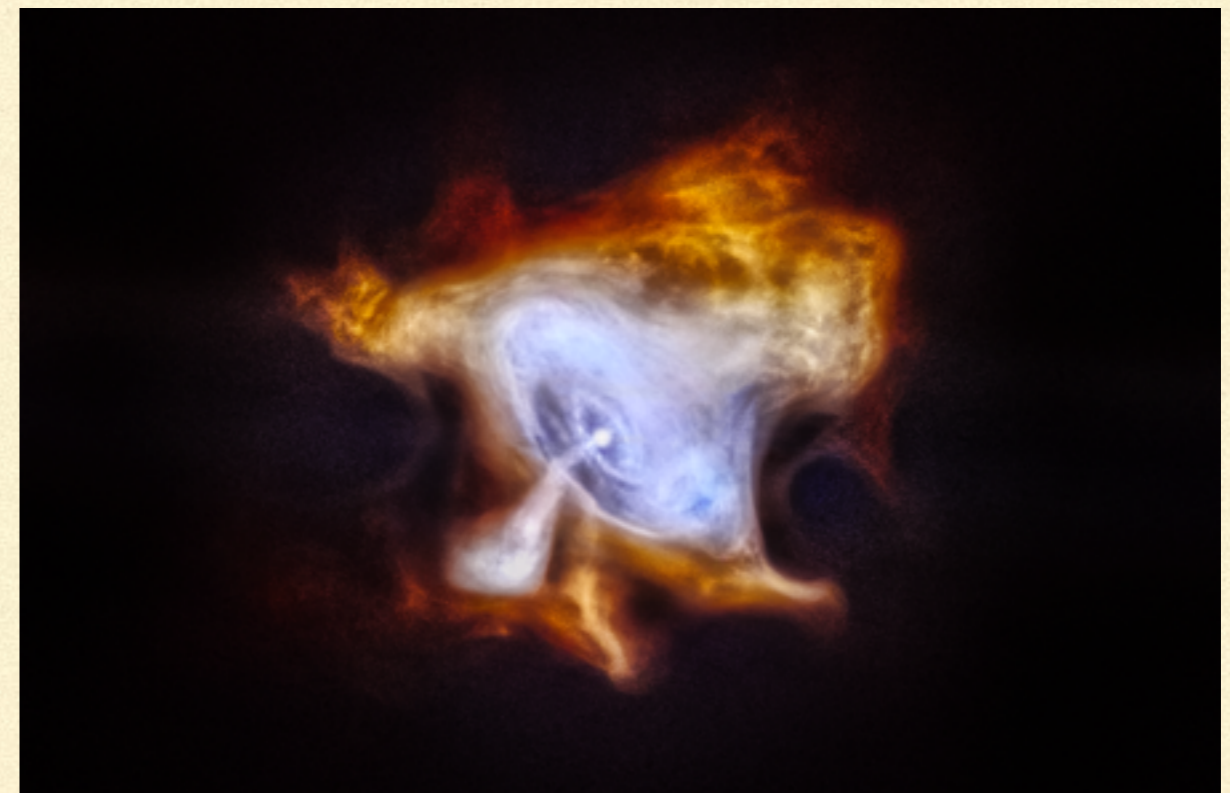
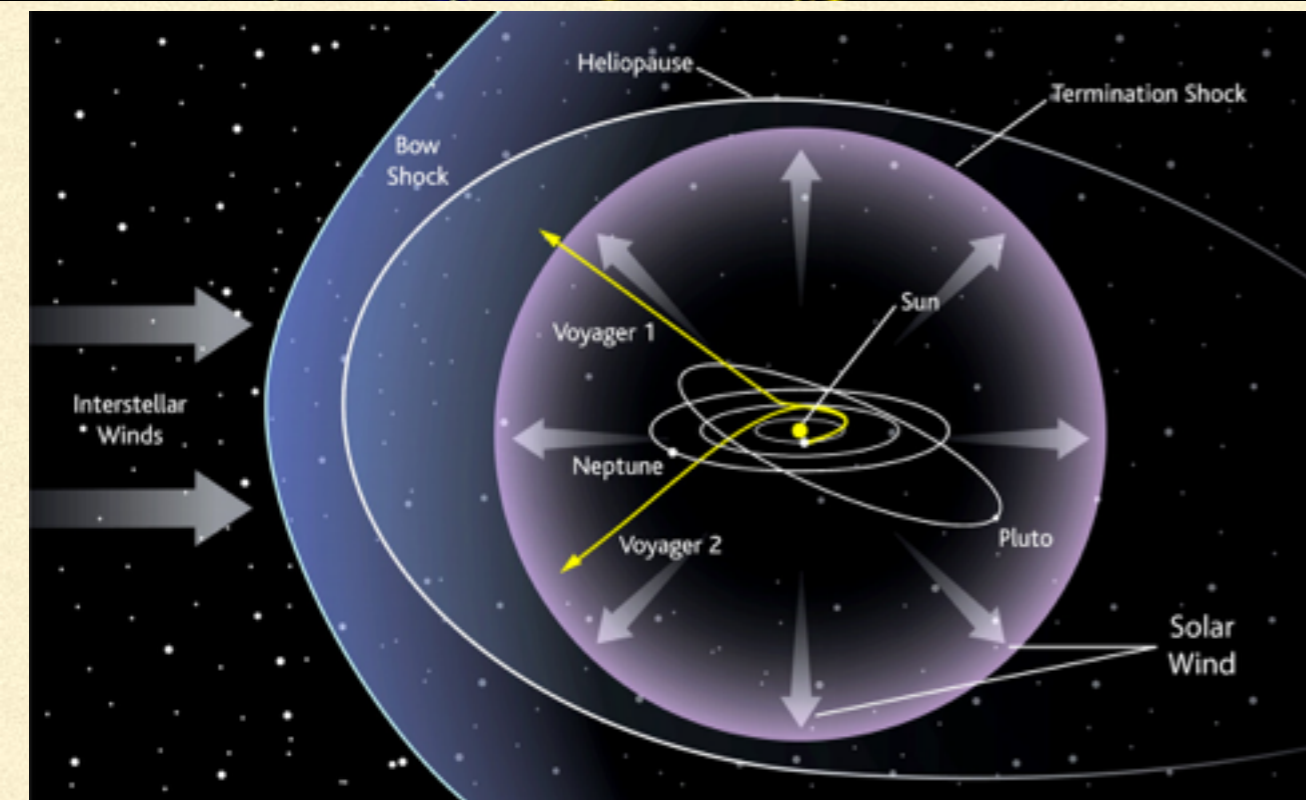
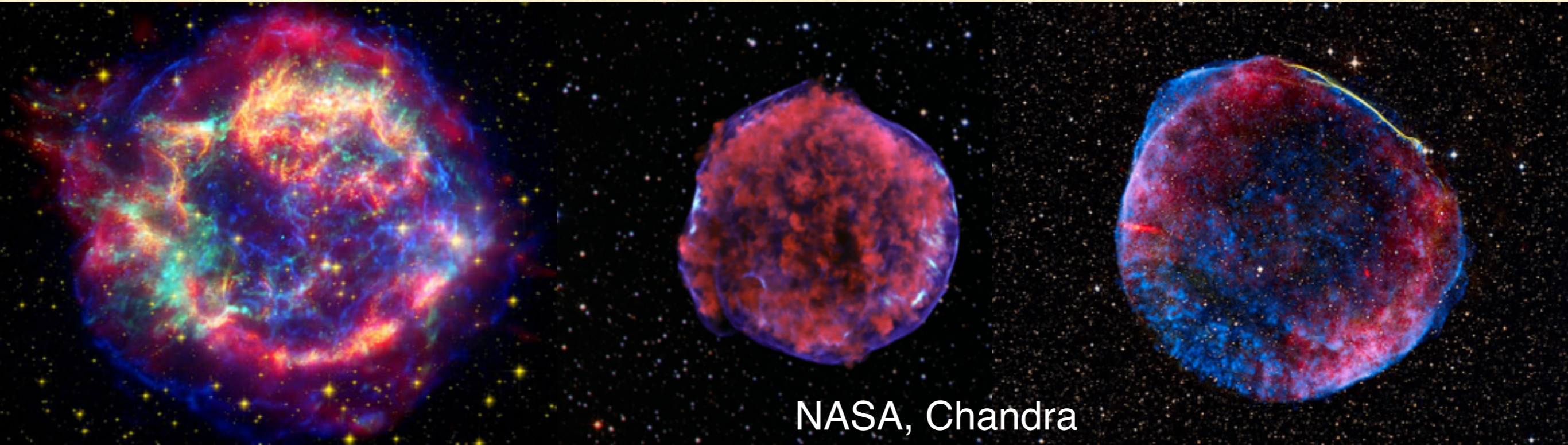
Axford, Leer and Skadron (1977)

Krymsky (1977)

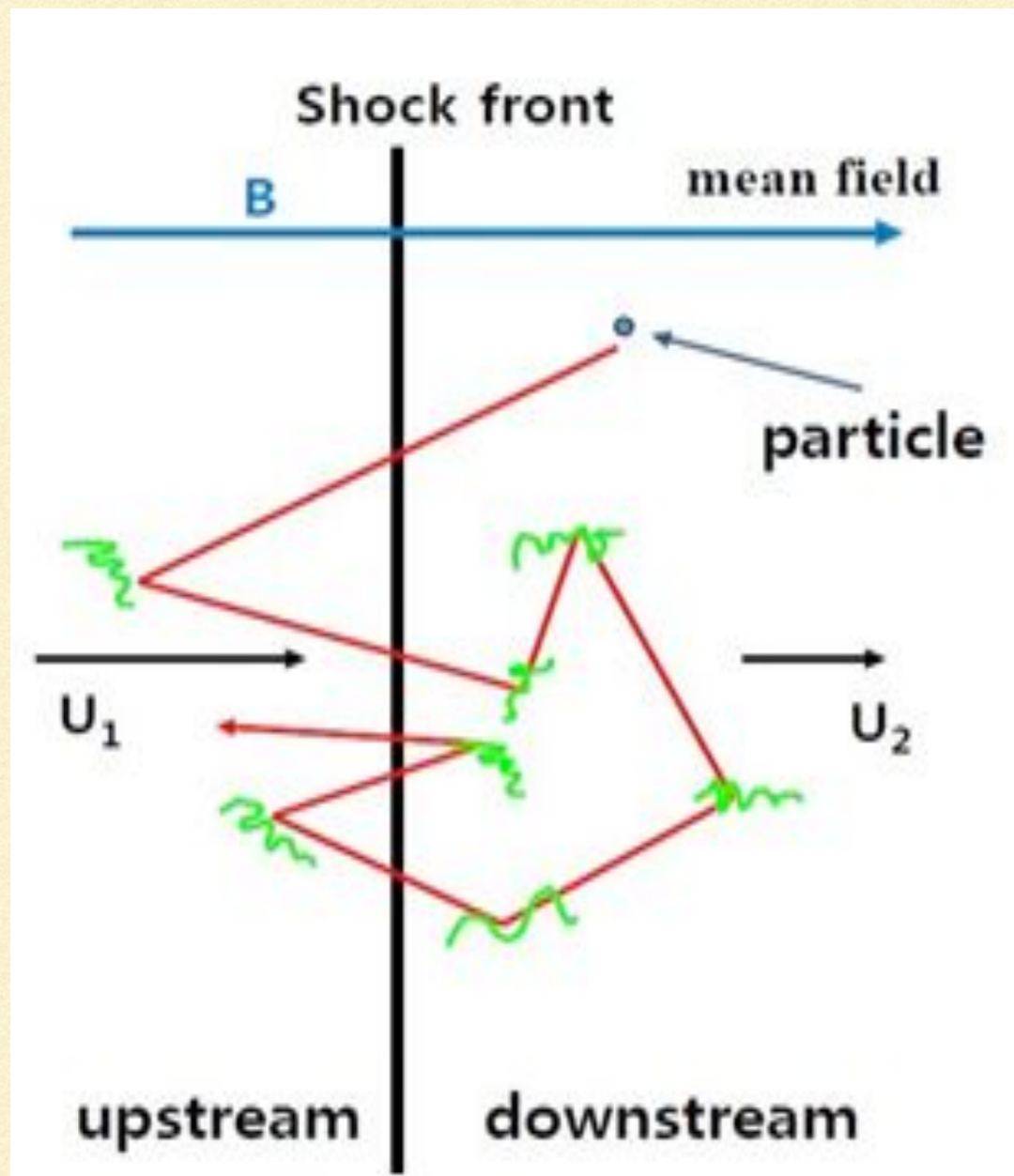
Bell (1978)

Blandford and Ostriker (1978)

Shocks are ubiquitous in astrophysical environments...

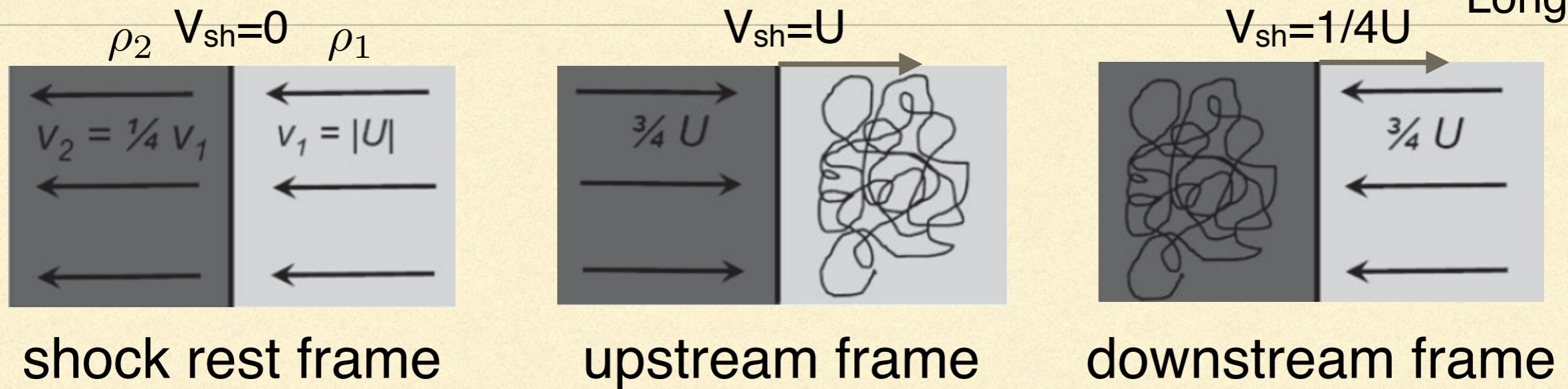


Diffusive shock acceleration



- An observer in upstream flow: I see downstream stuff moving toward me!
- An observer in downstream flow: the upstream stuff is approaching me!

Let's see if we can get the right power law from the shock.



- Shock jump condition (hydro shock)**

$$\rho_1 v_1 = \rho_2 v_2$$

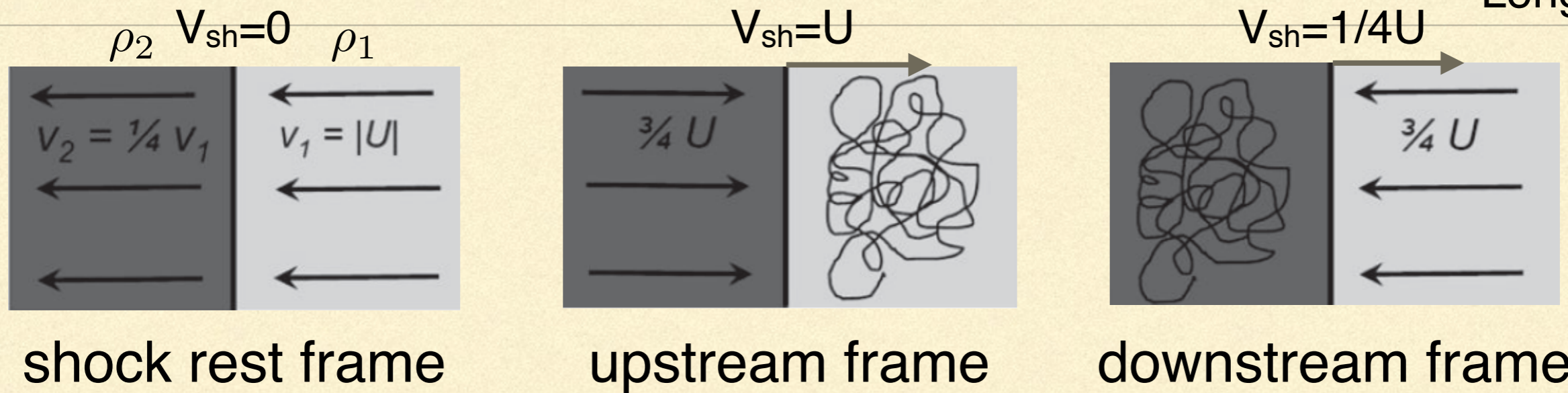
$$\rho_1 v_1^2 + P_1 = \rho_2 v_2^2 + P_2$$

$$\left(\frac{1}{2} \rho_1 v_1^2 + h_1 \right) v_1 = \left(\frac{1}{2} \rho_2 v_2^2 + h_2 \right) v_2$$

$$\frac{\rho_1}{\rho_2} = \frac{\gamma - 1}{\gamma + 1} + \frac{2}{(\gamma + 1)M^2}, \quad \frac{P_2}{P_1} = \frac{2\gamma M^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1}$$

For strong shock $M \gg 1$ $\frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1}$

Taking $\gamma = 5/3$ $\rho_2/\rho_1 = 4, v_2 = v_1/4$



• Energy gain for one shock crossing

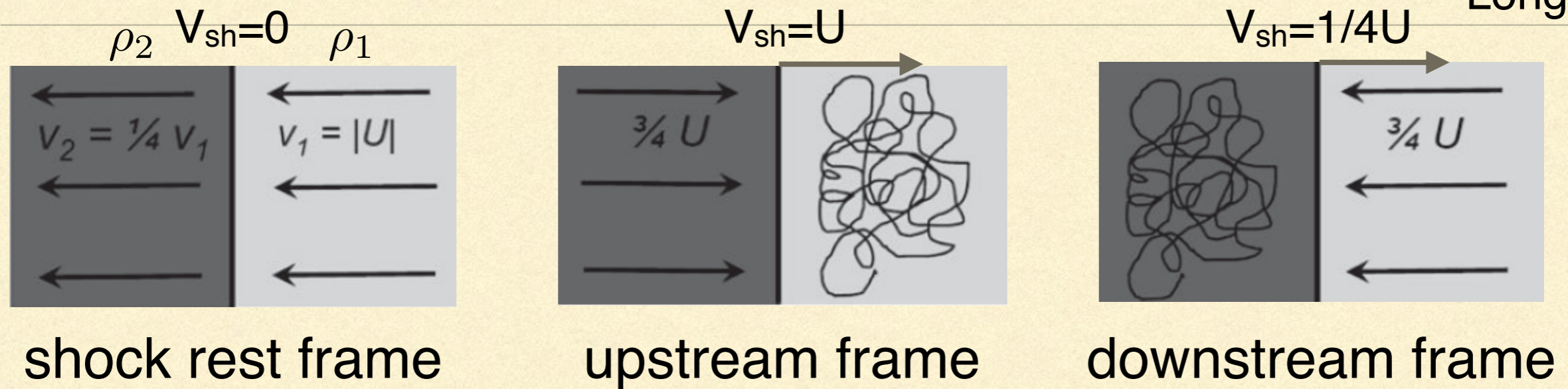
A particle in upstream with energy E enters downstream, energy measured in downstream is

$$E' = \gamma_V (E + p_x V) \approx E \left(1 + \frac{V}{c} \cos \theta \right)$$

Average over angle, the energy gain is

$$\frac{\Delta E}{E} = \int_0^{\pi/2} \frac{V}{c} \cos \theta \cdot 2 \cos \theta \sin \theta d\theta = \frac{2V}{3c}$$

Round trip energy gain $\frac{\Delta E}{E} = \frac{4V}{3c}$, $\beta = \frac{E}{E_0} = 1 + \frac{4V}{3c}$



- **Escape rate**

Number of particles crossing the shock per unit time is $Nc/4$

downstream the particles are swept away at a rate $Nv_2=NU/4$,
fraction of particle lost is thus U/c , so $P=1-U/c$

Finally, we get

$$\ln P = \ln(1 - U/c) = -U/c$$

$$\ln \beta = \ln(1 + 4V/3c) = 4V/3c = U/c$$

$$\ln P / \ln \beta = -1, N(E)dE \propto E^{-2}dE$$

-
- The power law index depends on the shock compression ratio $r \equiv \rho_2/\rho_1 = v_1/v_2$. In general the accelerated particles have a distribution (non-relativistic shock speed)

$$f(p) \propto p^{-q}, \quad q = \frac{3r}{r-1}$$

- For general Mach number, $q = \frac{3(\gamma+1)M^2}{2(M^2-1)}$
- When $M \gg 1$ and $\gamma=5/3$, we have $q=4$. Thus, for relativistic particles, $N(E) \propto E^{-2}$; for non-relativistic particles, $N(E) \propto E^{-1.5}$

More questions

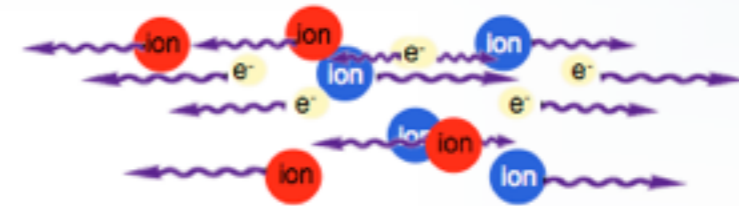
- How does a collisionless shock form?
- In the simple example, we used test particle approach. This may not be valid! Cosmic rays can contribute significantly to pressure and thus modify the shock jump condition. This will affect shock compression ratio and power law index.
- Injection problem
- Do we have enough turbulence to scatter the particles?

How collisionless shocks work

Collisionless plasma flows

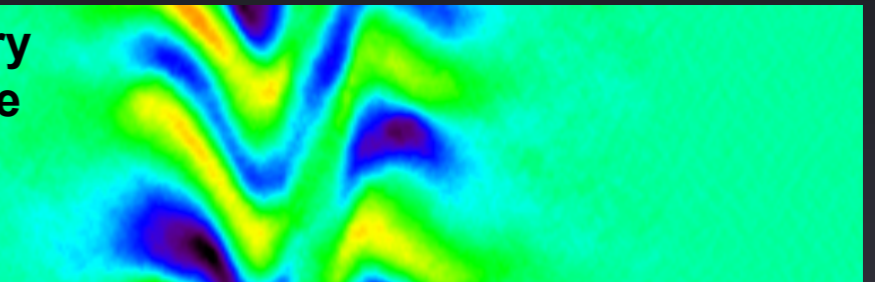


Coulomb mean free path is large



Do ions pass through without creating a shock?

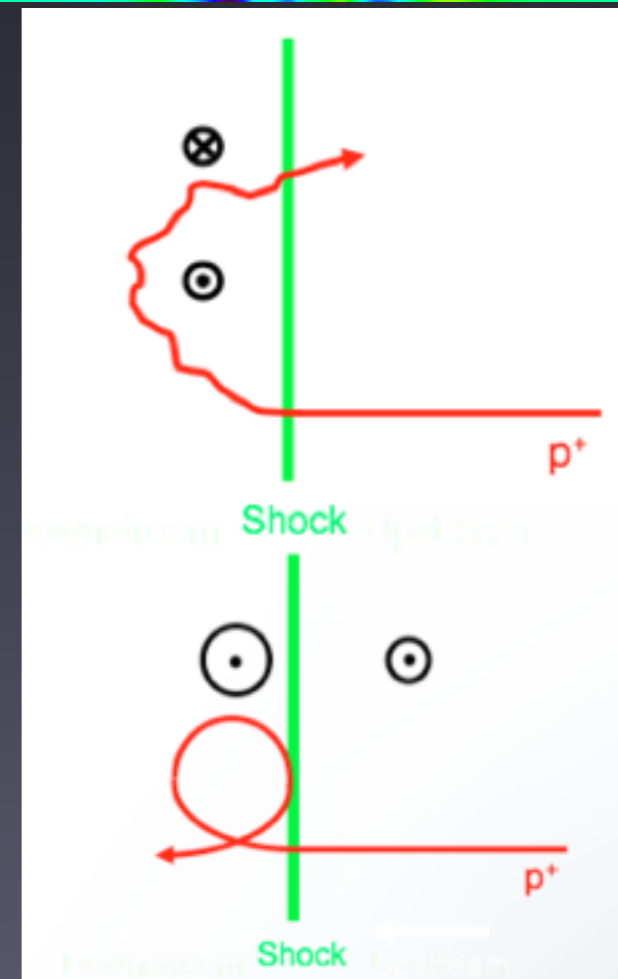
Filamentary
B fields are
created



Two main mechanisms for creating collisionless shocks:

1) For low initial B field, particles are deflected by self-generated magnetic fields (filamentation/Weibel instability)

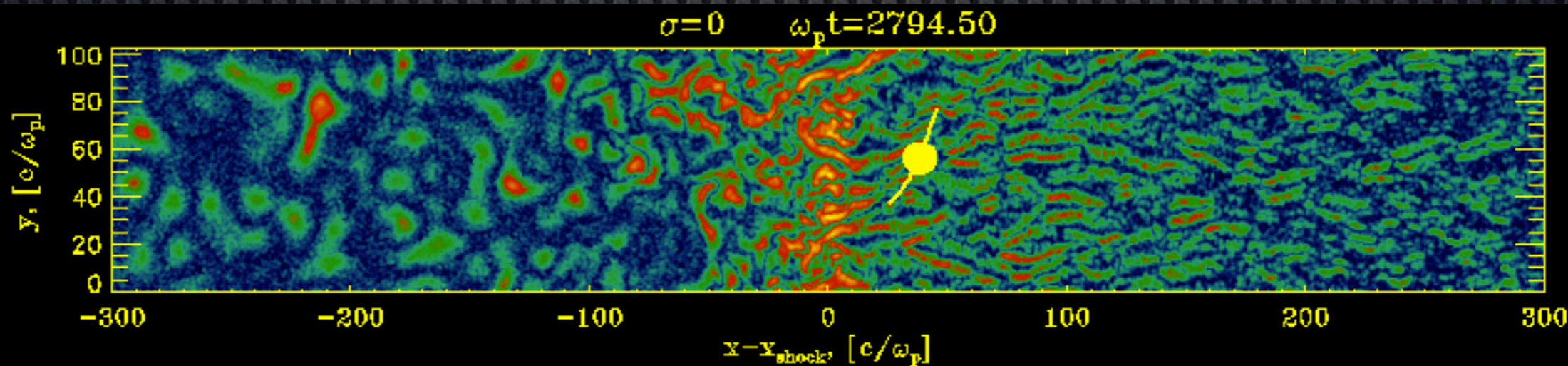
2) For large initial B field, particles are deflected by compressed pre-existing fields



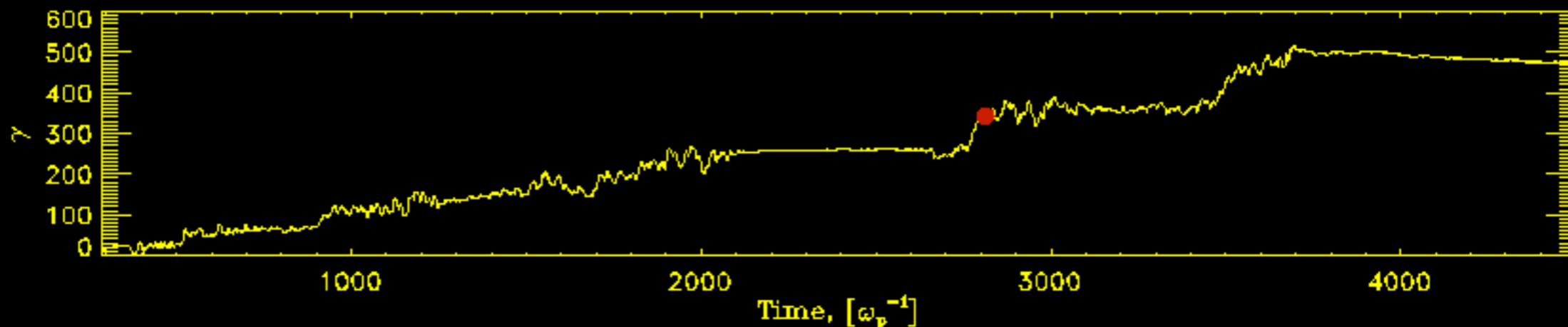
Particle acceleration

Self-generated magnetic turbulence scatters particles across the shock; each crossing results in energy gain -- Fermi process

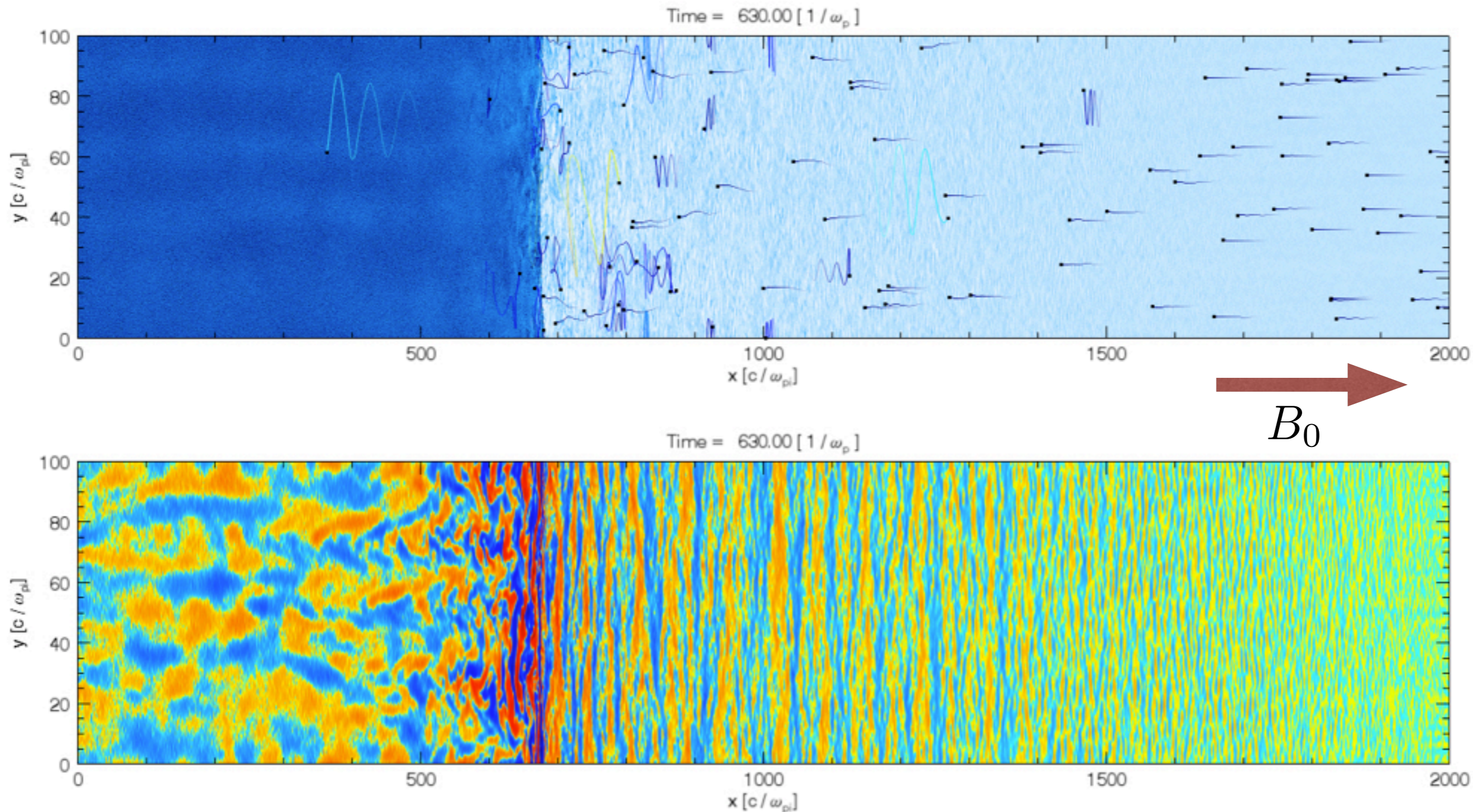
Magnetic filaments



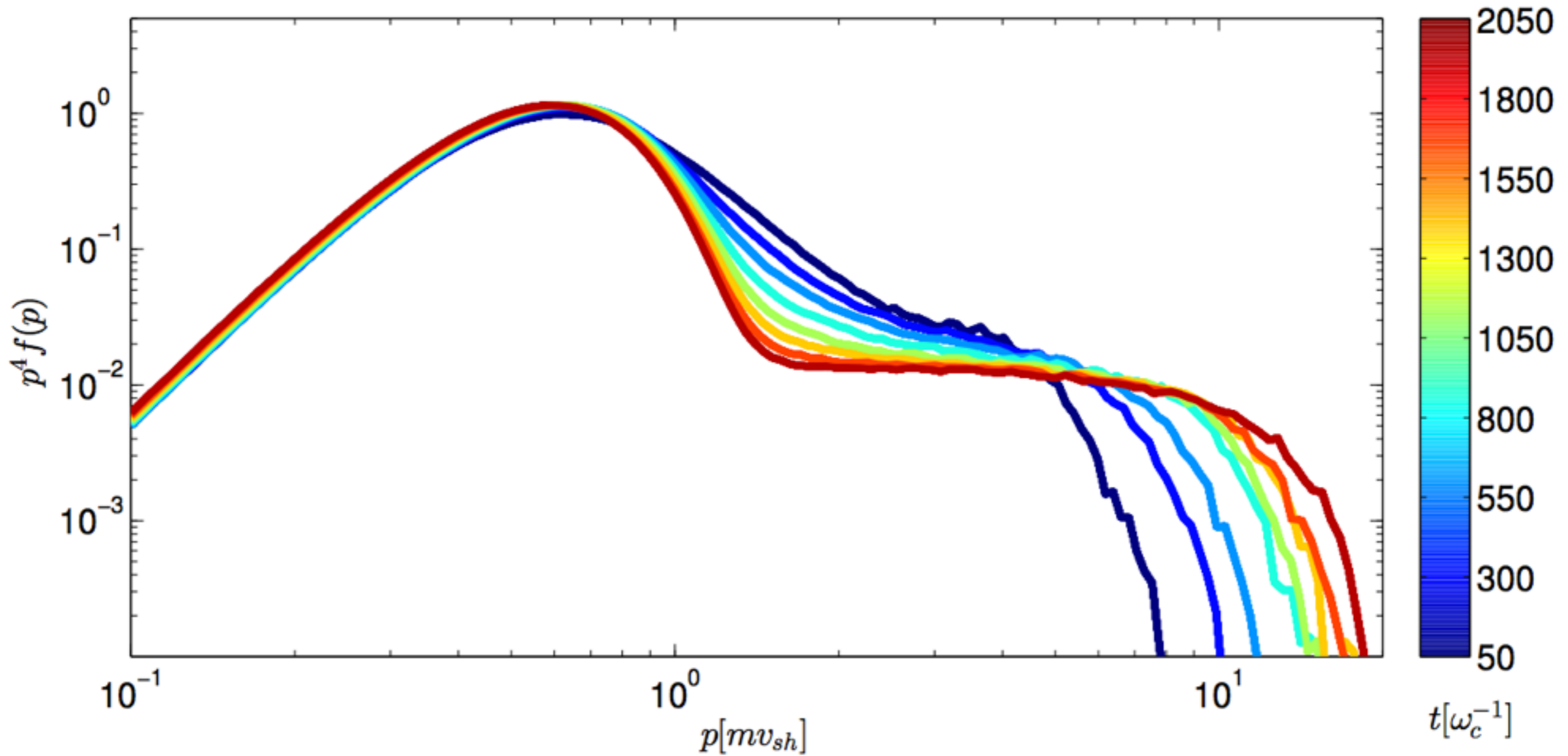
Particle energy



Hybrid simulation of magnetized, parallel shock, ions are scattered by self-generated waves.



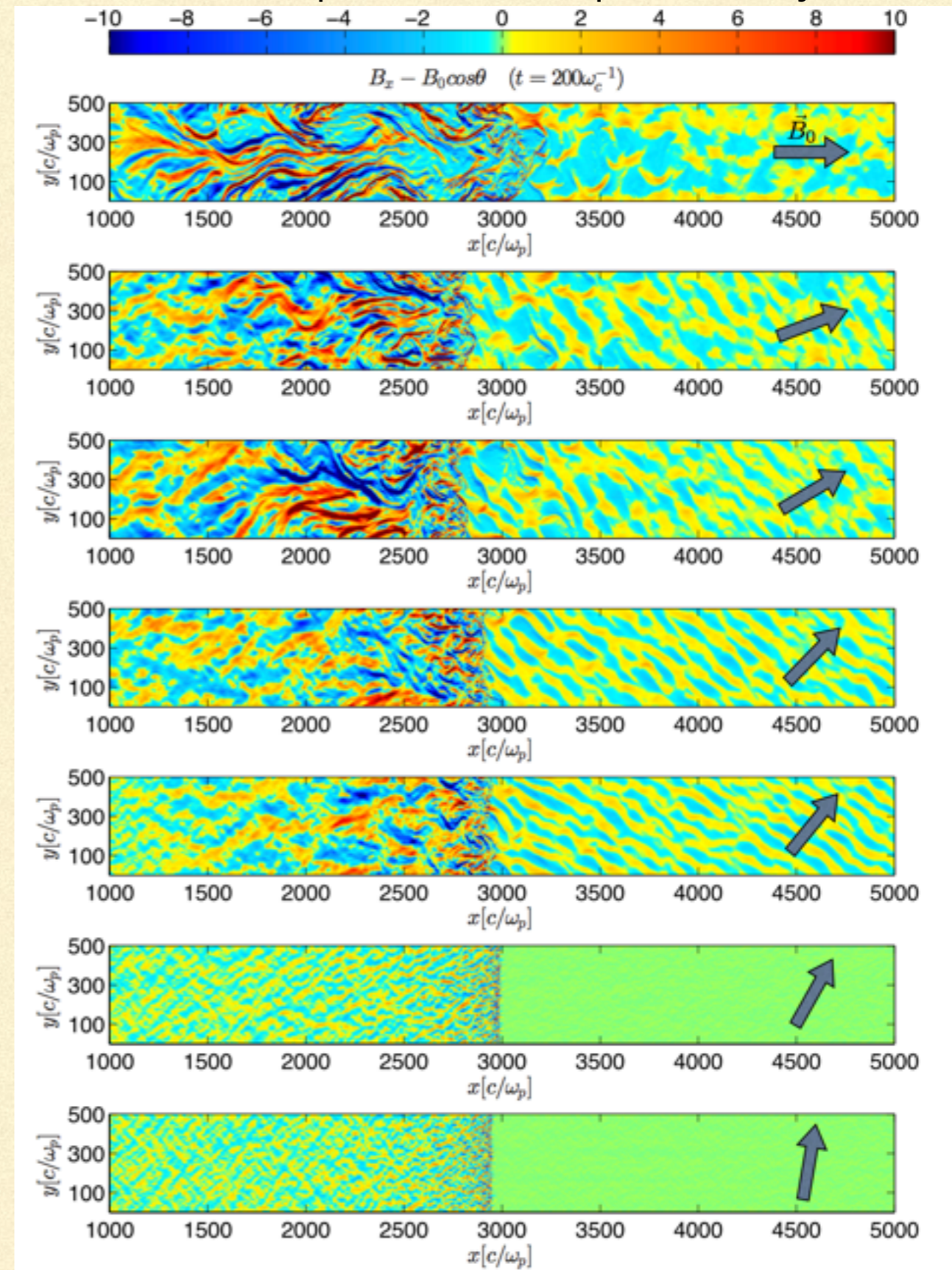
Particle spectrum



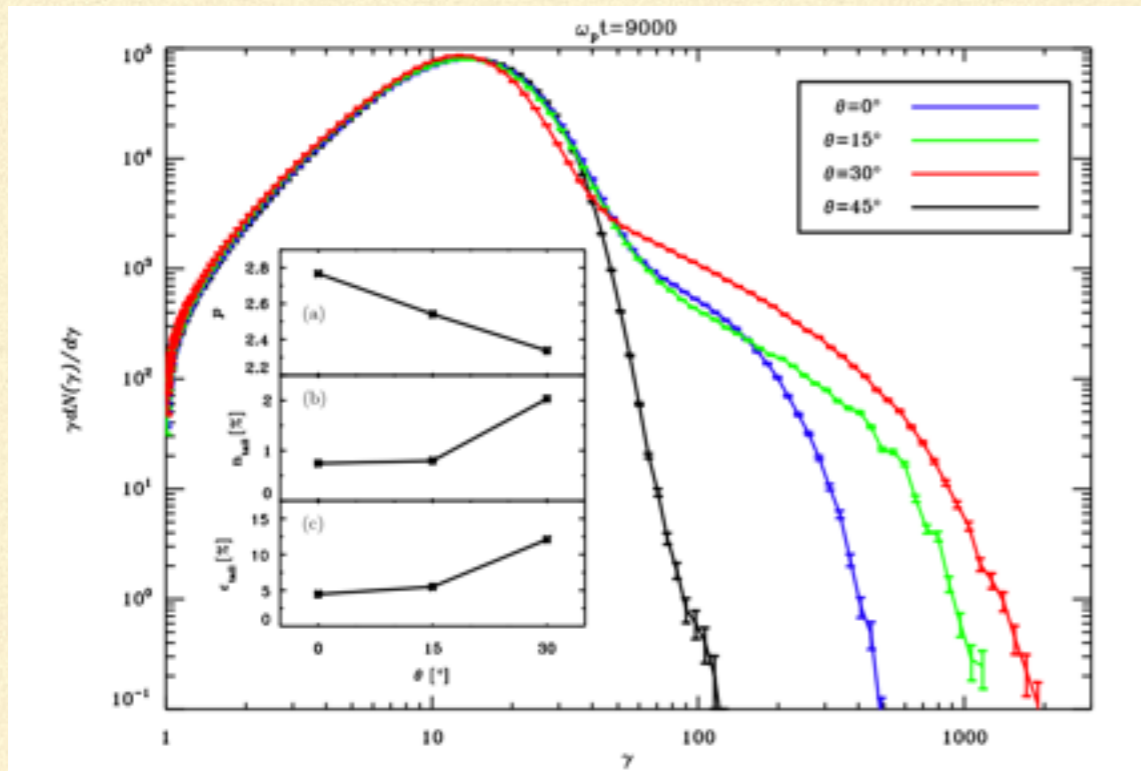
Caprioli and Spitkovsky 2014

Importance of scattering

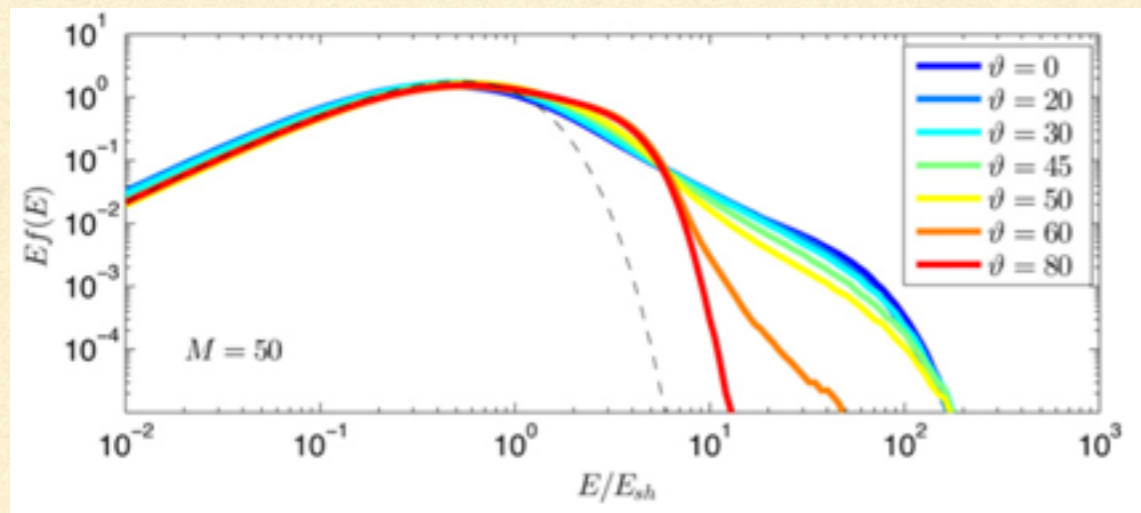
- Cosmic rays create their scatterers
 - Weibel instability (low magnetization shocks)
 - Bell instability $\lambda < r_L$
 - Resonant $\lambda \sim r_L$
 - Firehose instability $\lambda > r_L$
 - Cosmic ray filamentation
- Magnetic field amplification



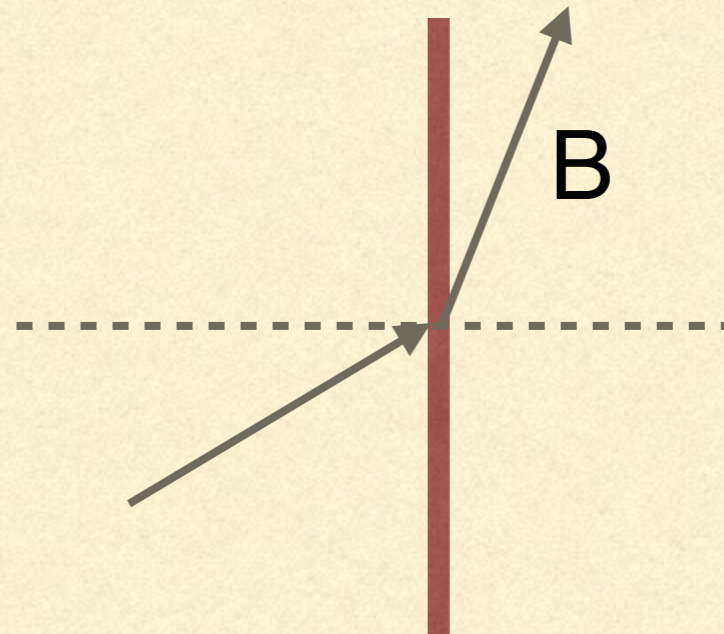
Superluminal and subluminal shocks



Sironi&Spitkovsky2009

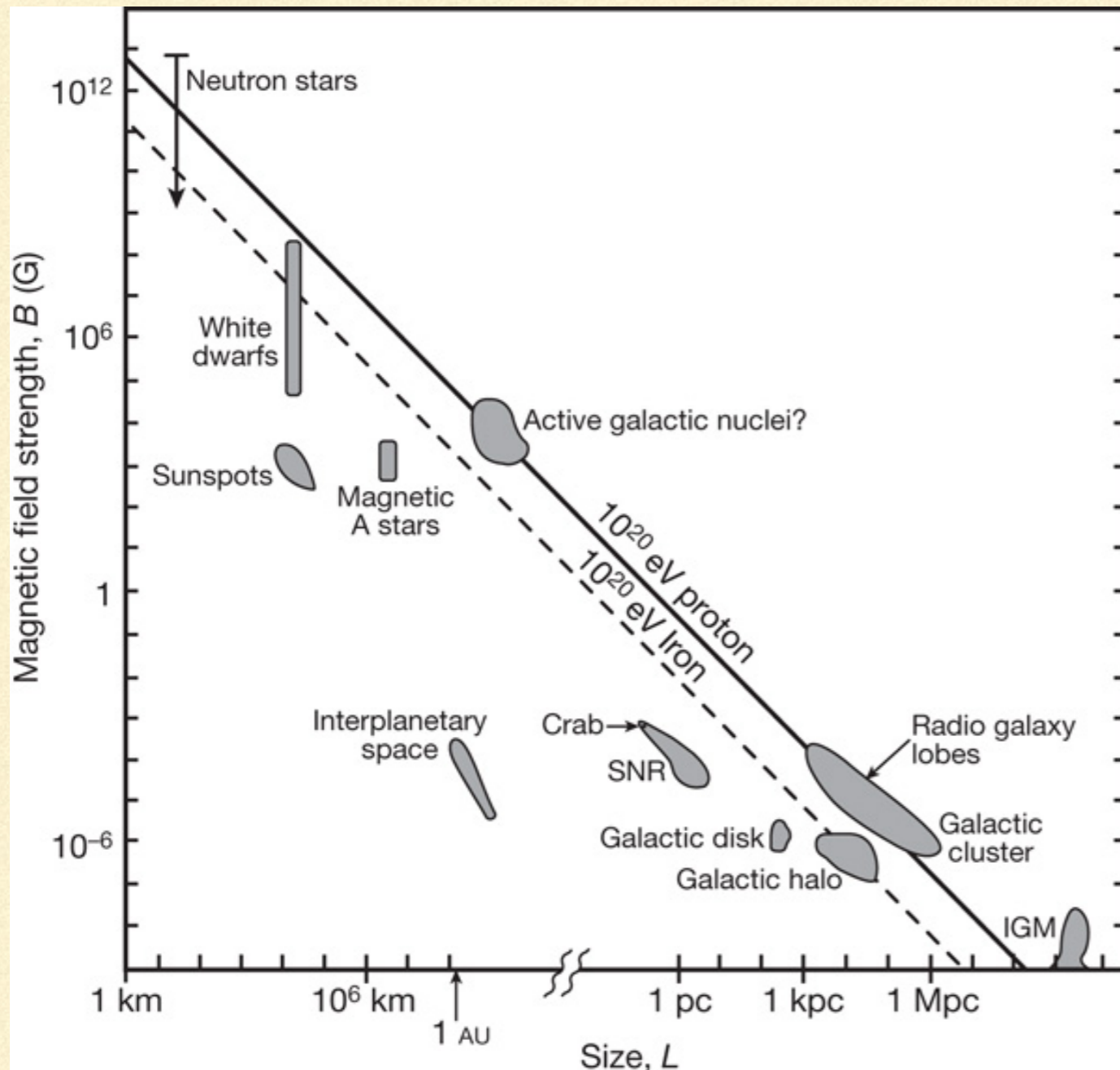


Caprioli&Spitkovsky2014



- High magnetization: particles slide along field lines
- Subluminal shock (quasi-parallel): particles can outrun the shock and return upstream → can generate turbulence upstream and allow diffusive shock acceleration
- Superluminal shock (quasi-perpendicular): particles cannot outrun the shock → cannot generate turbulence upstream, no DSA

Maximum energy attainable



$$E_{\max} = zeBUL$$

$$\frac{E_{\max}}{z\beta} = BL$$

- SNR, $B \sim 10^{-10}$ T, $U \sim 10^4 \text{ km s}^{-1}$, $t \sim 10^3 \text{ yr}$, $E_{\max} \sim 10^{14} \text{ eV}$
- Magnetic field amplification, could be $(3-10) \times 10^{-8} \text{ T}$

Hillas diagram, from P. M. Bauleo & J. R. Martino *Nature* **458**, 847-851 (16 April 2009)

Summary and Outlook

- Second order Fermi acceleration: can be important in turbulent medium
- Diffusive shock acceleration (first order Fermi): can be a viable mechanism to produce high energy particles, e.g. galactic cosmic rays. Necessary ingredients are:

- Shock formation and seed particle injection
- Enough turbulence to scatter particles across the shock many times

Parallel shocks are good for electron and ion acceleration, while perpendicular shocks mainly accelerate electrons.

- Active ongoing investigations
 - Energy partition between electrons and ions
 - Cosmic ray escape from shocks
 - How to produce UHECR

Further reading

- Very helpful introductory text:
 - Longair 2011, High energy astrophysics
- Good reviews:
 - Blandford, R., & Eichler, D. 1987, Physics Reports, 154, 1
 - Drury, L. O. 1983, Reports on Progress in Physics, 46, 973
- PIC and hybrid simulations
 - Caprioli, D., & Spitkovsky, A. 2014a, ApJ, 783, 91
 - Caprioli, D., & Spitkovsky, A. 2014b, ApJ, 794, 47
 - Caprioli, D., & Spitkovsky, A. 2014c, ApJ, 794, 46
 - Park, J., Caprioli, D., & Spitkovsky, A. 2015, PRL, 114, 85003
 - Sironi, L., & Spitkovsky, A. 2009, ApJ, 698, 1523
 - Sironi, L., & Spitkovsky, A. 2011, ApJ, 726, 75
 - Spitkovsky, A. 2008a, ApJL, 673, L39
 - Spitkovsky, A. 2008b, ApJL, 682, L5