

# Online Multi-Objective Optimization Using Gaussian Processes

Ryan Roussel, Adi Hanuka, Auralee Edelen, Luis Hidalgo

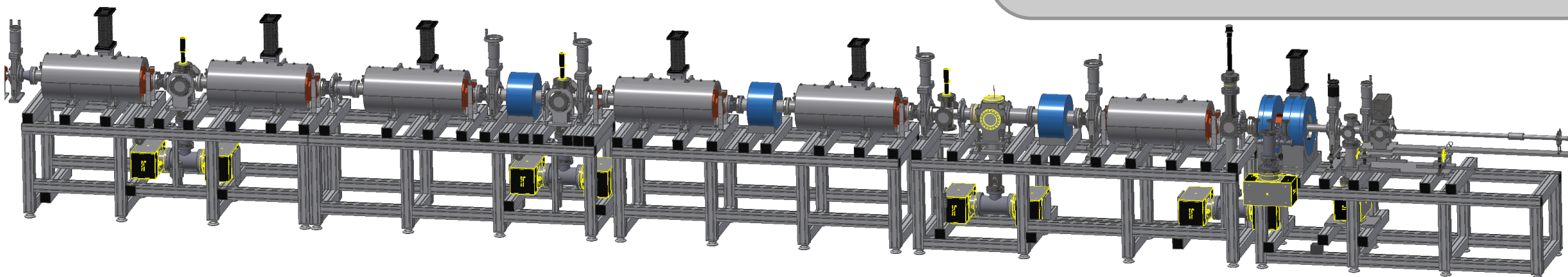
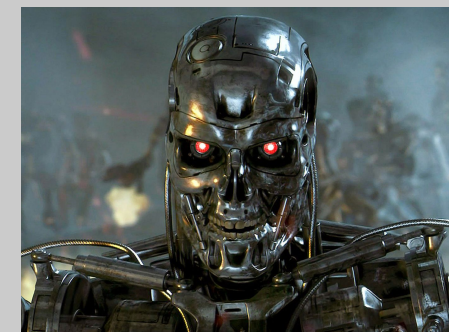


# Handing off Control/Tuning to the Machines



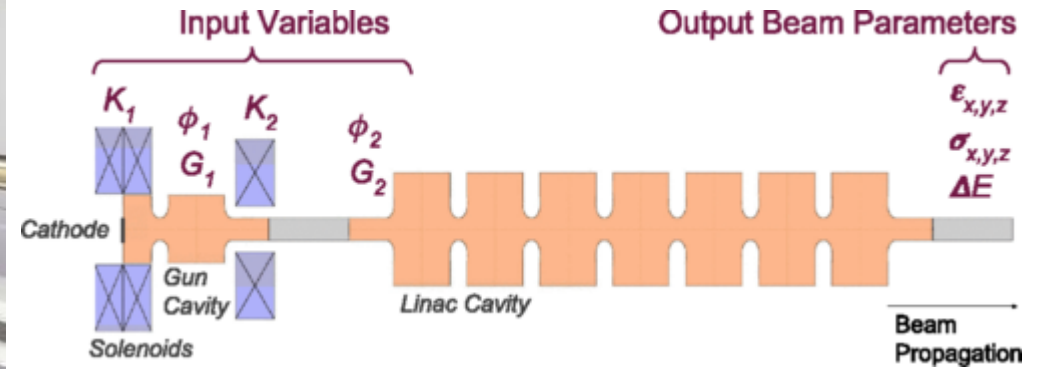
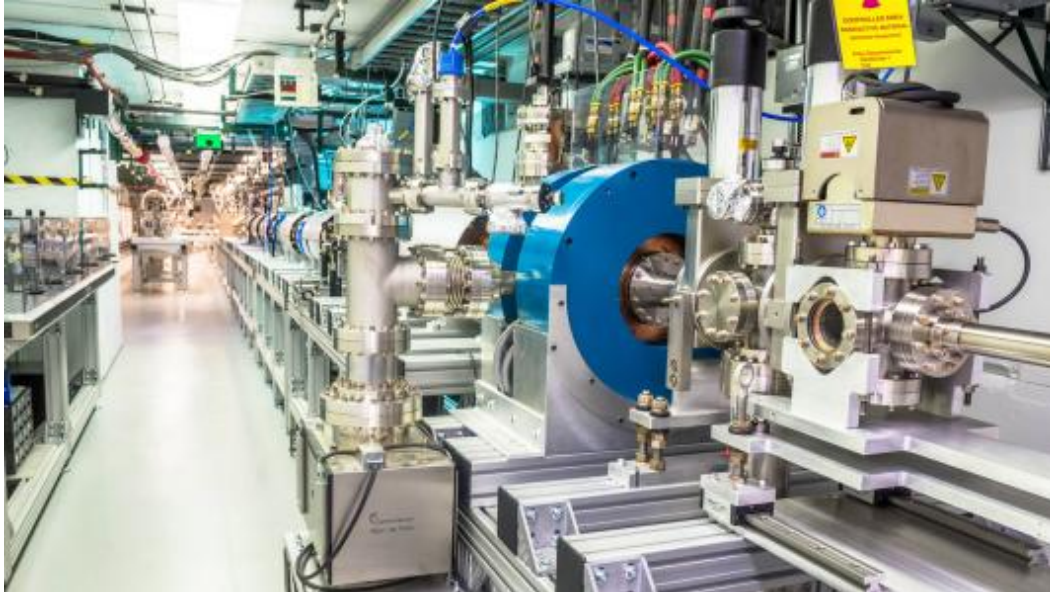
What do we want from a fully (or almost) autonomous system for controlling an accelerator?

- Able to optimize towards **multiple objectives** at once
- Satisfy multiple operating **constraints**
- **Quickly** find reasonable solutions in extremely high dimensional systems (> 30D, < 1 hr tuning time)
- Handle parameter/observation **noise and drift**
- **Learn** from past experiments and simulations





# Photoinjector Optimization



For online photoinjector optimization we wish to simultaneously:

- Minimize **emittances (3x)**
- Minimize **bunch sizes (3x)**
- Minimize **energy spread (1x)**

**7 objectives**

Tuning knobs:

- Solenoid strengths (**2x**)
- RF Amplitudes (**2x**)
- RF Phases (**2x**)

**6 input parameters**

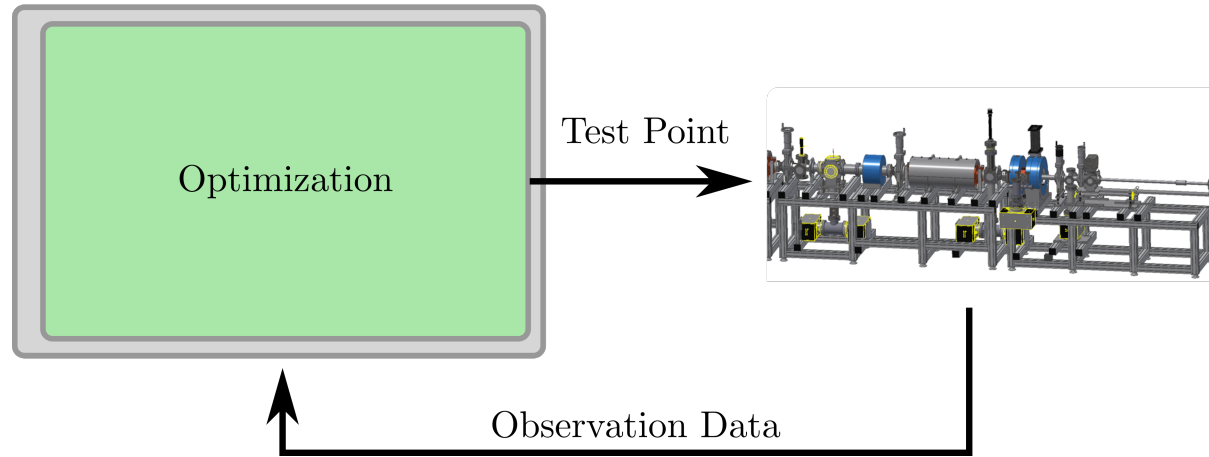
Edelen, Auralee, et al. PRAB 23.4 (2020): 044601.



# Normal Accelerator Optimization

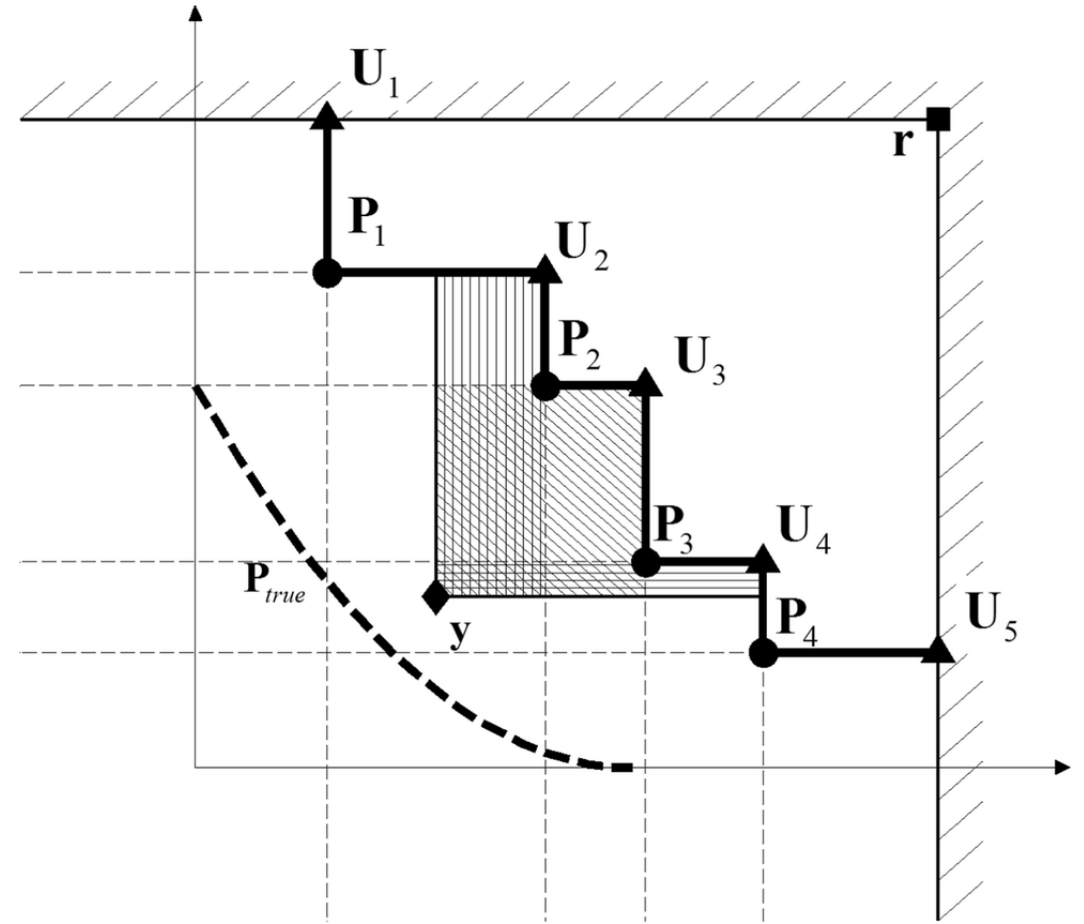
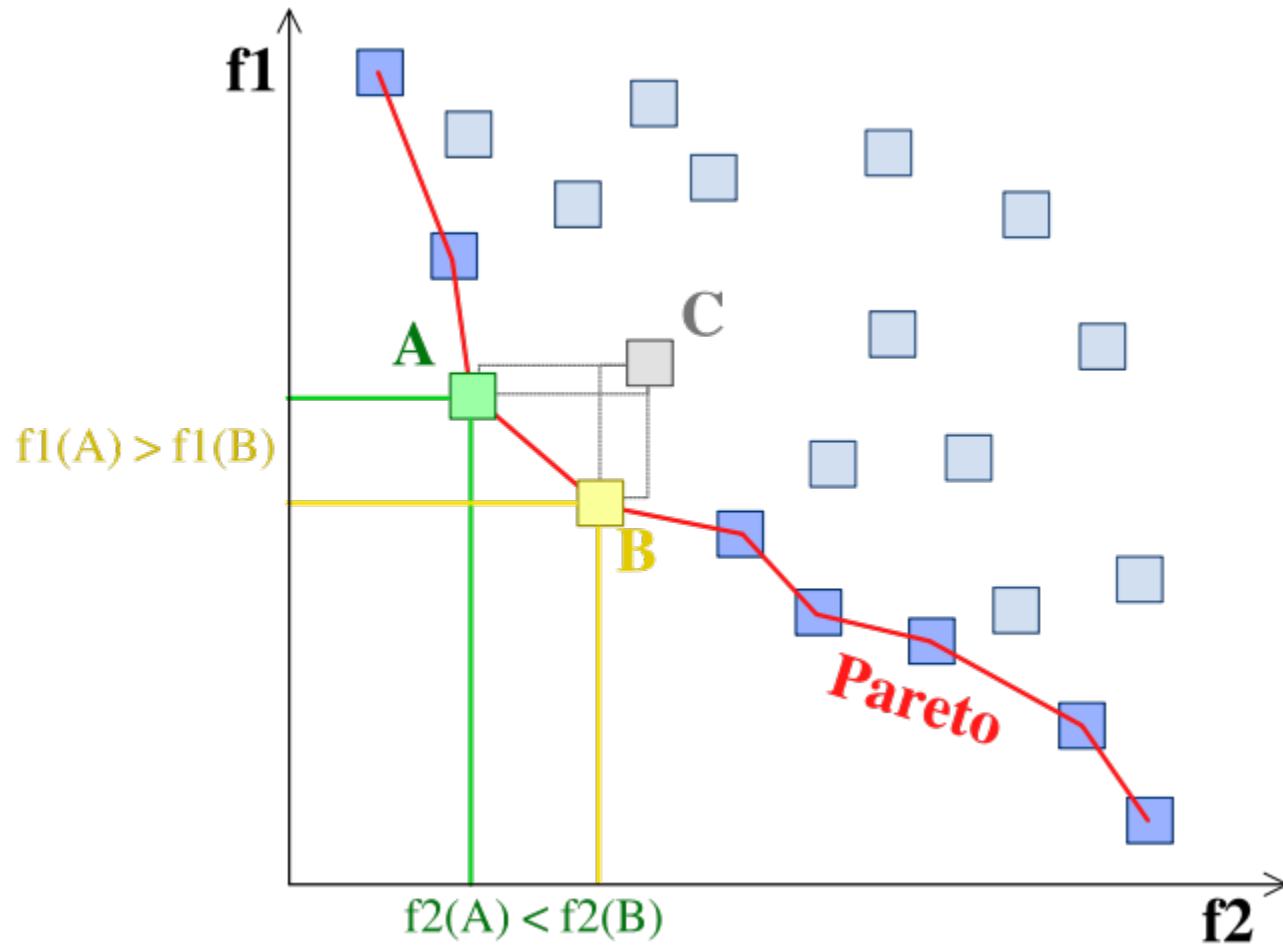


## Normal Optimization Algorithm





# Multiple Objectives: Pareto Front



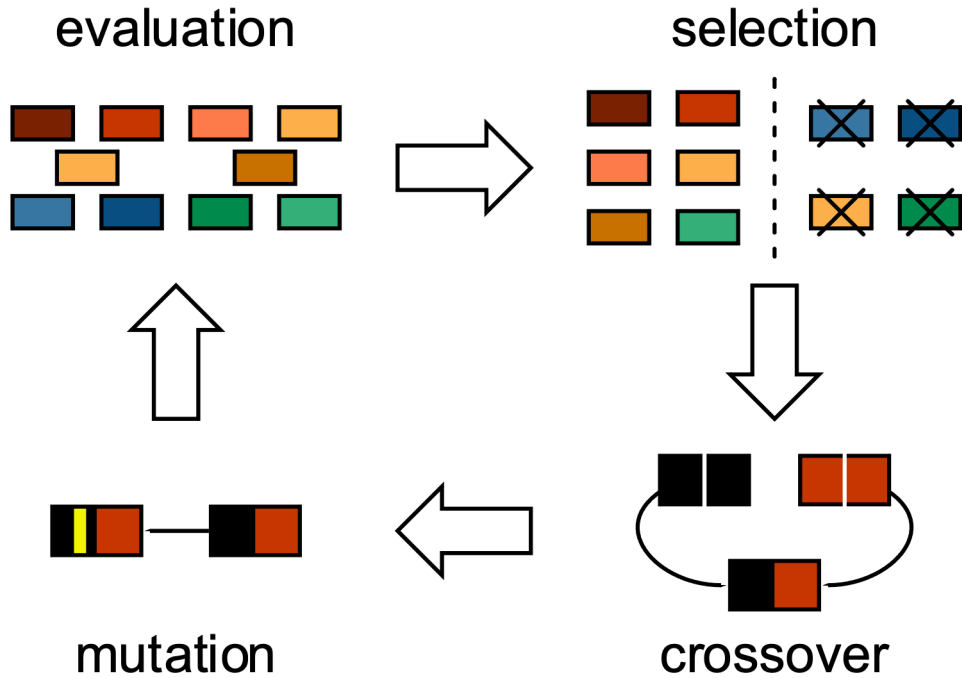
Li, Zheng, et al. *Structural and Multidisciplinary Optimization* 58.5 (2018): 1961-1979.



# Genetic Optimization of PF

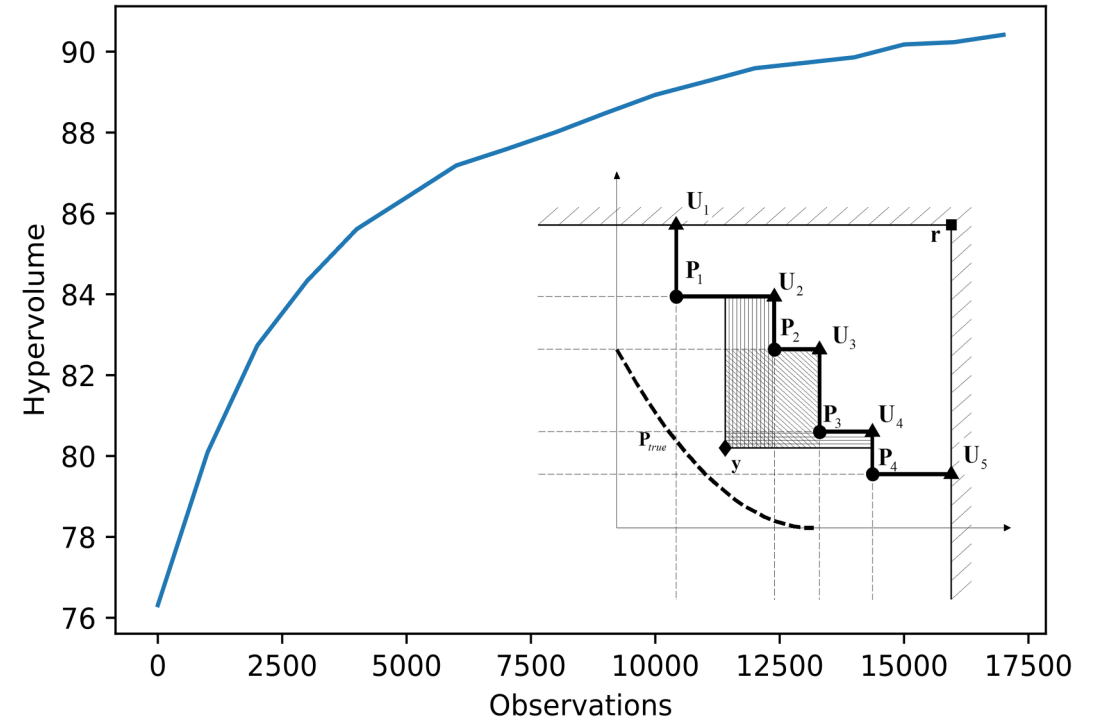


## NSGA-II



<https://www.strong.io/blog/evolutionary-optimization>

175 generations, 100 individuals



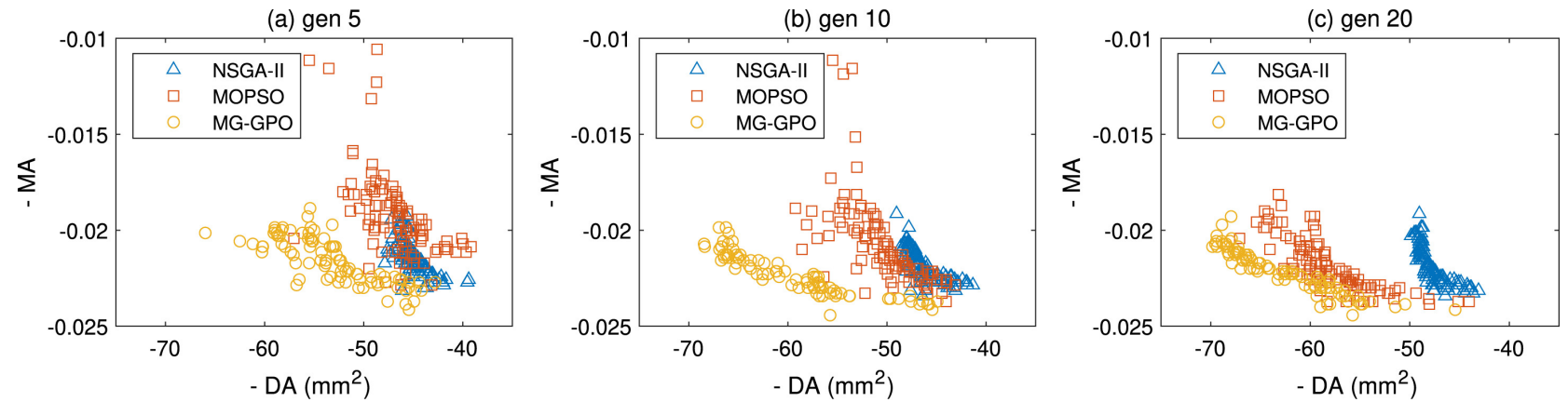
At 5 seconds per observation, optimization time > 24 hrs



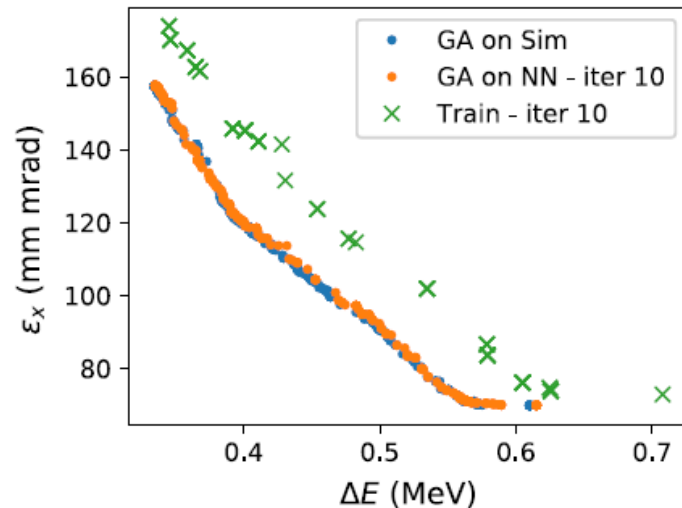
# Hybrid Genetic Optimization at SLAC



NSGA-II + Gaussian processes to evaluate GA created candidates



Song, Minghao, et al. NIMA (2020): 164273.



Edelen, Auralee, et al. PRAB 23.4 (2020): 044601.

Iterated neural network optimized by NSGA-II to propose ideal points

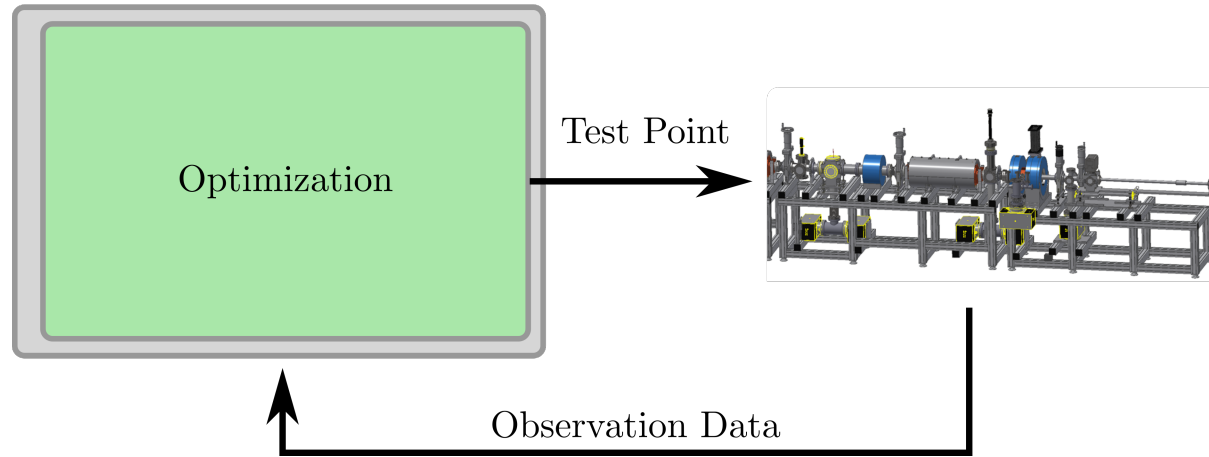
**Large improvements** in optimization speedup, but both methods rely on **batch observations**, suited best for **parallel optimization**



# Accelerator Optimization w/Surrogates



## Normal Optimization Algorithm



Can we do the same optimization with **less observations**?  
Need an **efficient** method!

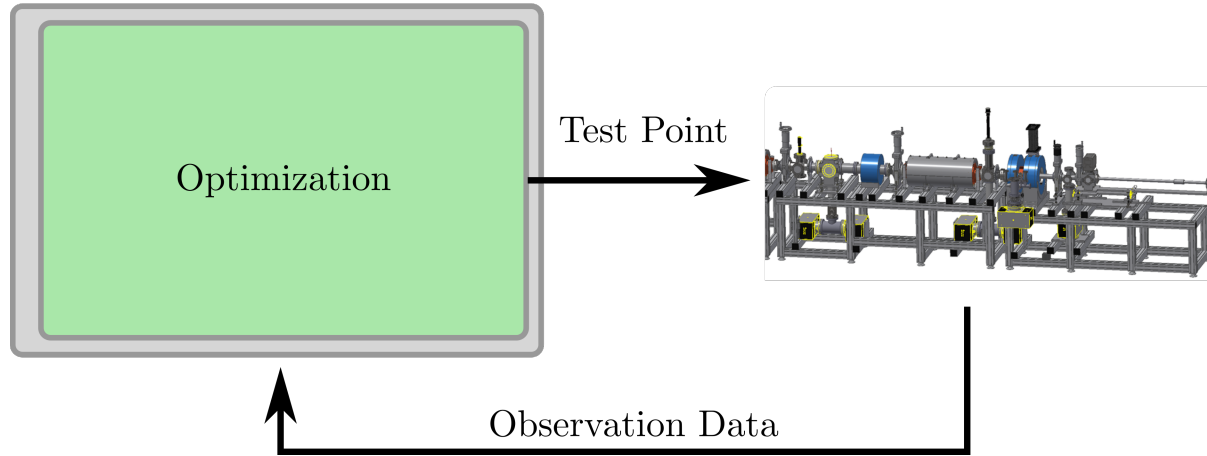




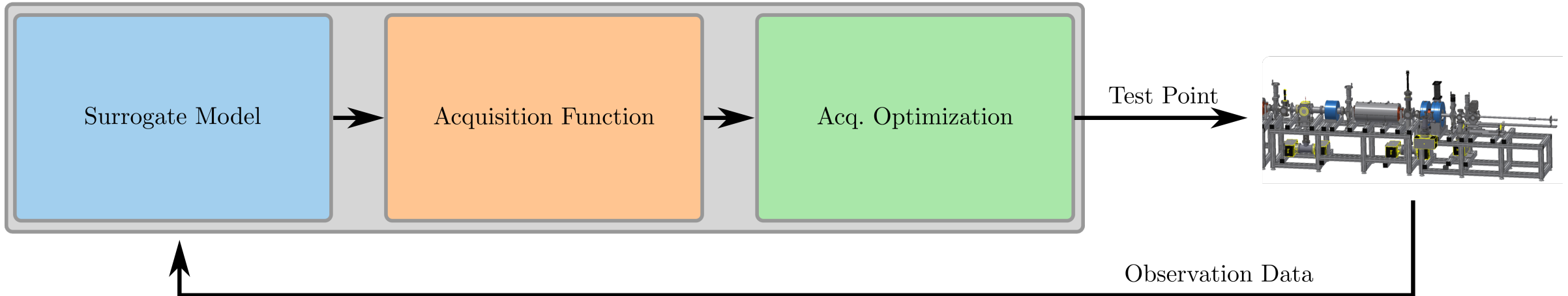
# Accelerator Optimization w/Surrogates



### Normal Optimization Algorithm

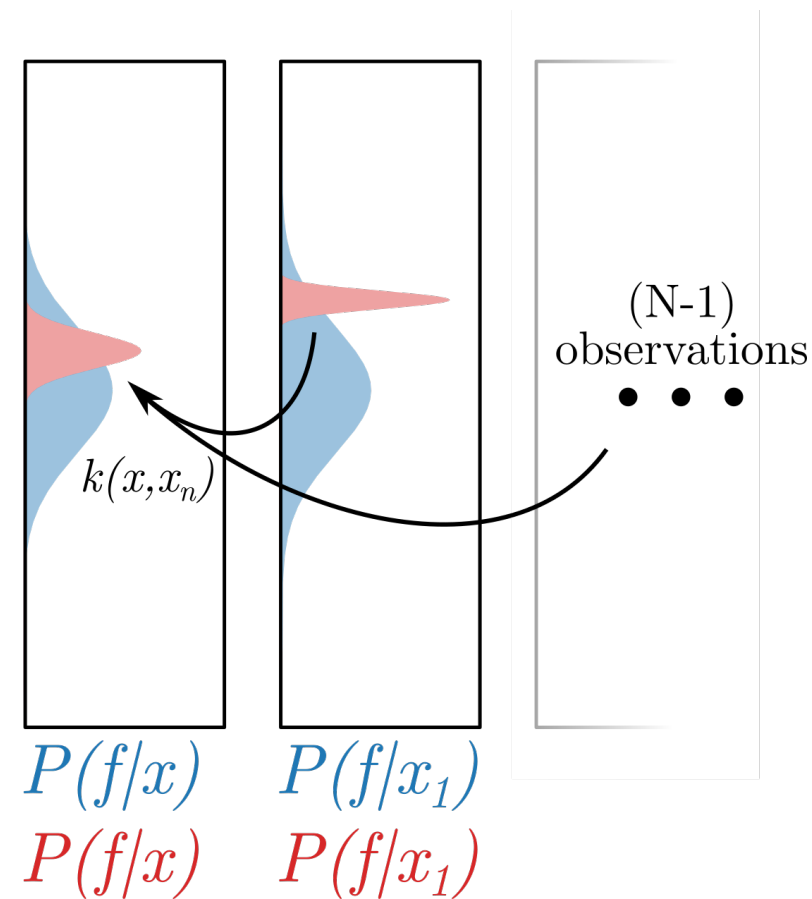
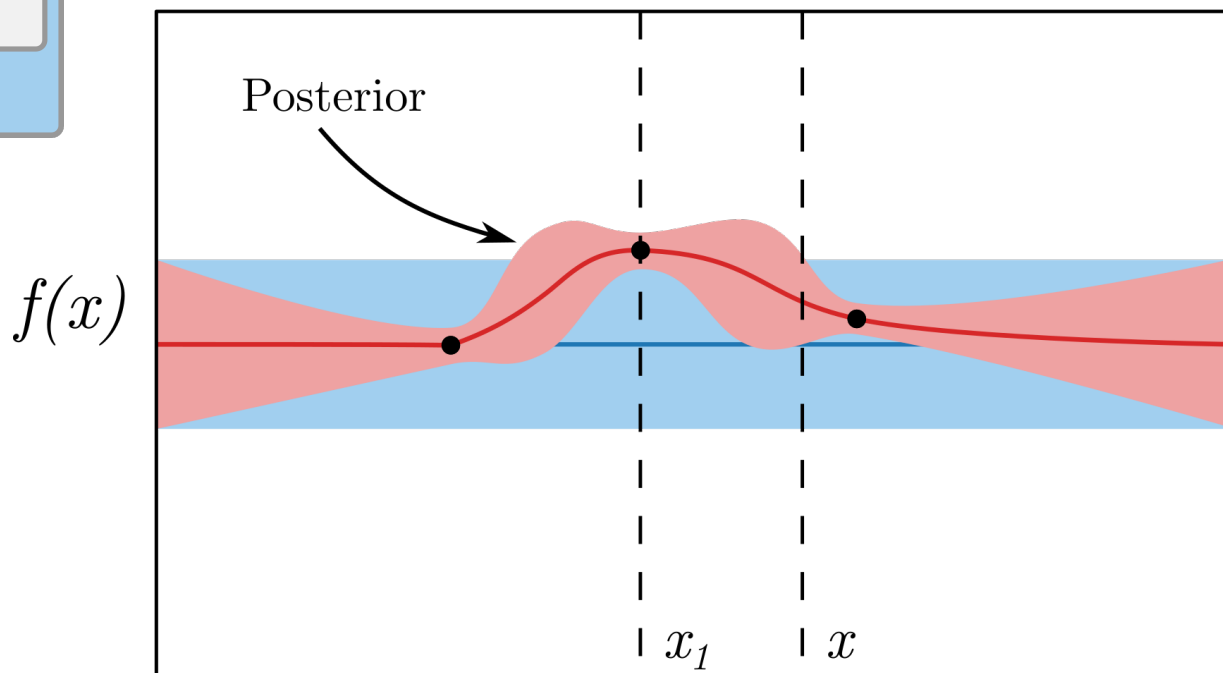
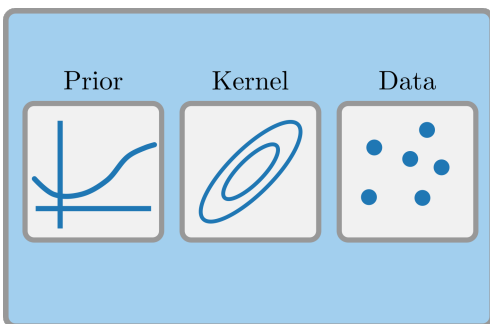


### Bayesian Optimization Algorithm





# Gaussian Processes as a Surrogate



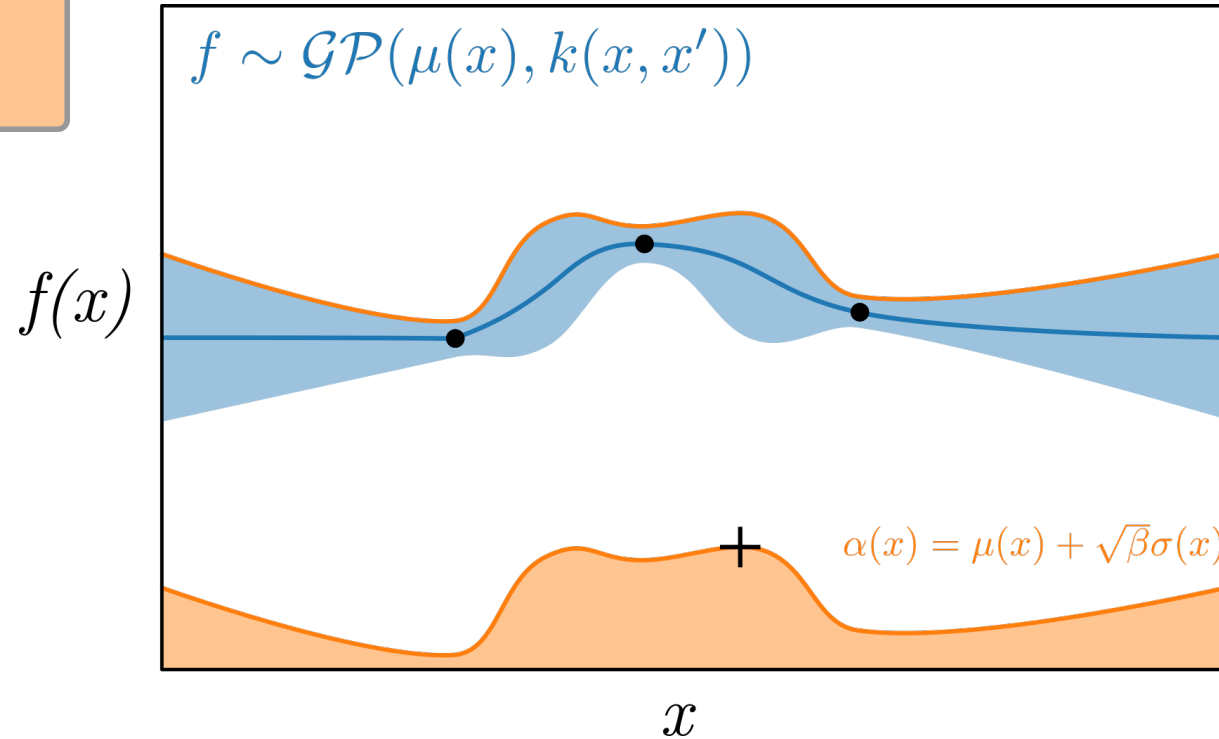
$$k(x, x_n) = \sigma e^{-\frac{|x-x_n|^2}{2\lambda^2}}$$



# Gaussian Processes for Optimization

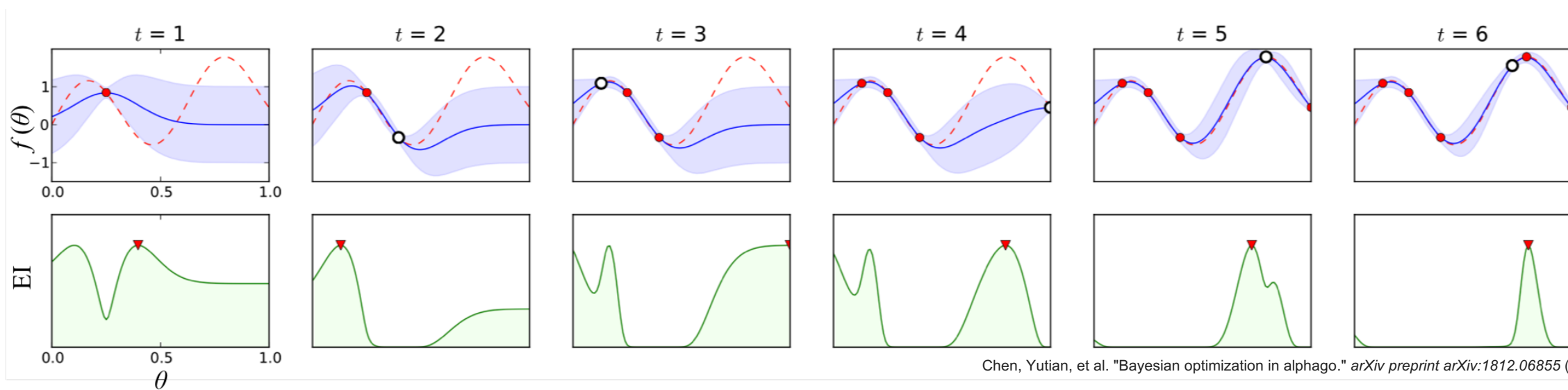


Acquisition Function





# Gaussian Processes for Optimization



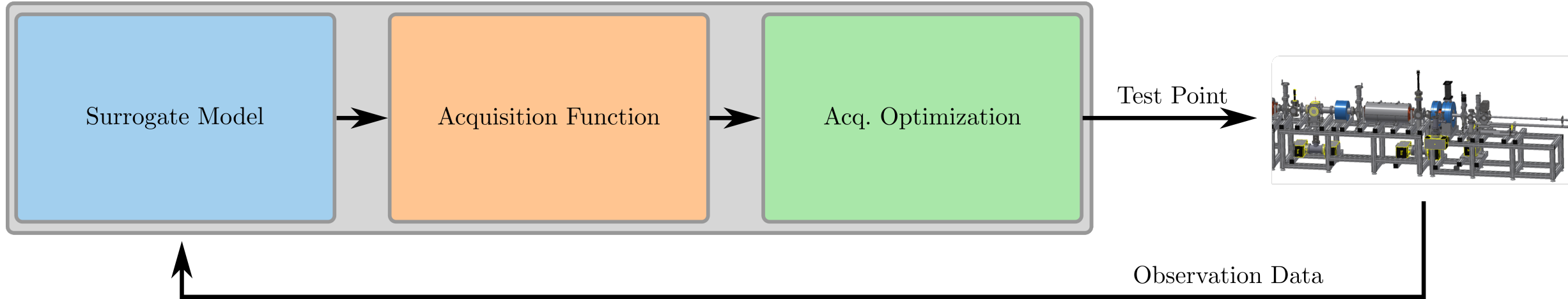
Chen, Yutian, et al. "Bayesian optimization in alphago." *arXiv preprint arXiv:1812.06855* (2018).



# Gaussian Processes for Optimization

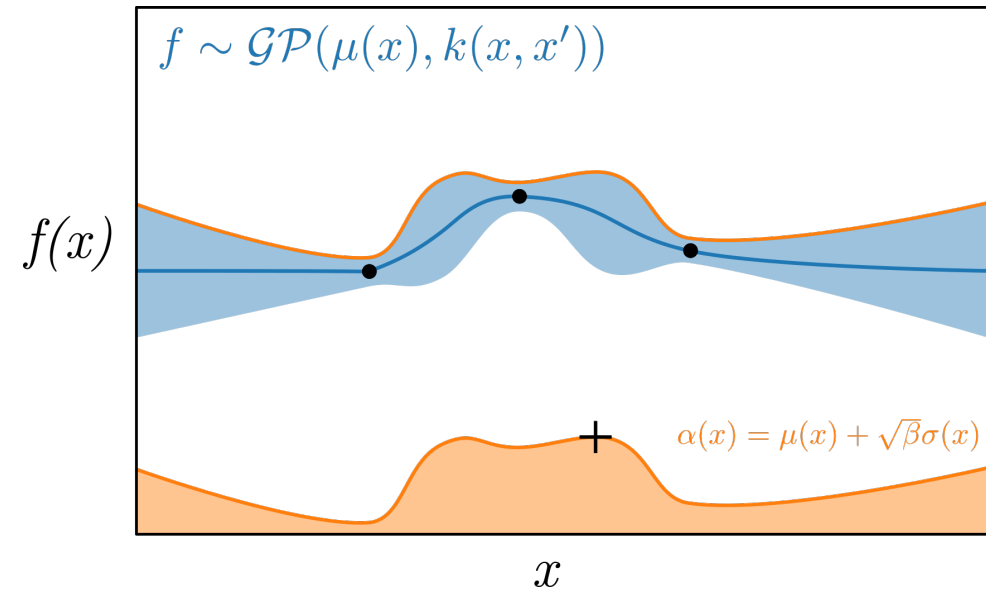


## Bayesian Optimization Algorithm



### Benefits:

- Specify tradeoff between **exploration** and **exploitation**
- Inherently **improves model accuracy** in **regions of interest**
- Enables parallel optimization strategies

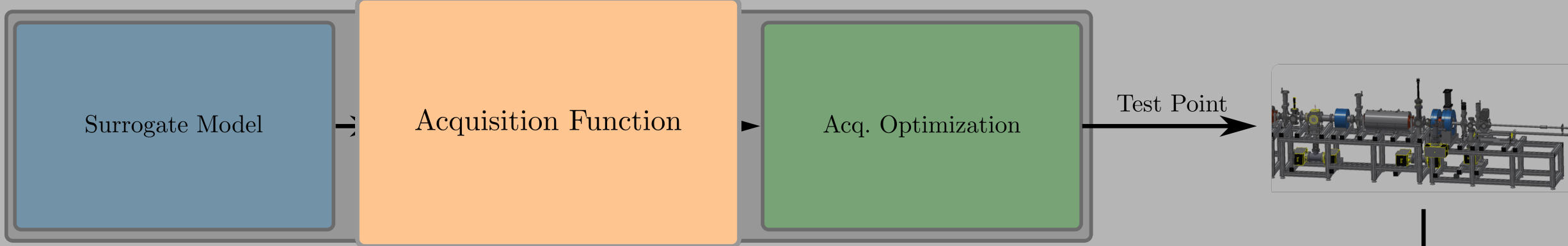




# Gaussian Processes for Optimization



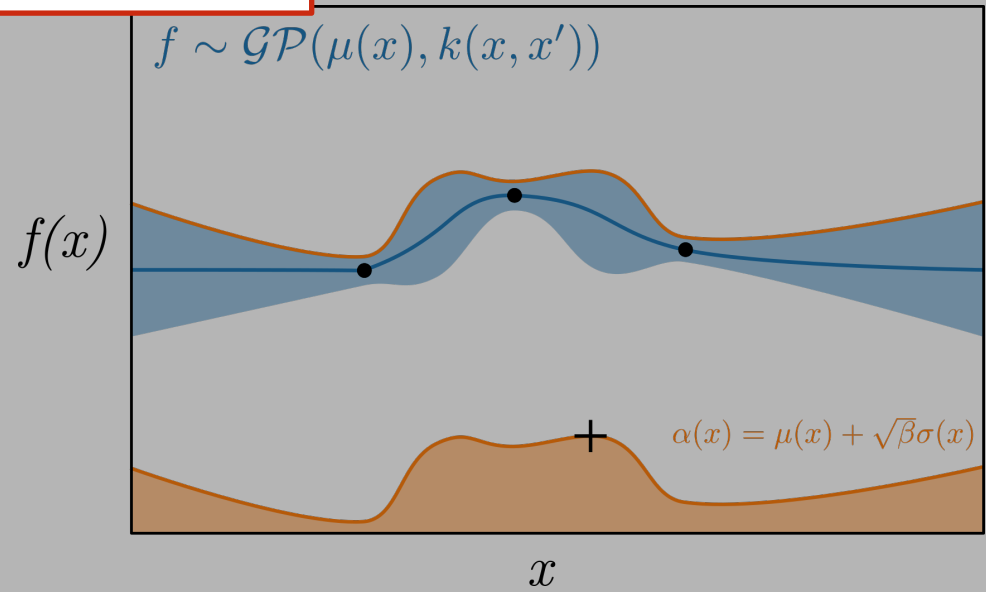
## Bayesian Optimization Algorithm



**Adapt this for multi-objective problems**

Benefits:

- Specify tradeoff between **exploration** and **exploitation**
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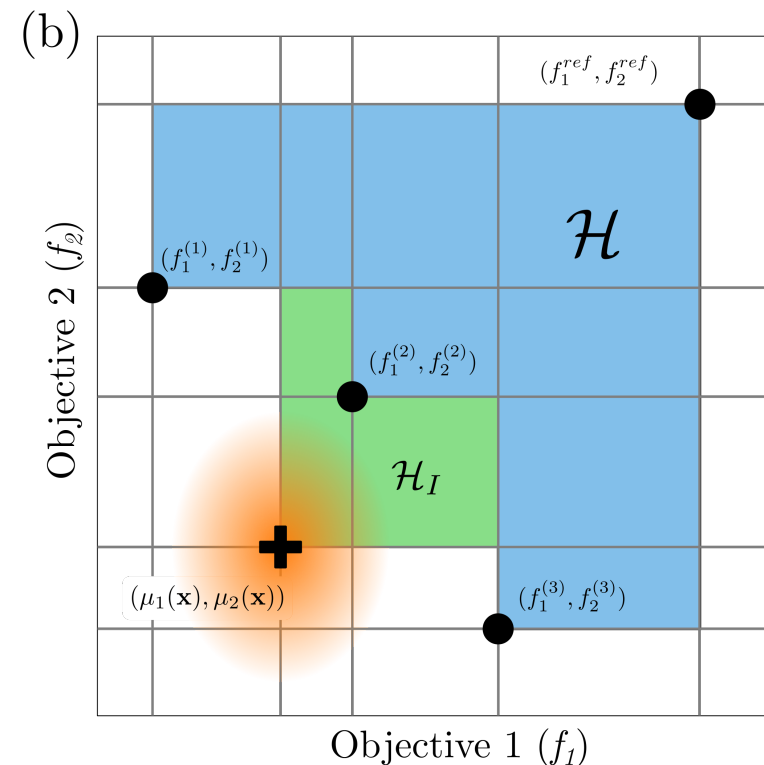
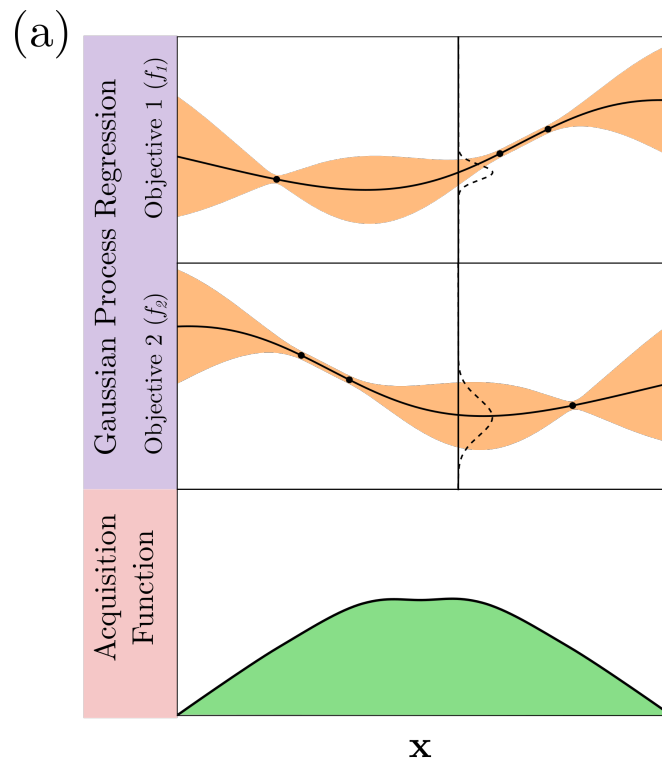
# Extension to Multiple Objective Optimization



We wish to find the set of **Pareto-optimal points** in objective space by **maximizing** the contained **Pareto front hypervolume**

- Each objective has a GP surrogate model
- Using the surrogates we calculate the **Expected Hypervolume Improvement (EHVI)** as a function of the input
- Find a point that maximizes the EHVI and use as our next measurement point

This allows us to find the Pareto front **w/ a small number of measurements** unlike genetic or swarm optimization methods



$$\alpha_{EHVI}(\boldsymbol{\mu}, \boldsymbol{\sigma}, \mathcal{P}, \mathbf{r}) := \int_{\mathbb{R}^P} \text{HVI}(\mathcal{P}, \mathbf{y}, \mathbf{r}) \cdot \xi_{\boldsymbol{\mu}, \boldsymbol{\sigma}}(\mathbf{y}) d\mathbf{y}$$

Yang, Kaifeng, et al. *Swarm and evolutionary computation* 44 (2019): 945-95

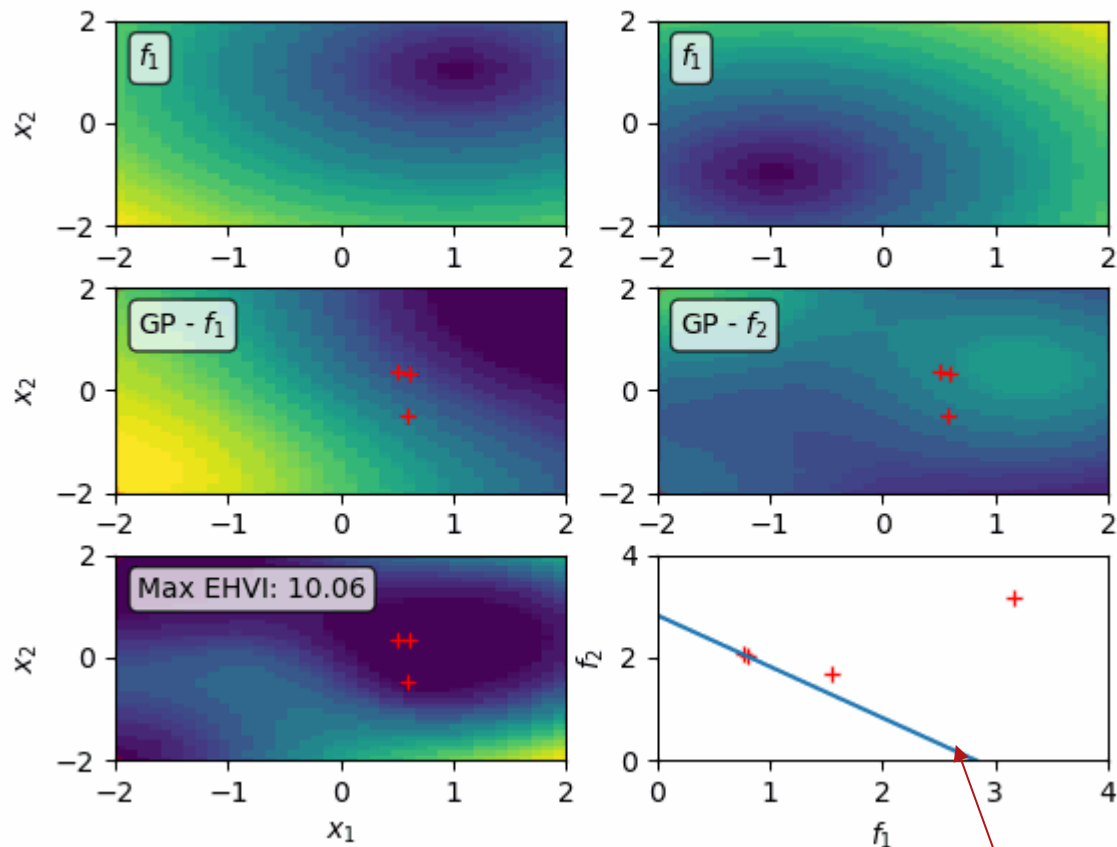


# Two Objective Example Optimization

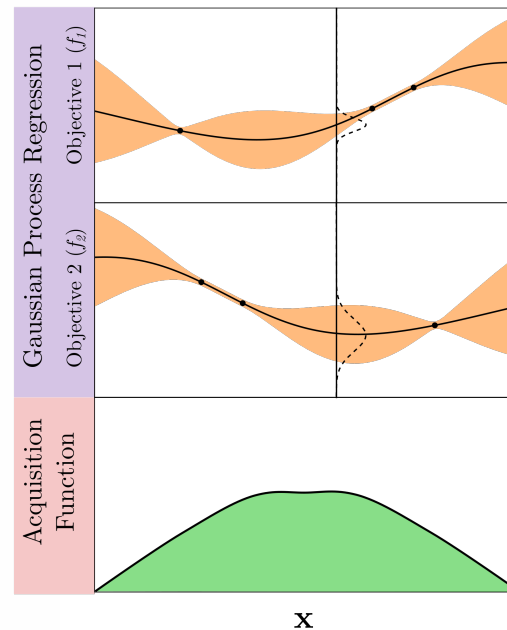


$$f_1 = |\mathbf{x} - (1,1)|^2$$

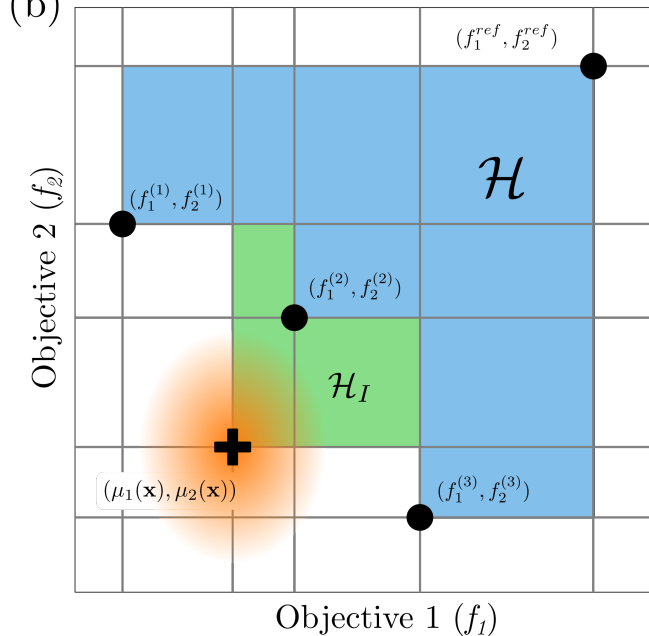
$$f_2 = |\mathbf{x} - (-1,-1)|^2$$



(a)



(b)



Red cross – observation points Analytical Pareto front

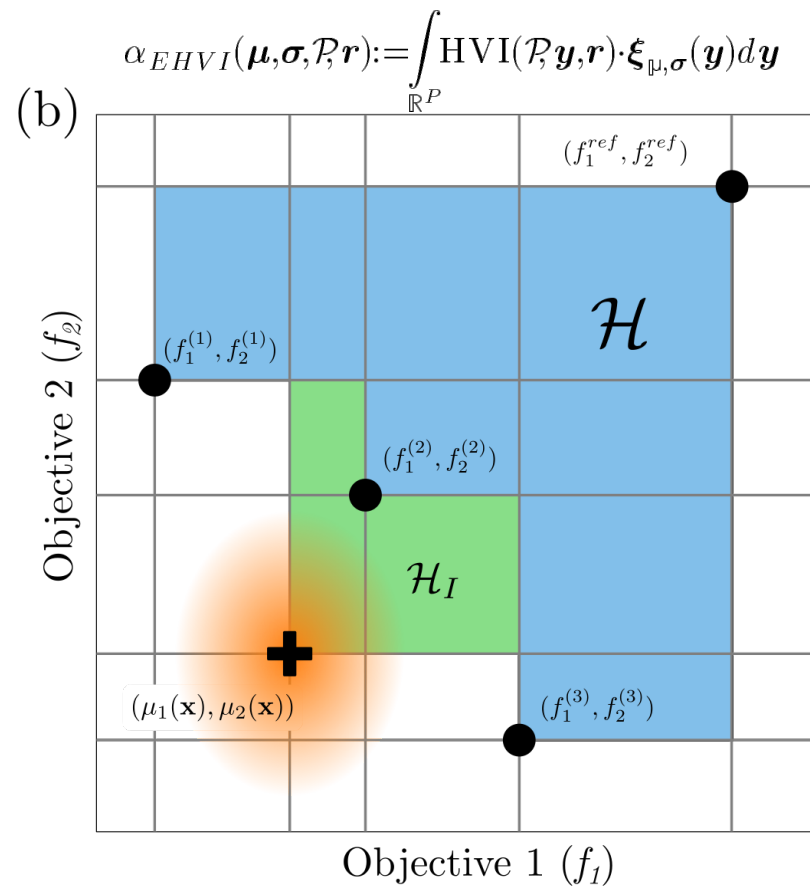
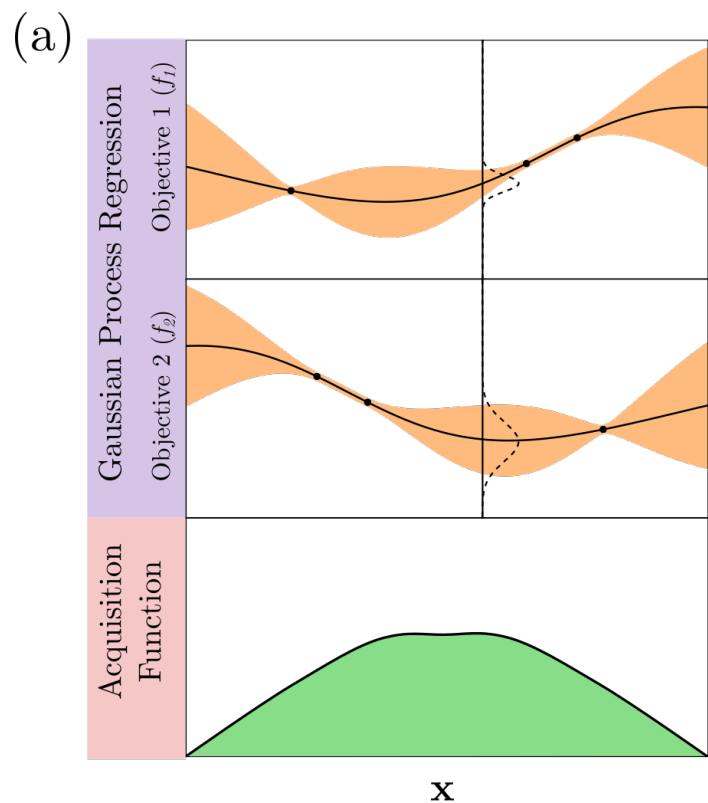




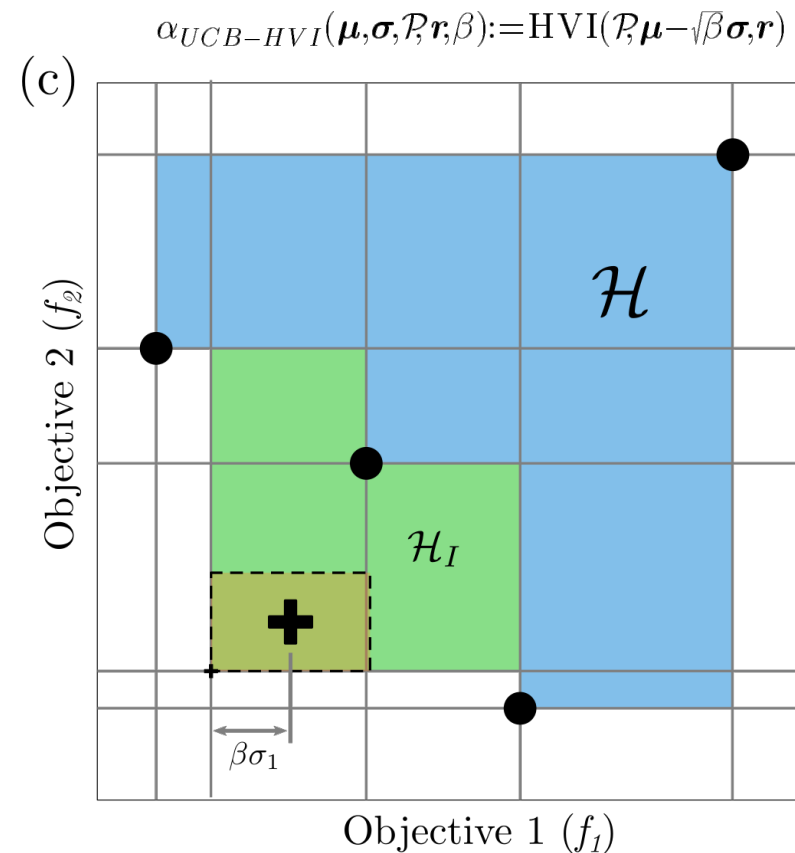
# Alternative Hypervolume Improvement Metrics



Large number of objectives -> too expensive for EHVI calculation  $\sim \mathbf{O(N^D)}$



**Slow! >>**

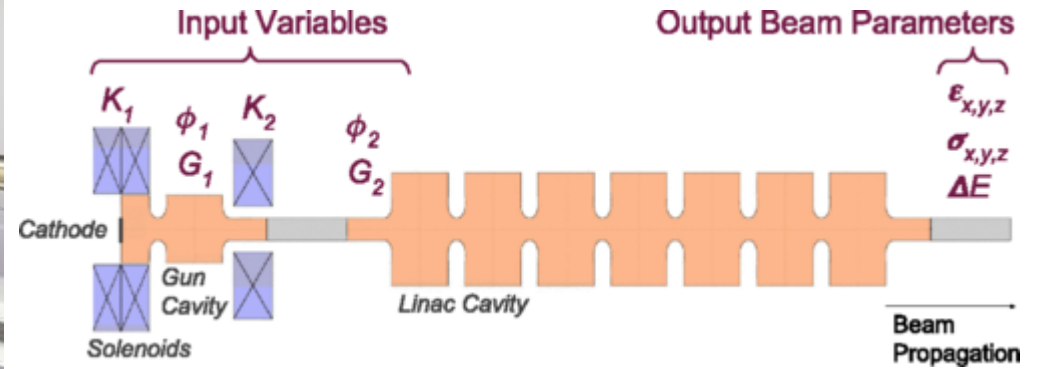
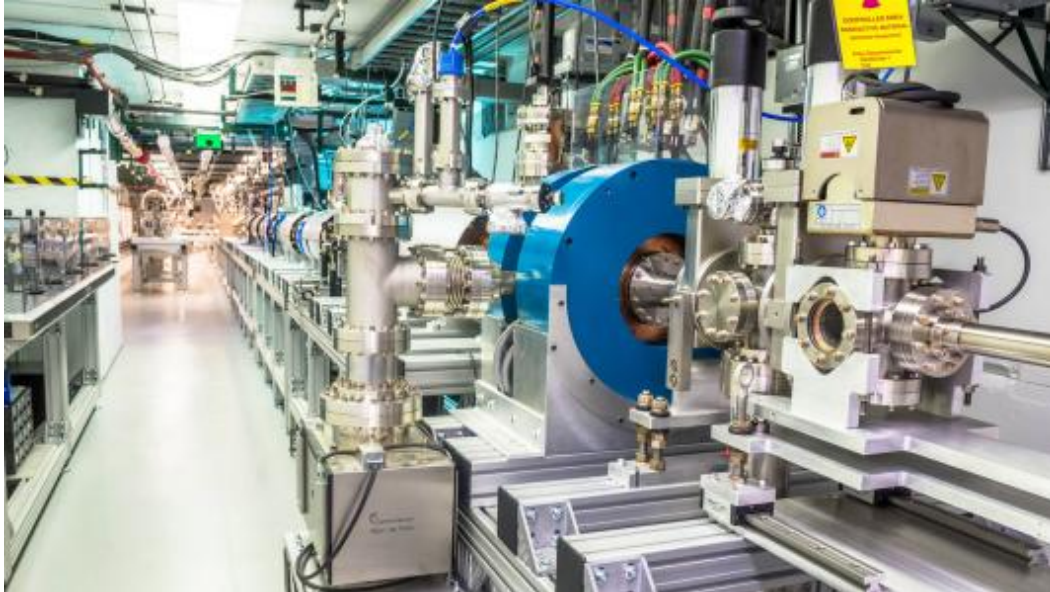


**Fast! >>>>>>>>>>**

While, Lyndon, Lucas Bradstreet, and Luigi Barone. *IEEE Transactions on Evolutionary Computation* 16.1 (2011): 86-95.



# Photoinjector Optimization



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Edelen, Auralee, et al. PRAB 23.4 (2020): 044601.



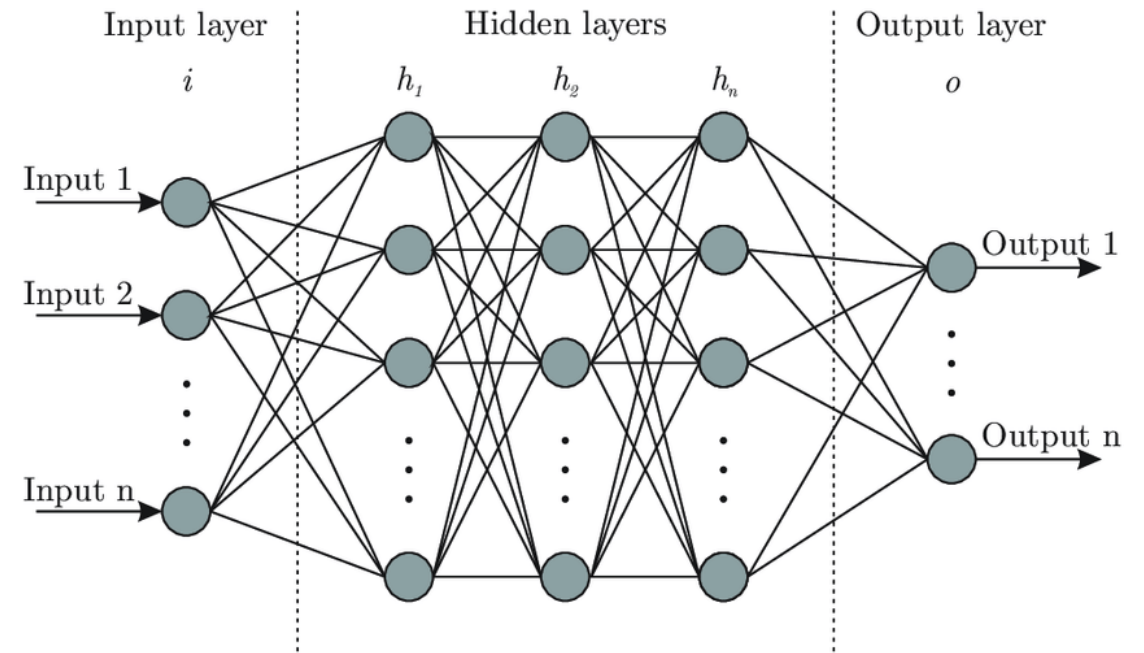
# Simulated Photoinjector Optimization



OPAL simulation of AWA photoinjector  
**6 mins on HPC cluster**



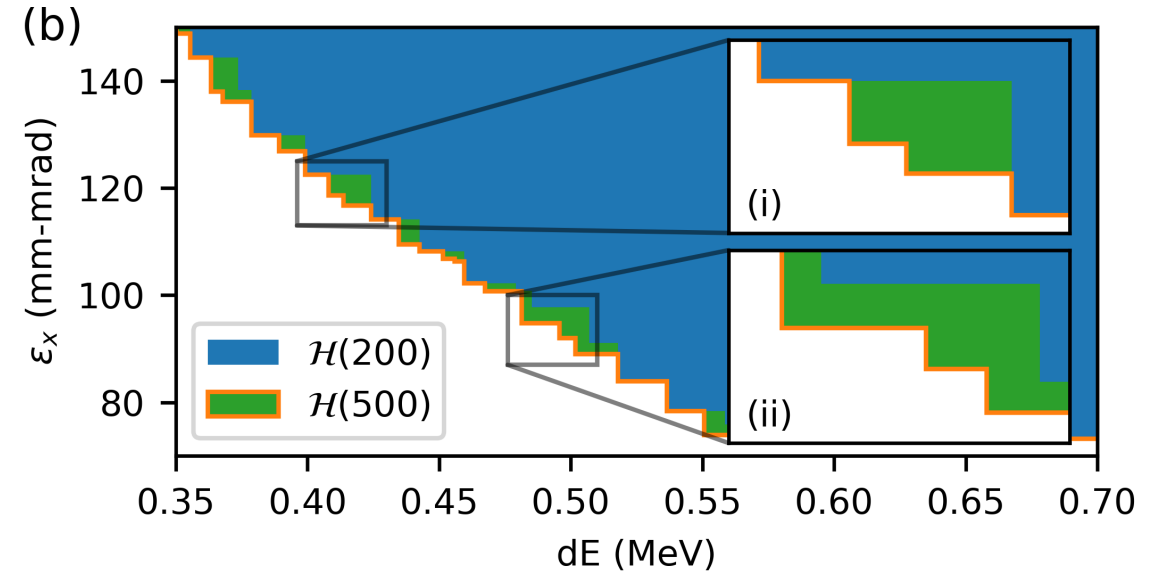
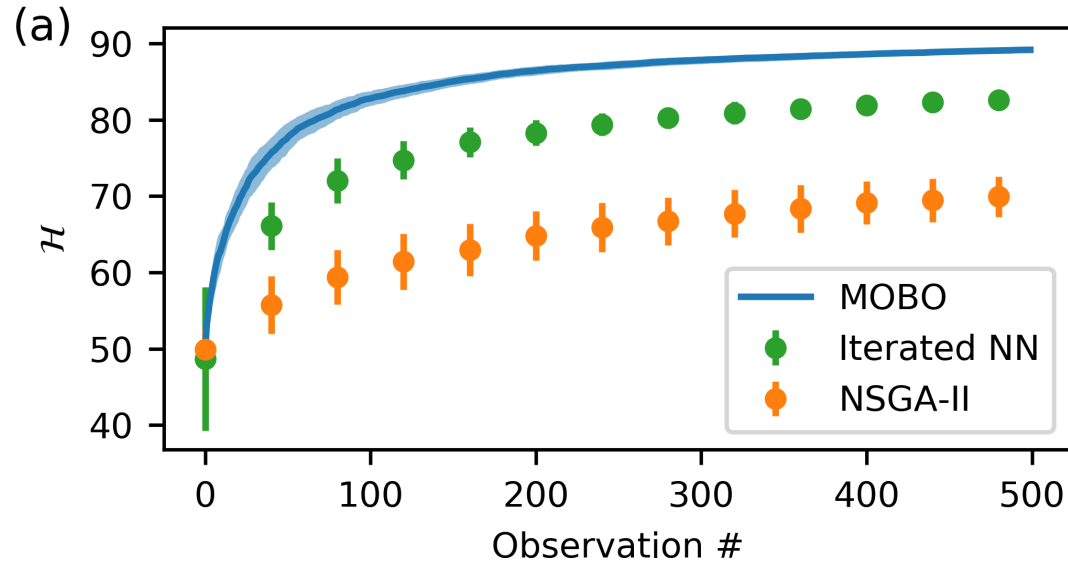
NN surrogate of AWA photoinjector  
**< 1 sec on a laptop**



Edelen, Auralee, et al. PRAB 23.4 (2020): 044601.



# Simulated Photoinjector Optimization



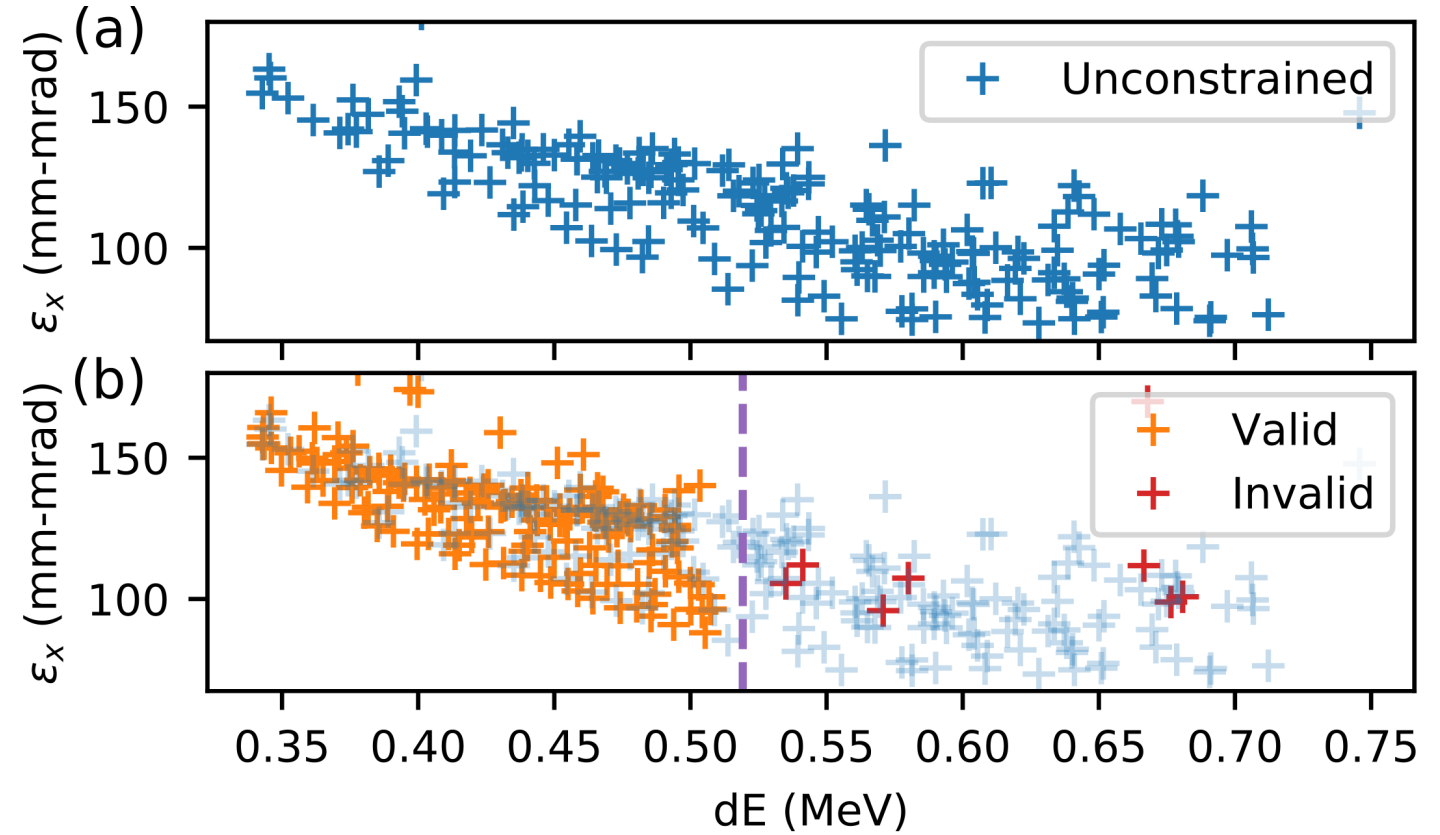
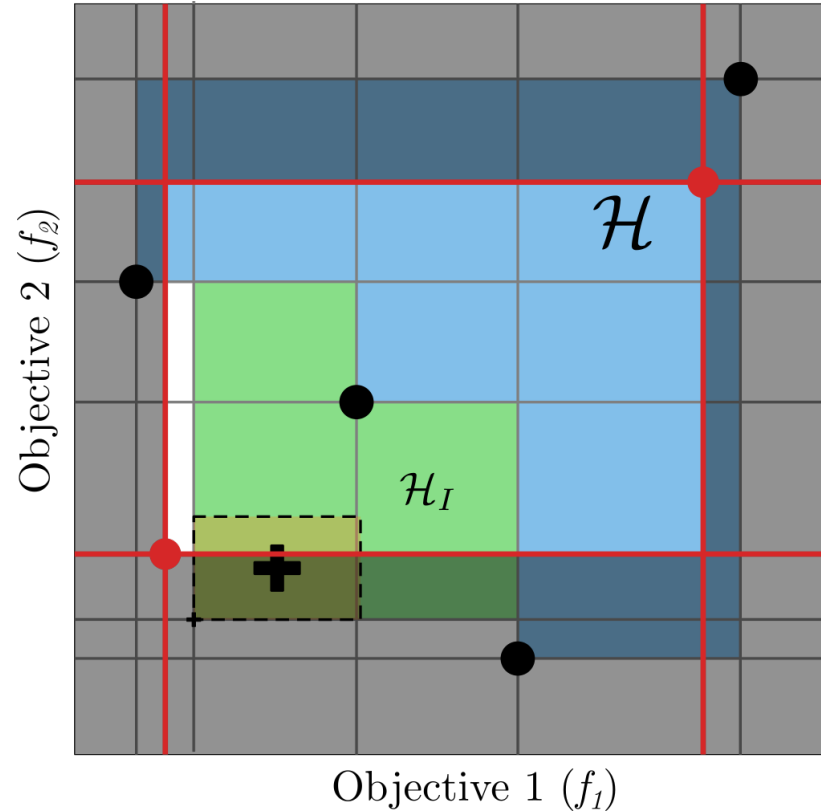
- 10 optimization runs
- 20 initial points each
- Pk hypervolume  $\sim 90$  in  $< 500$  steps (NSGA-II  $\sim 17.5k$ ) factor of 35x speedup, tuned in  $< 45$  mins!



# Adding Objective Preferences



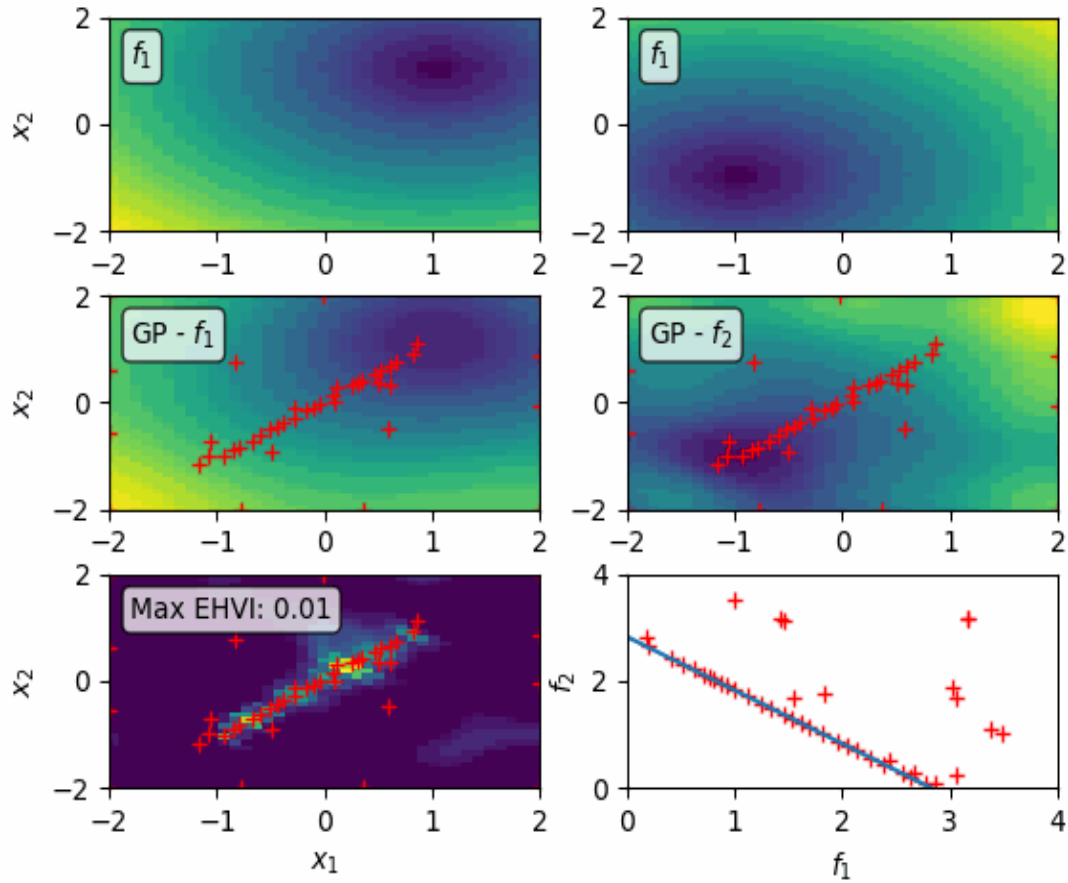
Preferences: only calculate HV inside objective region



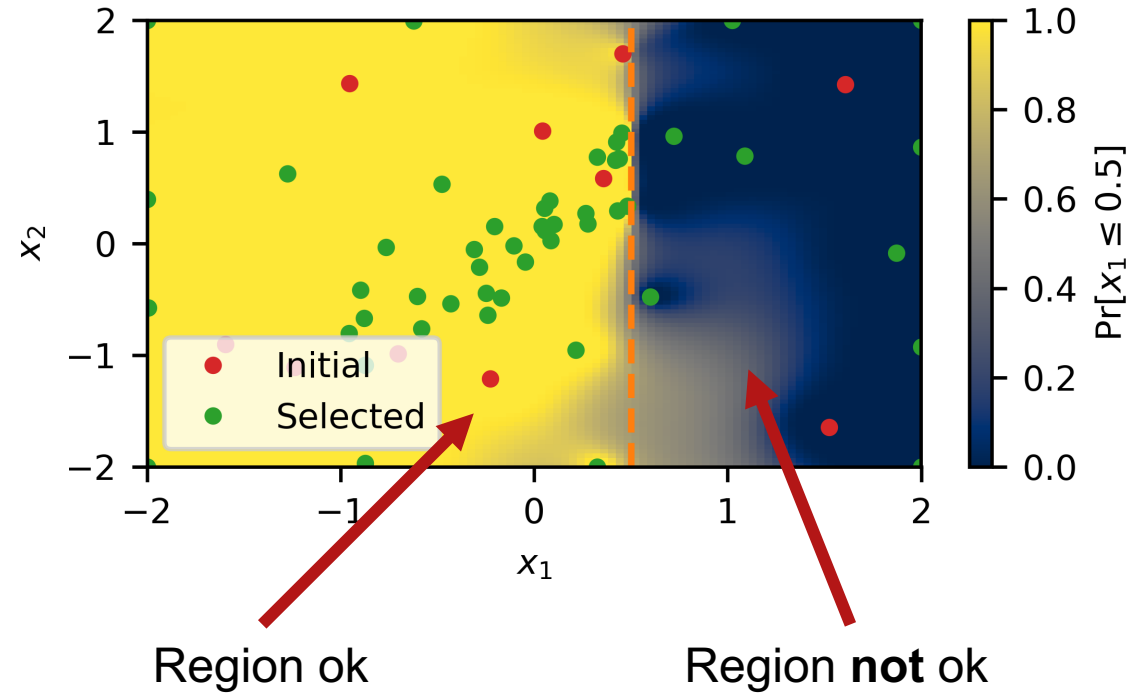
$$\alpha_{TUCB-HVI}(\mu, \sigma, \mathcal{P}, \beta, \mathbf{A}, \mathbf{B}) := \begin{cases} HVI(\mathcal{P}, \mathbf{y}, \mathbf{B}) & \mathbf{y} \in \mathcal{T} \\ 0 & \text{otherwise} \end{cases}$$



# Adding Constraints



Constraints: model probability that a constraint is satisfied



$$P_g(\mathbf{x}) := \Pr[g(\mathbf{x}) \leq h] = \int_{-\infty}^h p(g(\mathbf{x}) | \mathcal{D}_g) dg(\mathbf{x})$$

$$\hat{\alpha}(\mathbf{x}) = \alpha(\mathbf{x}) P_g(\mathbf{x})$$



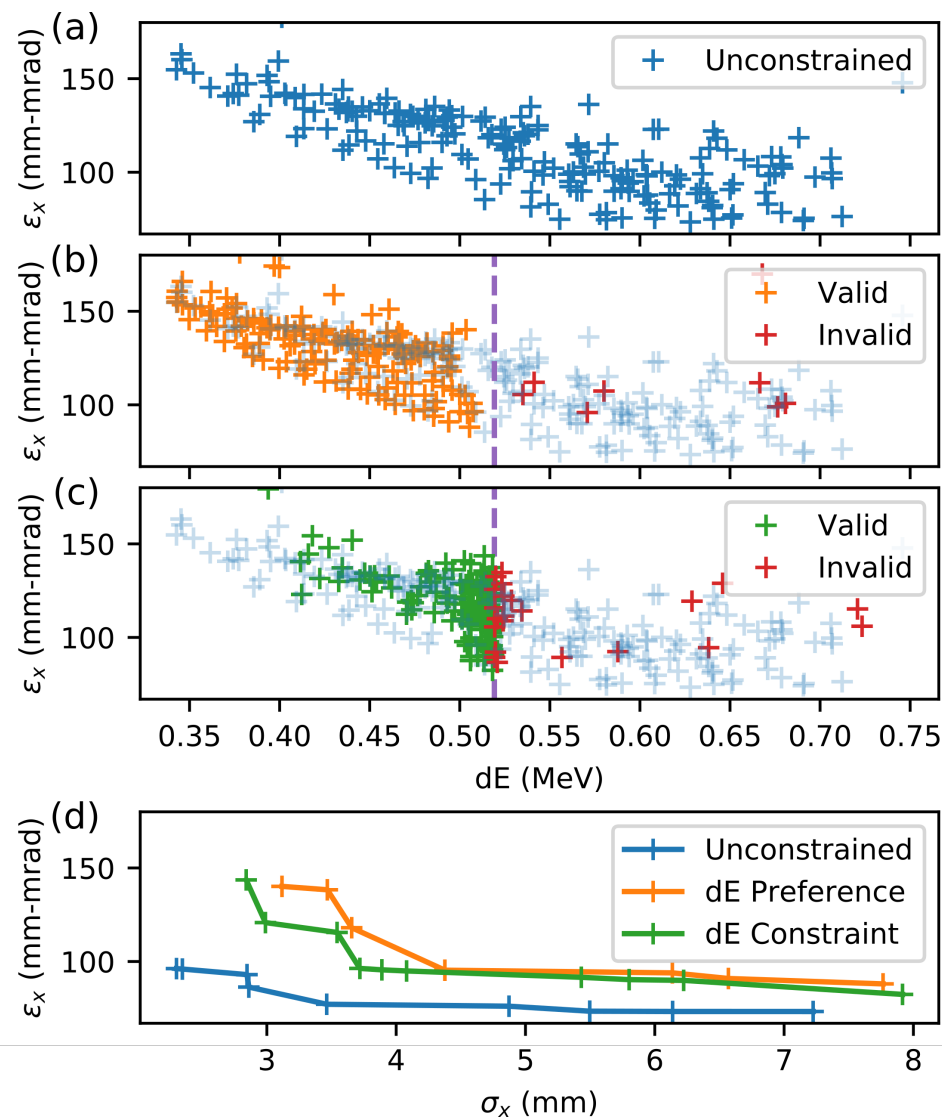
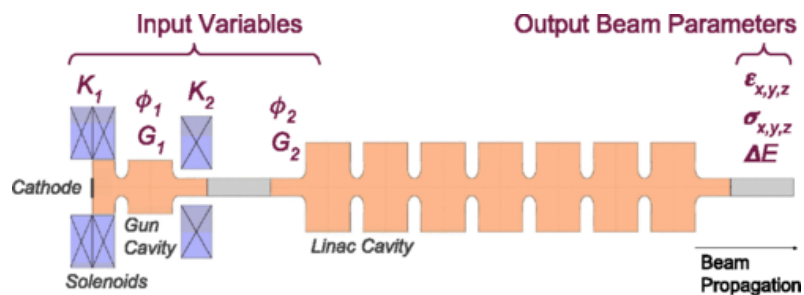
# Adding Preferences vs. Constraints



Control

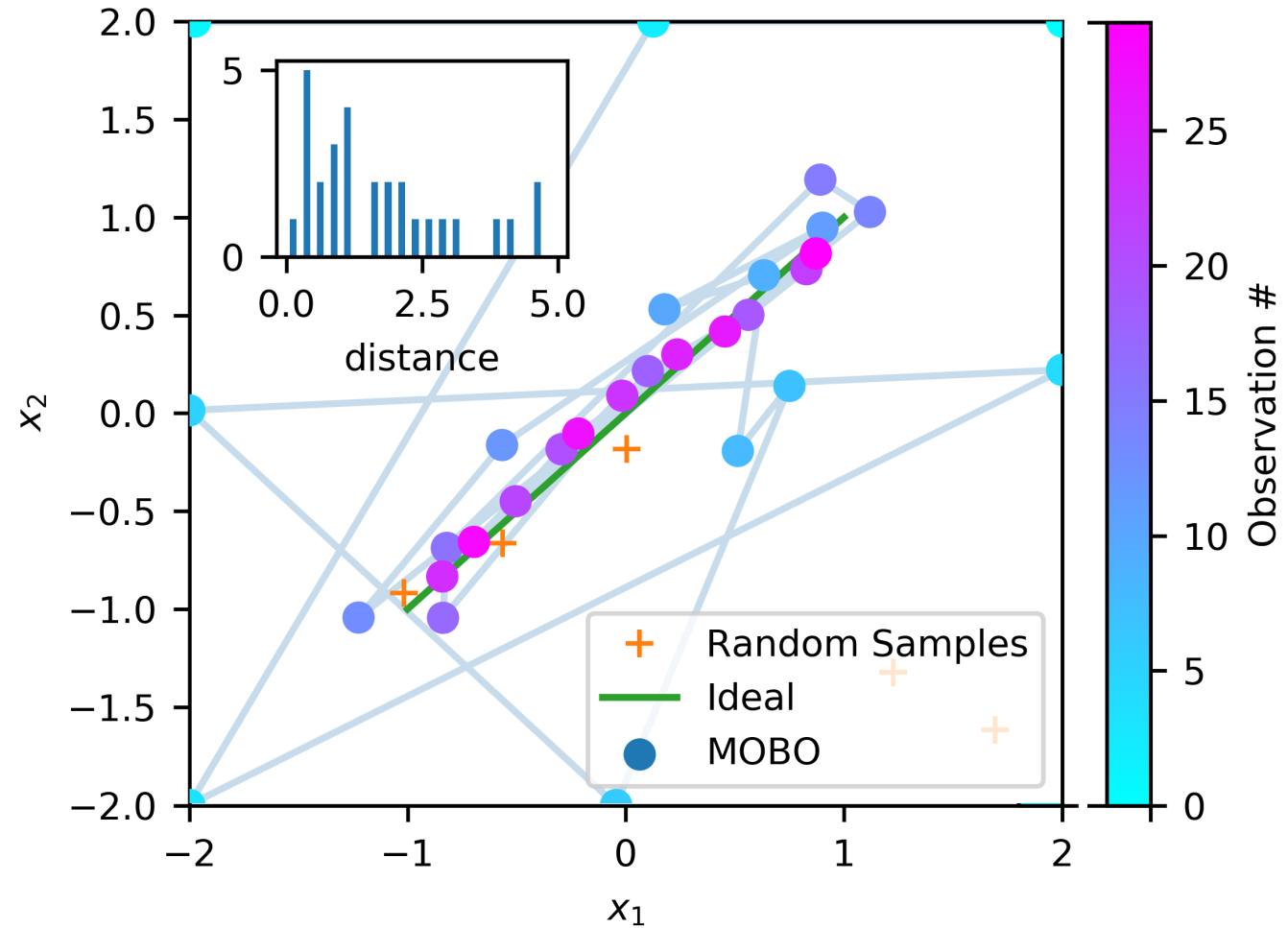
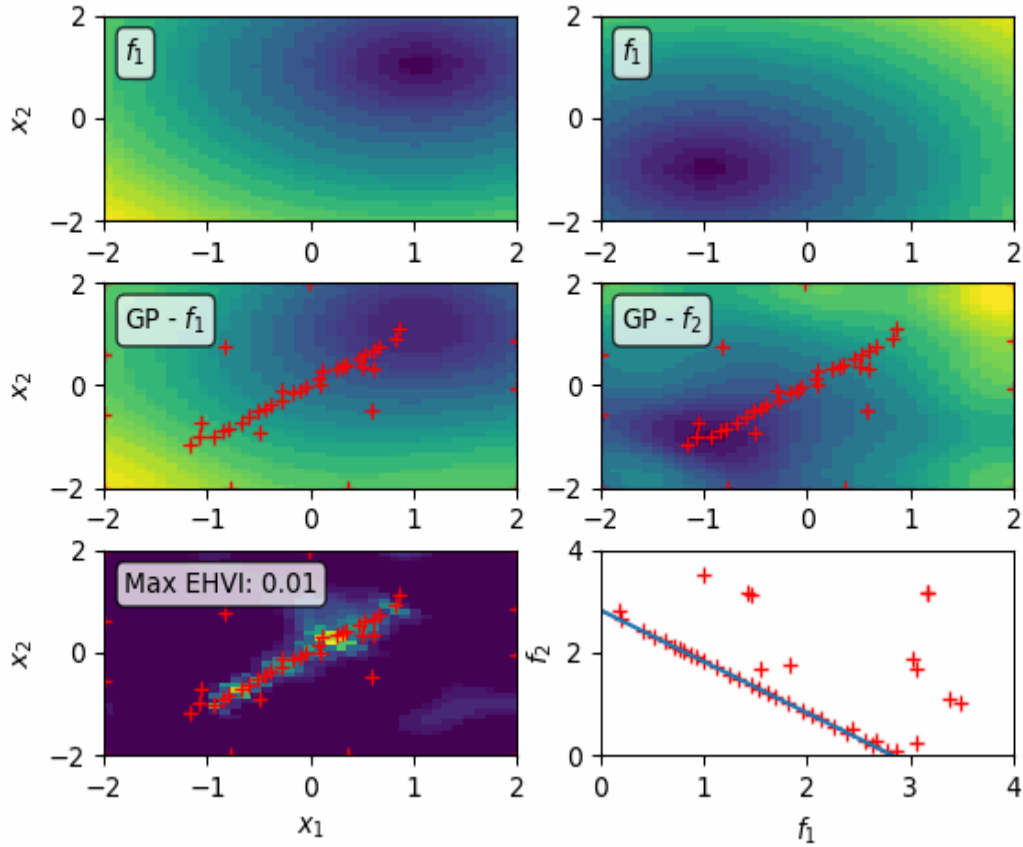
Preference:  $dE < 0.52$  MeV

Constraint:  $dE < 0.52$  MeV





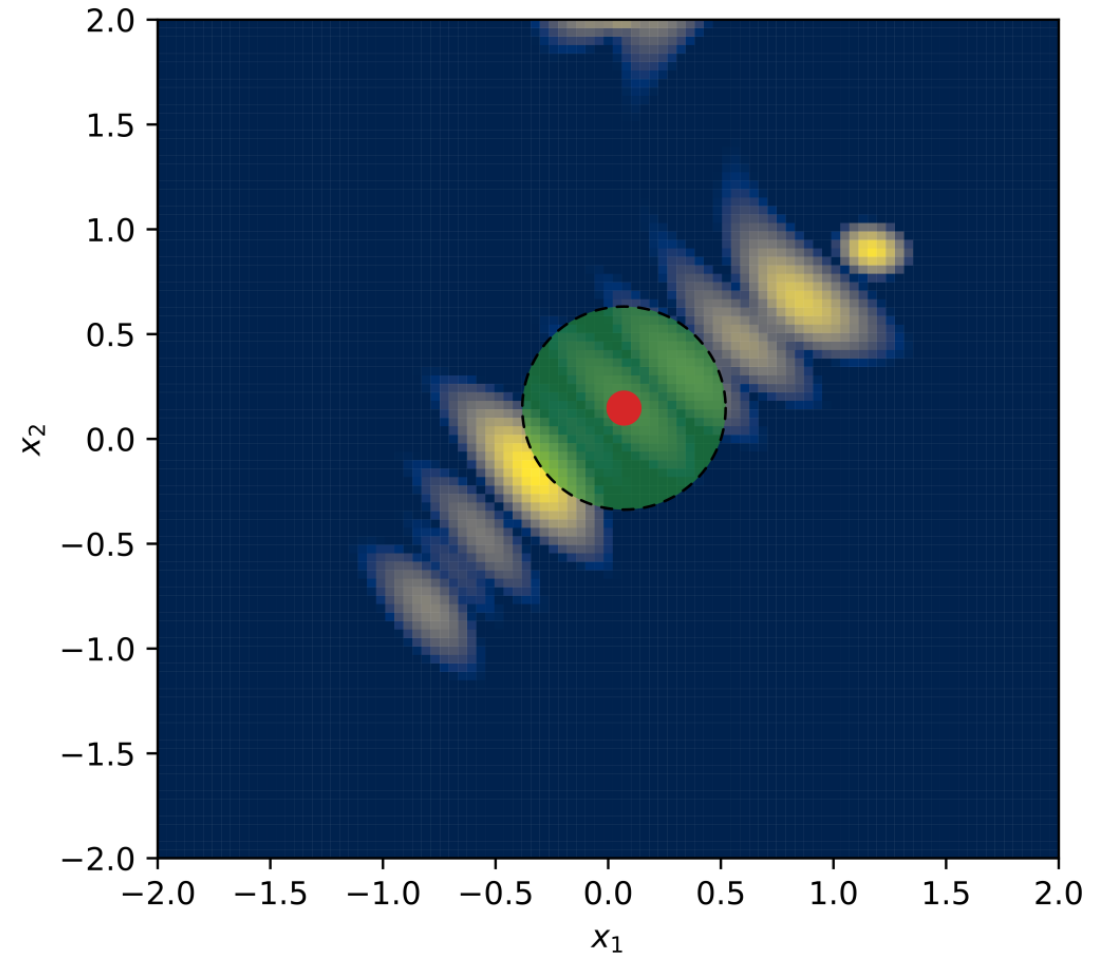
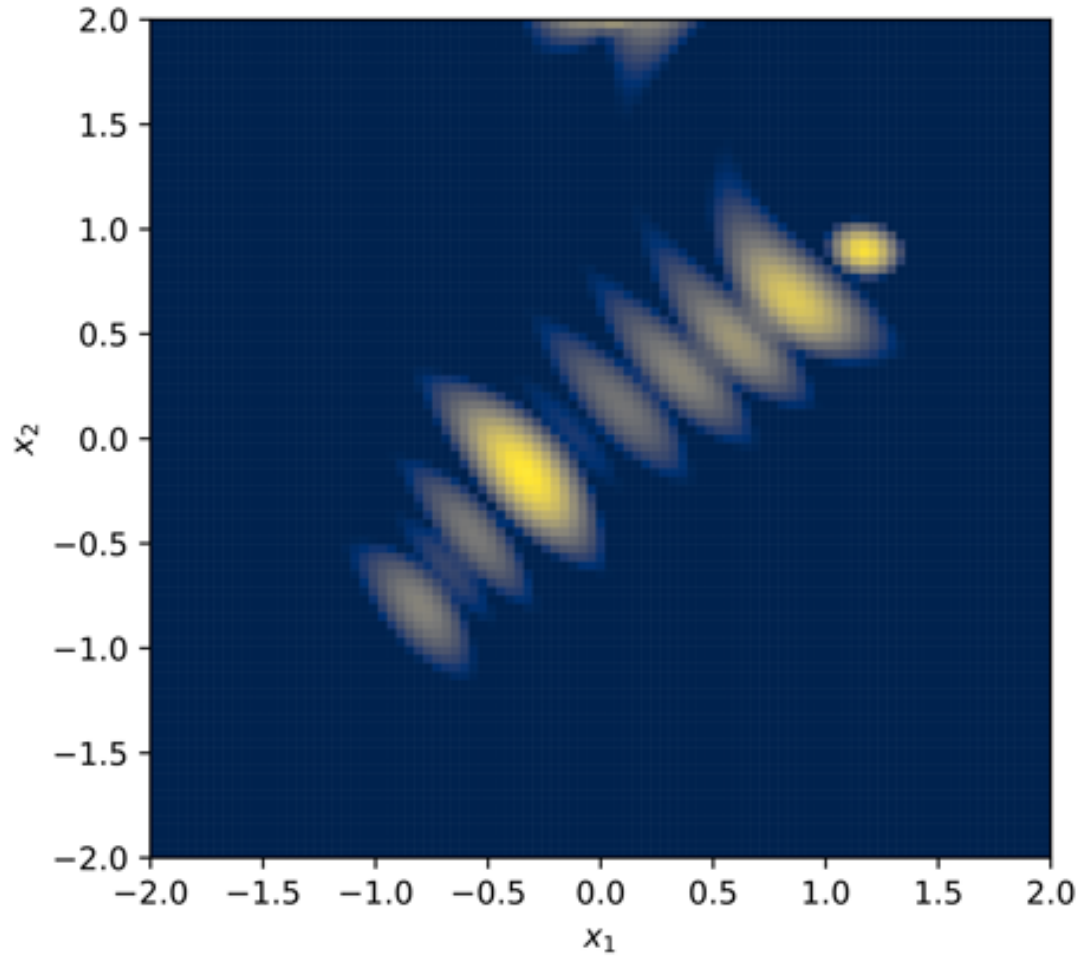
# Smooth (Localized) Exploration





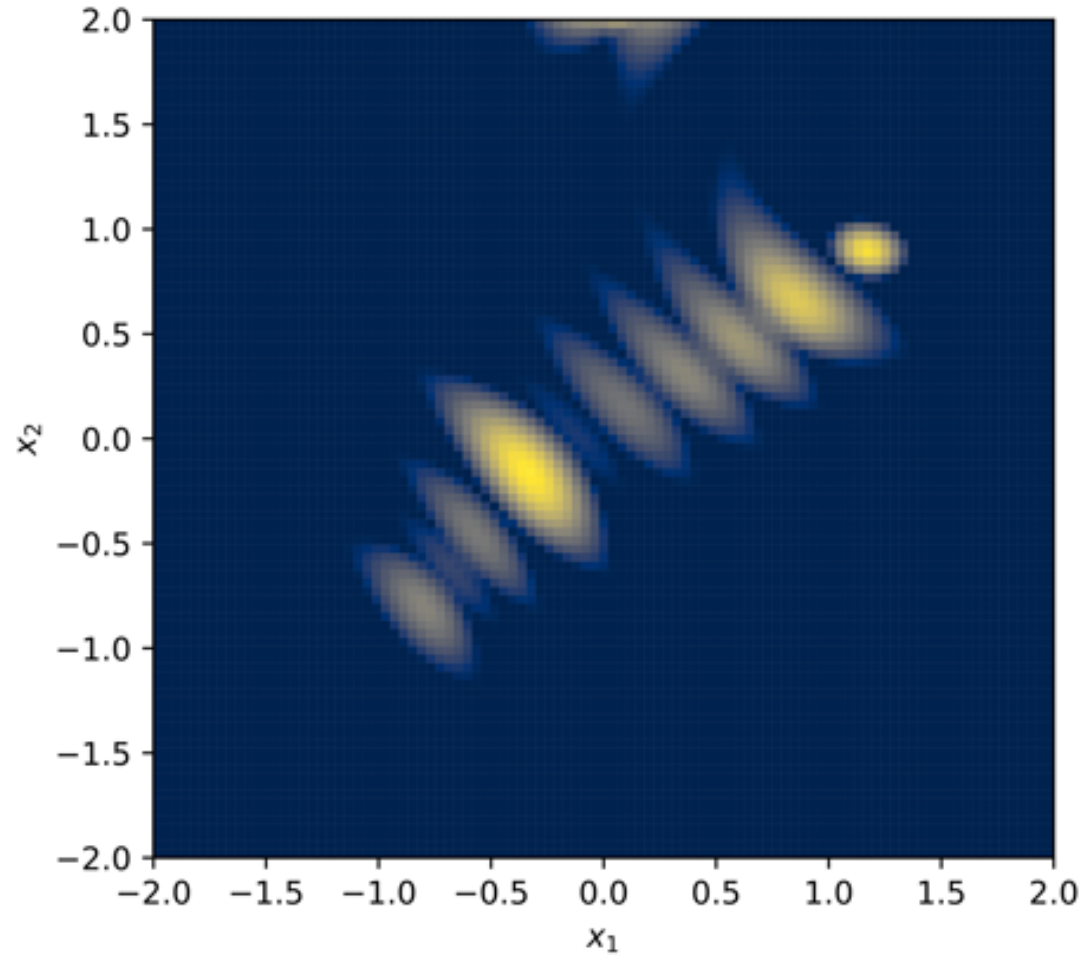


# Smooth (Localized) Exploration

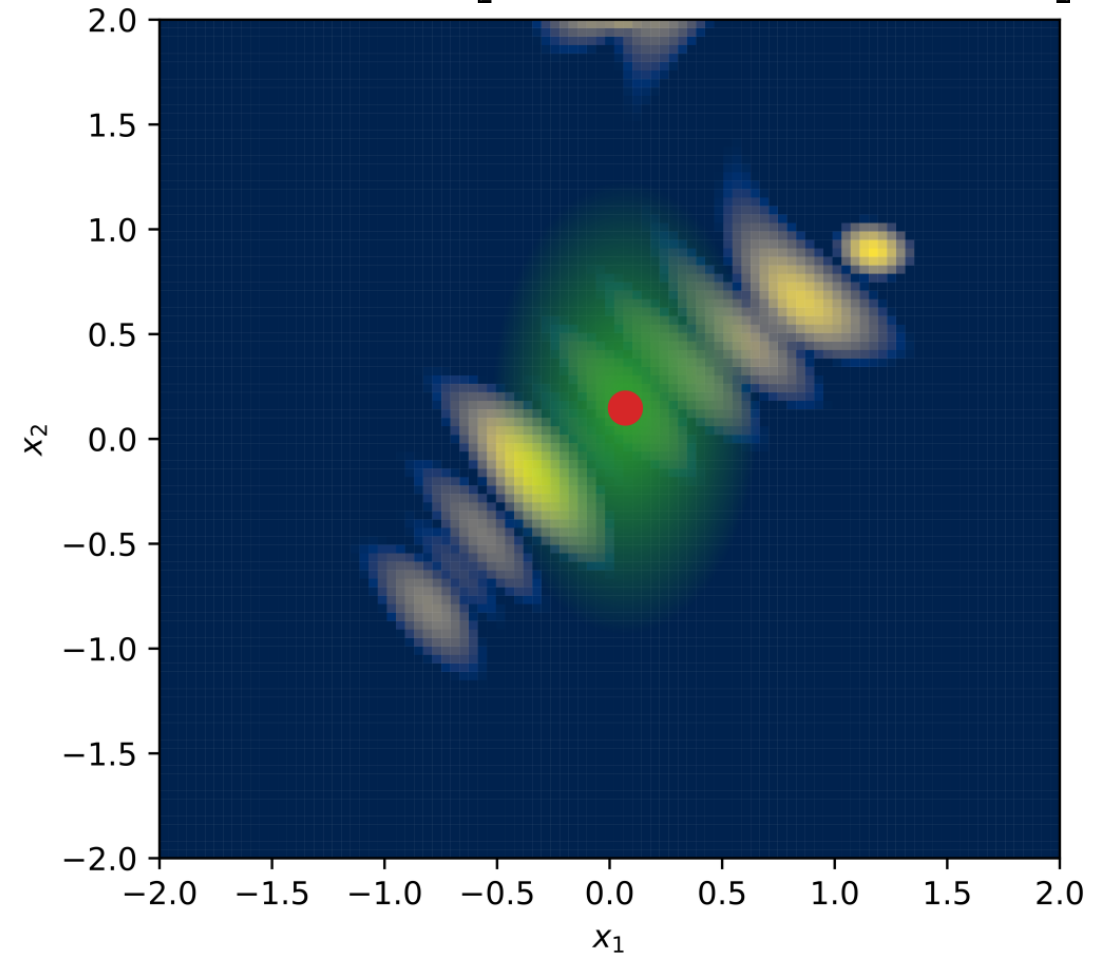




# Smooth (Localized) Exploration

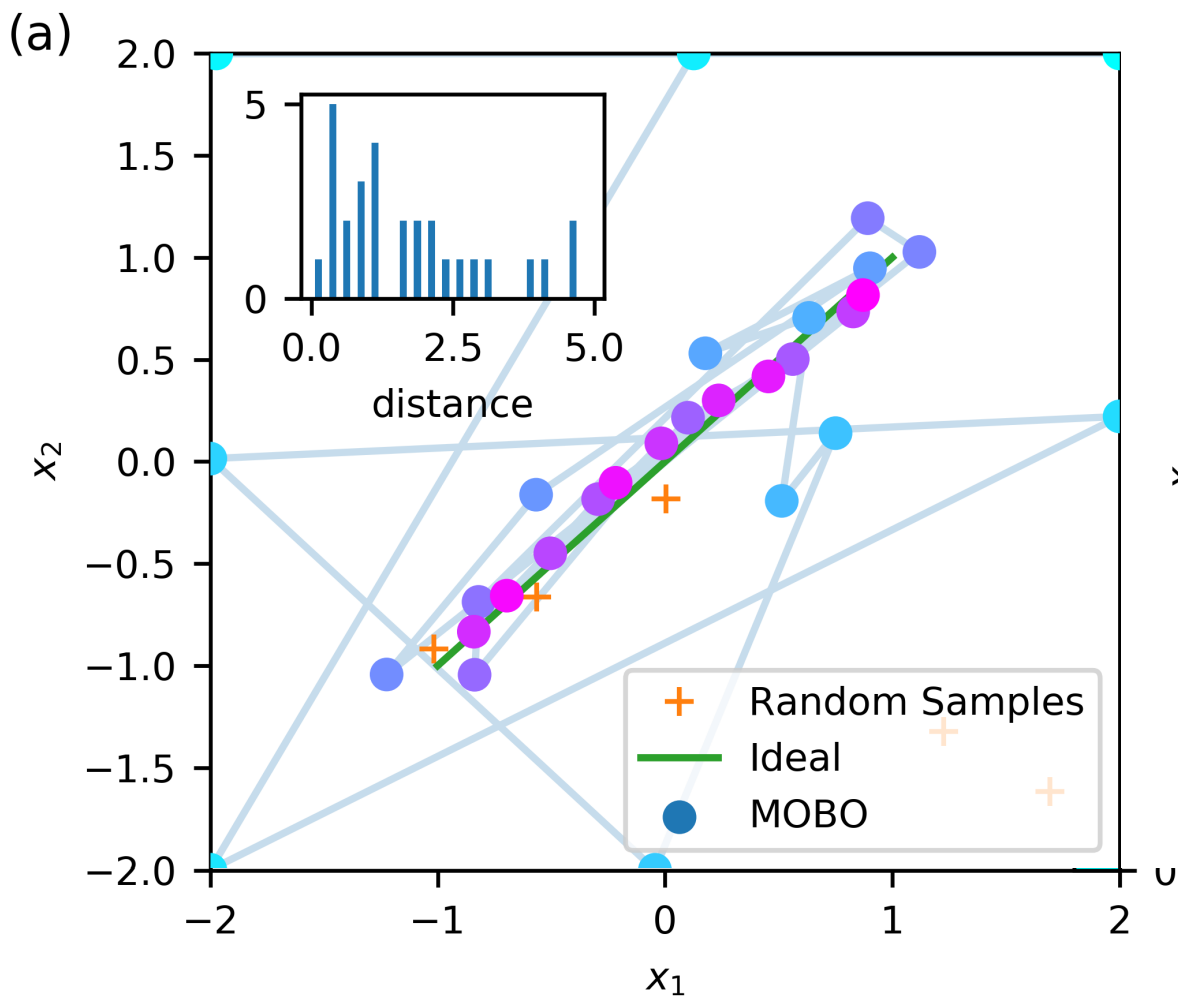


$$\tilde{\alpha}(\mathbf{x}, \mathbf{x}_0) = \alpha(\mathbf{x}) \exp\left[-\frac{1}{2}(\mathbf{x} - \mathbf{x}_0)^T \boldsymbol{\Sigma}(\mathbf{x} - \mathbf{x}_0)\right]$$

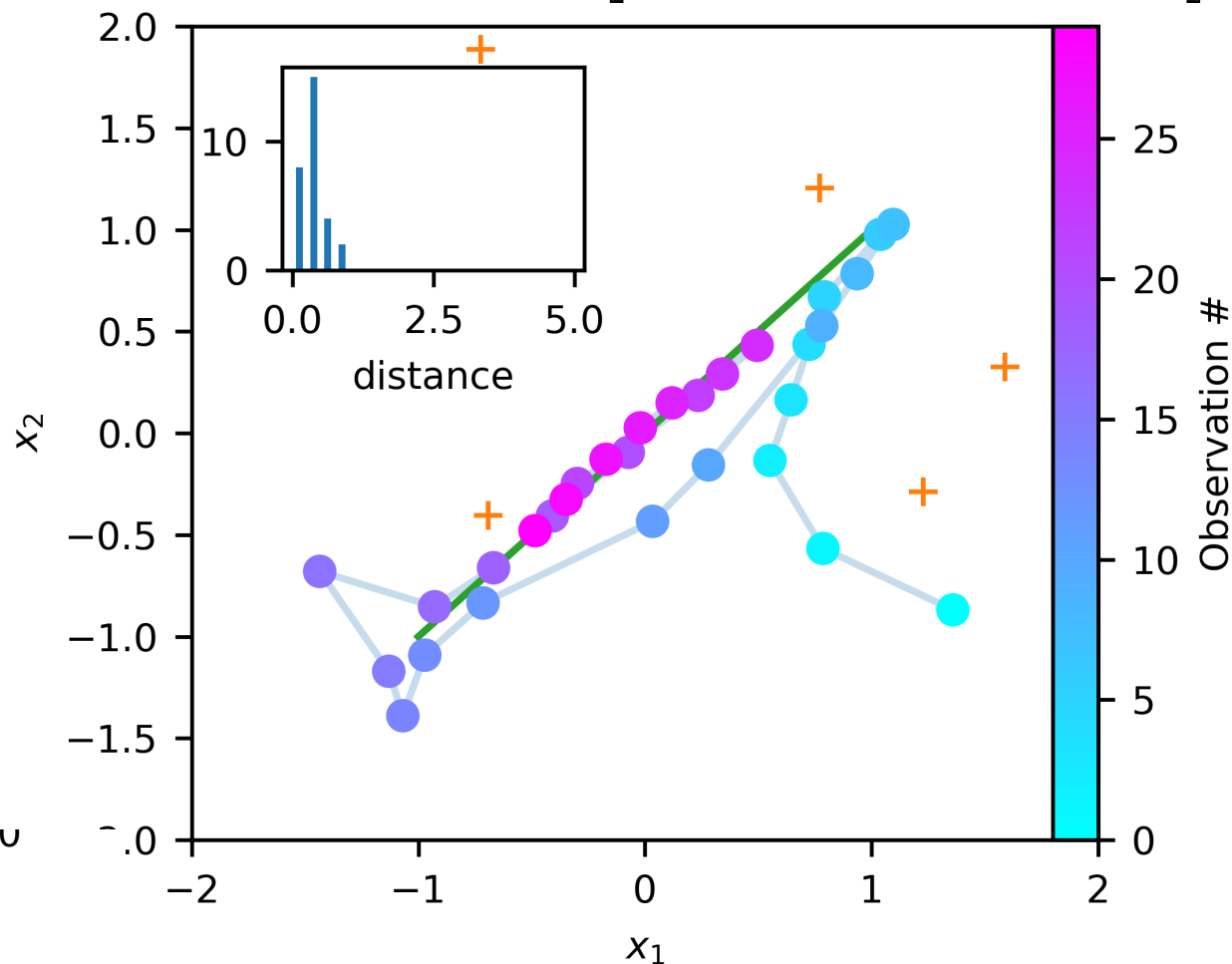




# Smooth (Localized) Exploration



$$\tilde{\alpha}(\mathbf{x}, \mathbf{x}_0) = \alpha(\mathbf{x}) \exp\left[-\frac{1}{2}(\mathbf{x} - \mathbf{x}_0)^T \Sigma (\mathbf{x} - \mathbf{x}_0)\right]$$





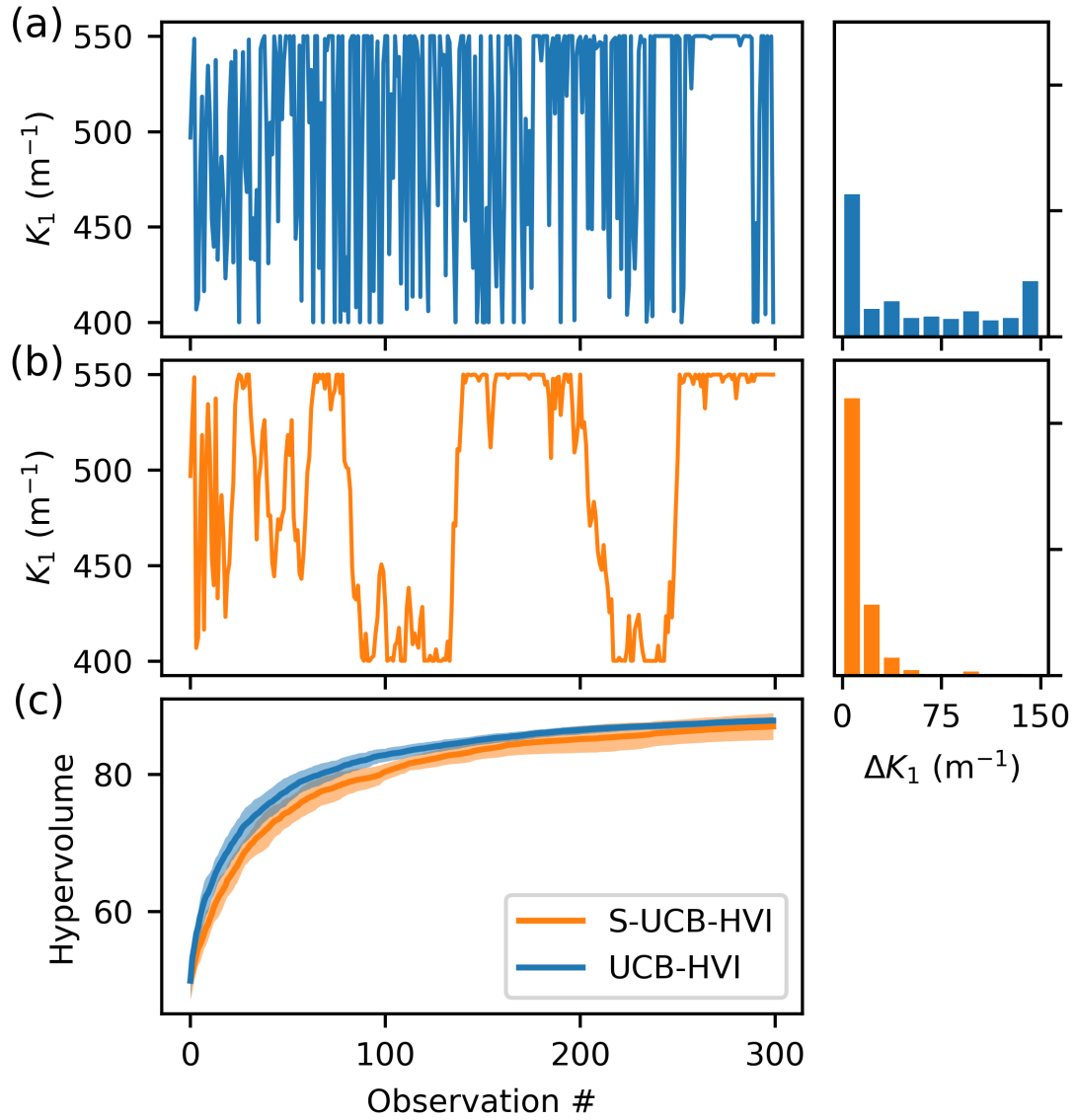
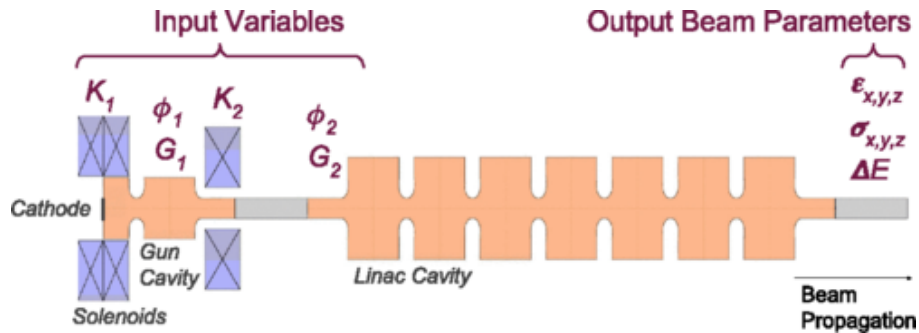
# Smooth Exploration (AWA)



No modification

Localized acquisition function

$$\tilde{\alpha}(\mathbf{x}, \mathbf{x}_0) = \alpha(\mathbf{x}) \exp\left[-\frac{1}{2}(\mathbf{x} - \mathbf{x}_0)^T \Sigma (\mathbf{x} - \mathbf{x}_0)\right]$$

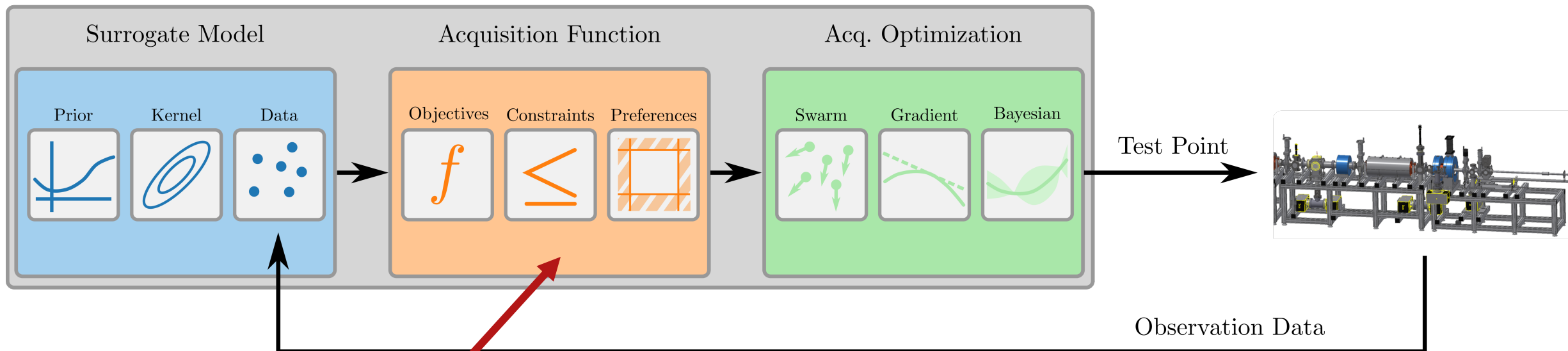




# Gaussian Processes for Accelerator Optimization



## Bayesian Optimization Algorithm



This work

Cornell University

arXiv.org > physics > arXiv:2010.09824

Physics > Accelerator Physics

[Submitted on 19 Oct 2020]

**Multi-Objective Bayesian Optimization for Accelerator Tuning**

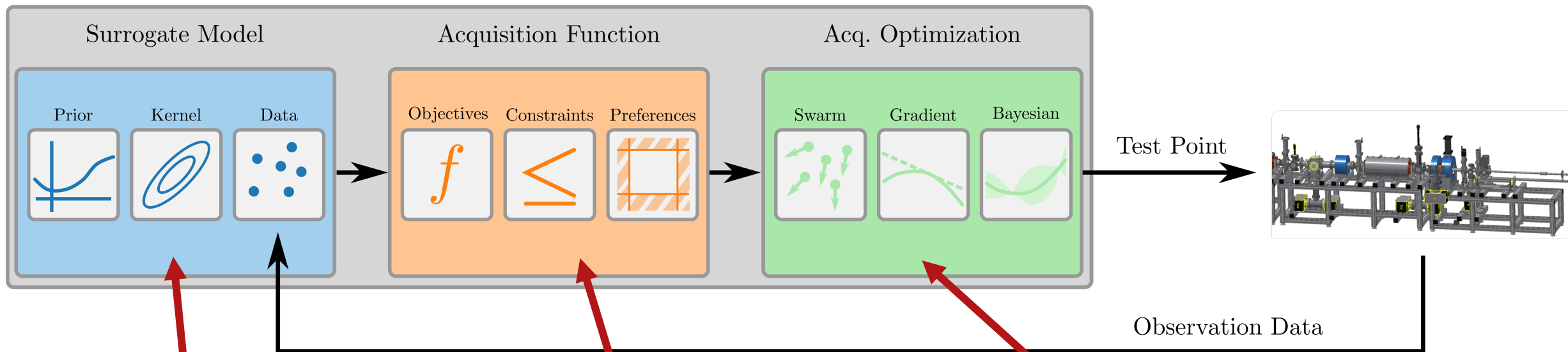
Ryan Roussel, Adi Hanuka, Auralee Edelen



# Gaussian Processes for Accelerator Optimization



## Bayesian Optimization Algorithm



- Improve predictive accuracy
- multi-fidelity simulation results
  - neural network engine
  - manifold GP

- Expand capabilities
- Include time dependent drift and noise

- Improve optimization speed
- seeded swarm optimization
  - hierarchical Bayesian optimization

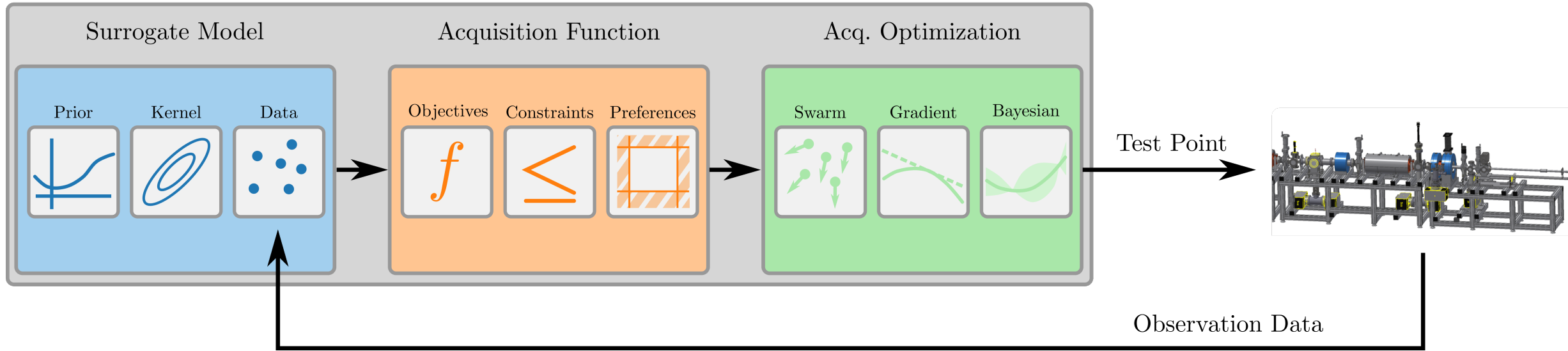
Future work



# Conclusion



## Bayesian Optimization Algorithm



Experimental demonstration coming soon!

