COSMIC RAY PHYSICS: DIFFUSION AND ACCELERATION

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LECTURE I

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OUTLINE

- **Historical note on Cosmic Rays**

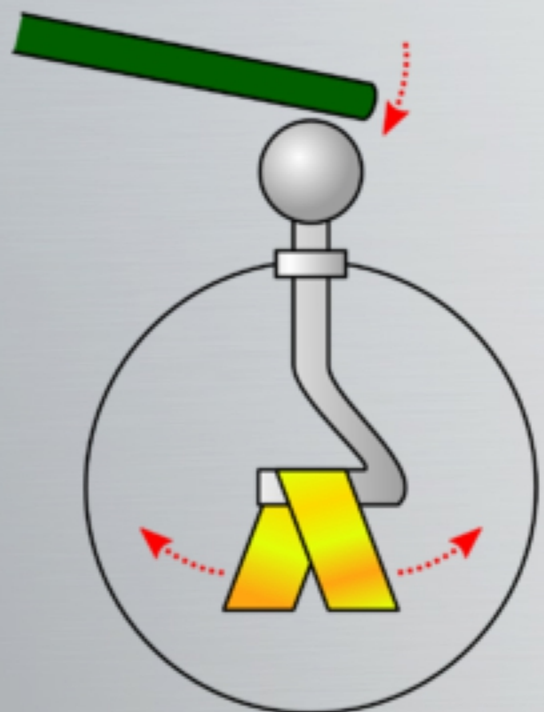
- **The SNR-CR connection**
  - ***Why SNRs?***
  - ***Propagation in the Galaxy***

- **Diffusive motion**
  - ***Motion of particles in perturbed magnetic field***

- **From diffusion to energy gain**
  - ***First and second order Fermi acceleration***
  - ***The nature of collisionless shocks***

HISTORICAL NOTES

THE COSMOS IN AN ELECTROSCOPE



IT WAS KNOWN SINCE THE END OF 1800 THAT AN ELECTROSCOPE WOULD NOT STAY CHARGED INDEFINITELY

HOW WOULD THE CHARGE FLOW AWAY?

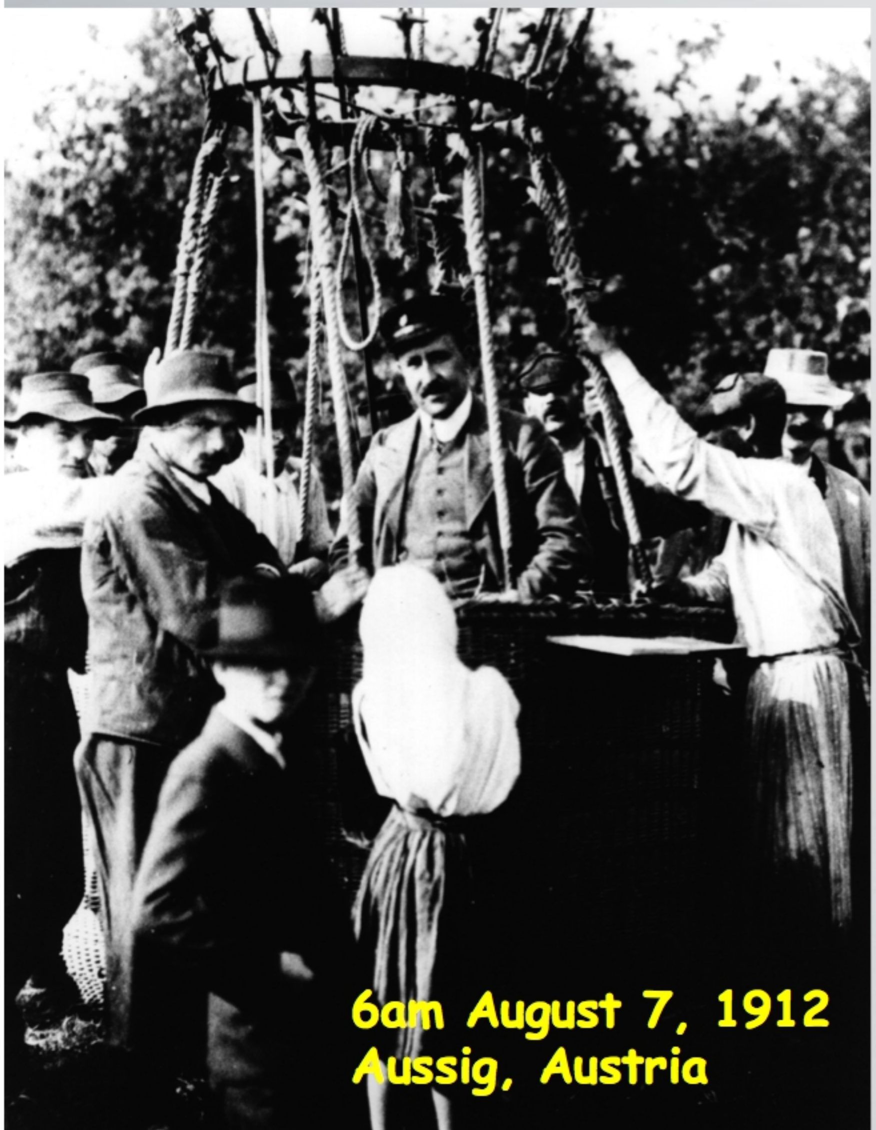
THE AIR MUST HAVE A RESIDUAL IONIZATION, SO THAT THE CHARGE WOULD BE ABLE TO LEAVE THE ELECTROSCOPE...

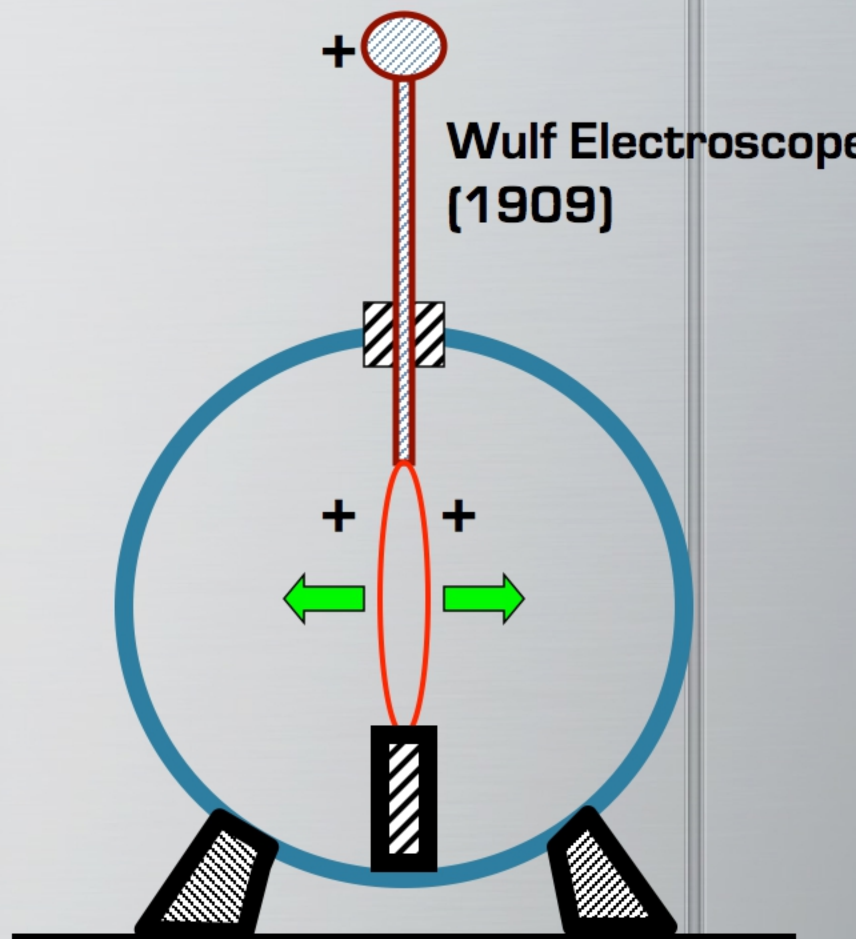
BUT THEN WHAT IS THE SOURCE OF SUCH A RESIDUAL IONIZATION?

1895: X-RAYS (ROENGTEN)
1896: RADIOACTIVITY (BECQUEREL)

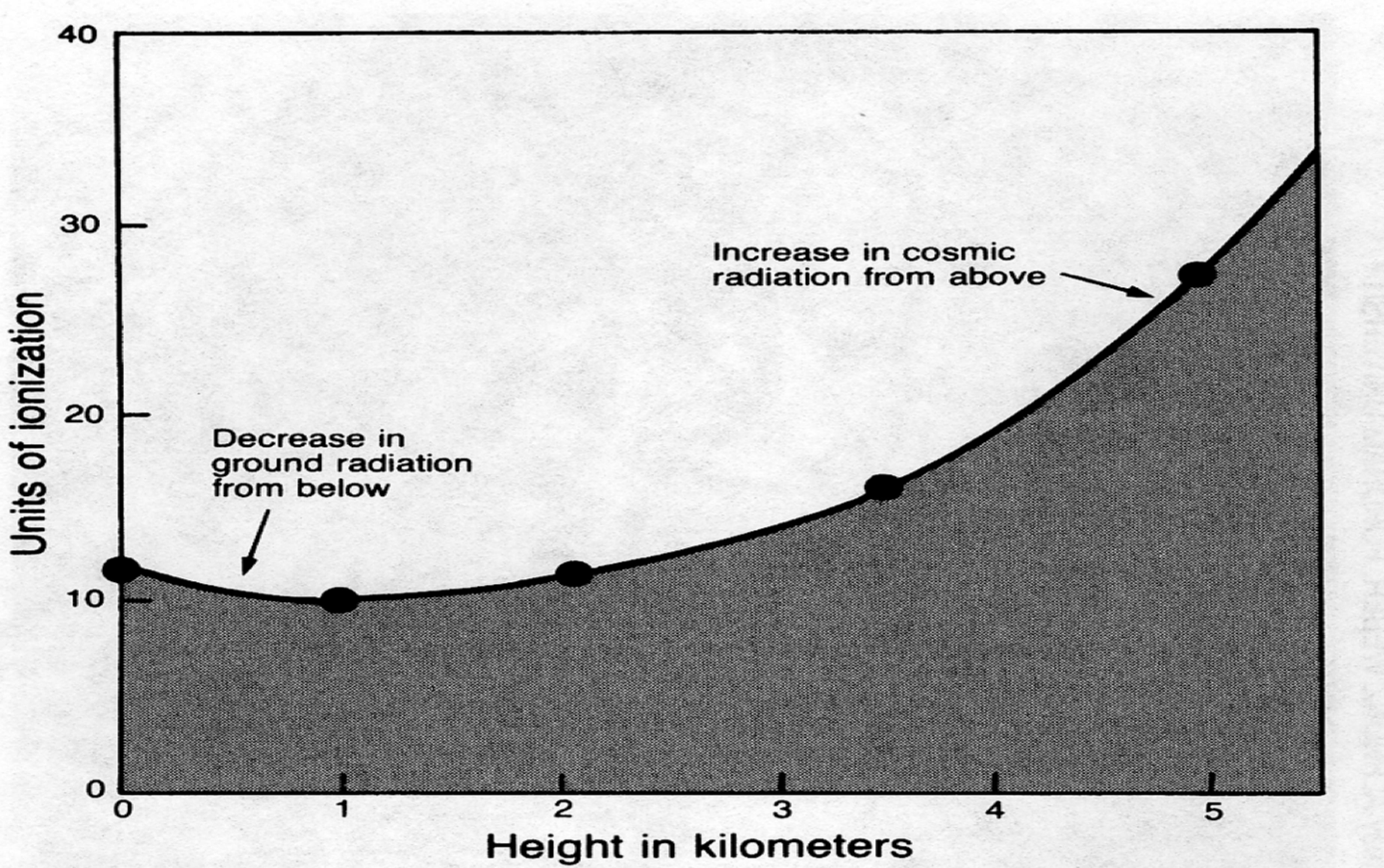
BUT IONIZATION REMAINED WHEN THE ELECTROSCOPE WAS IN A LEAD OR WATER CAVITY: **HIGHLY PENETRATING RADIATION**

Victor F. Hess: the 1912 flight





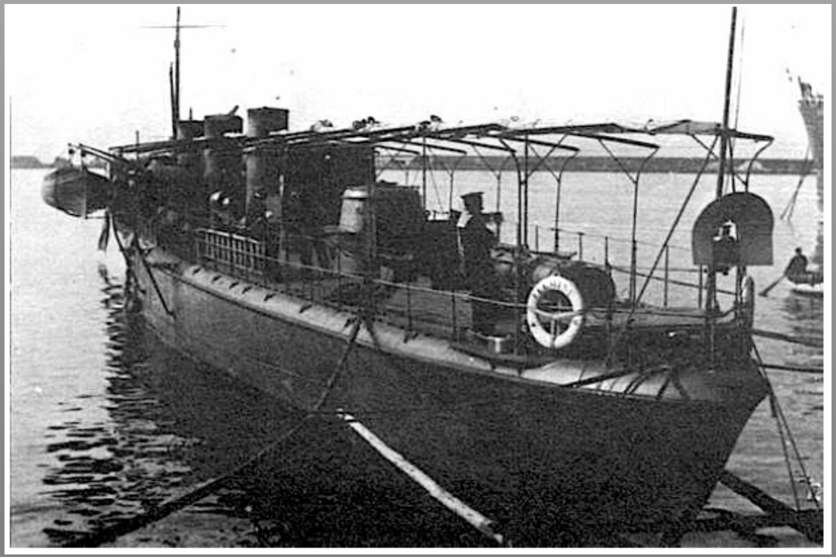
COSMIC Rays



40
30
20
10
0
Units of ionization
Increase in cosmic radiation from above
Decrease in ground radiation from below
0
1
2
3
4
5
Height in kilometers

1911: Domenico Pacini

One year before Hess, in 1911, D. Pacini observed the radiation strength to decrease when going from the surface to a few meters underwater (both in the sea and in the lake), thus demonstrating that such radiation could not come from the Earth.



The cacciatorpediniere “Fulmine”, used by Pacini for his measurements on the sea.

MILLIKAN's THEORY

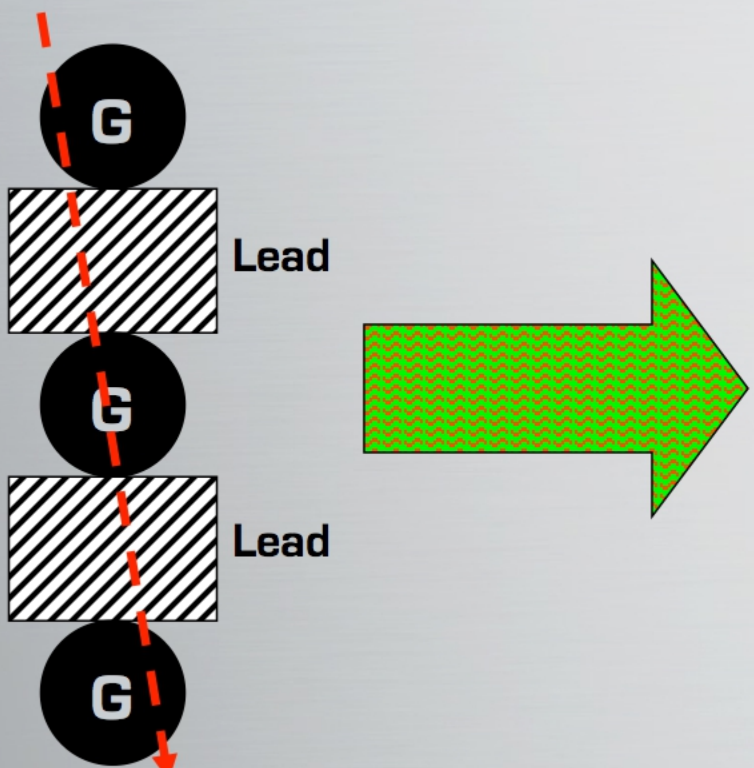
Cosmic Rays (as Millikan called them) are gamma rays as the birth cry of elements heavier than hydrogen

Millikan found that the absorption curve of CR was not compatible with one absorption length, but rather could be fit with a combination of three absorption lengths: 300, 1250 and 2500 g/cm^2, corresponding, according to Compton Theory to gamma ray energies of 26, 110 and 220 MeV

4 p → He	ΔM=27 MeV	**OK**
14 p → N	ΔM=108 MeV	**OK**
12 p → C	**ΔM=85 MeV**	**?**
16 p → O	ΔM=129 MeV	**OK**
28 p → Si	ΔM=239 MeV	**OK**

But gamma rays do not go through lead!

Bruno Rossi had performed several experiments with his coincidence Geiger counters and found that CR could penetrate even 1m of lead



COSMIC RAYS
ARE NO GAMMA RAYS
And
THEIR ENERGY IS > GeV

East-West Effect

In the 30s Bruno Rossi had predicted that if CR were charged then one should see a latitude dependence of the flux, depending upon the sign of the charge.

THE EFFECT WAS DISCOVERED BY COMPTON IN 1930 AND CONFIRMED BY ROSSI IN 1931.

MOST COSMIC RAYS WERE POSITIVELY CHARGED PARTICLES

- **1930:** B. Rossi in Arcetri predicts the East-West effect
- **1932:** Carl Anderson discovers the positron in CR
- **1934:** Bruno Rossi detects coincidences even at large distance from the center…first evidence of extensive showers!
- **1937:** Seth Neddermeyer and Carl Anderson discover the muon
- **1938-39:** Auger detects first extensive air showers with energy up to $10^{13\text{-}14}$ eV
- **1940's:** Boom of particle physics discoveries in CR
- **1962:** UHECRs by Linsley & Scarsi
- **1966:** Penzias and Wilson discover the CMB

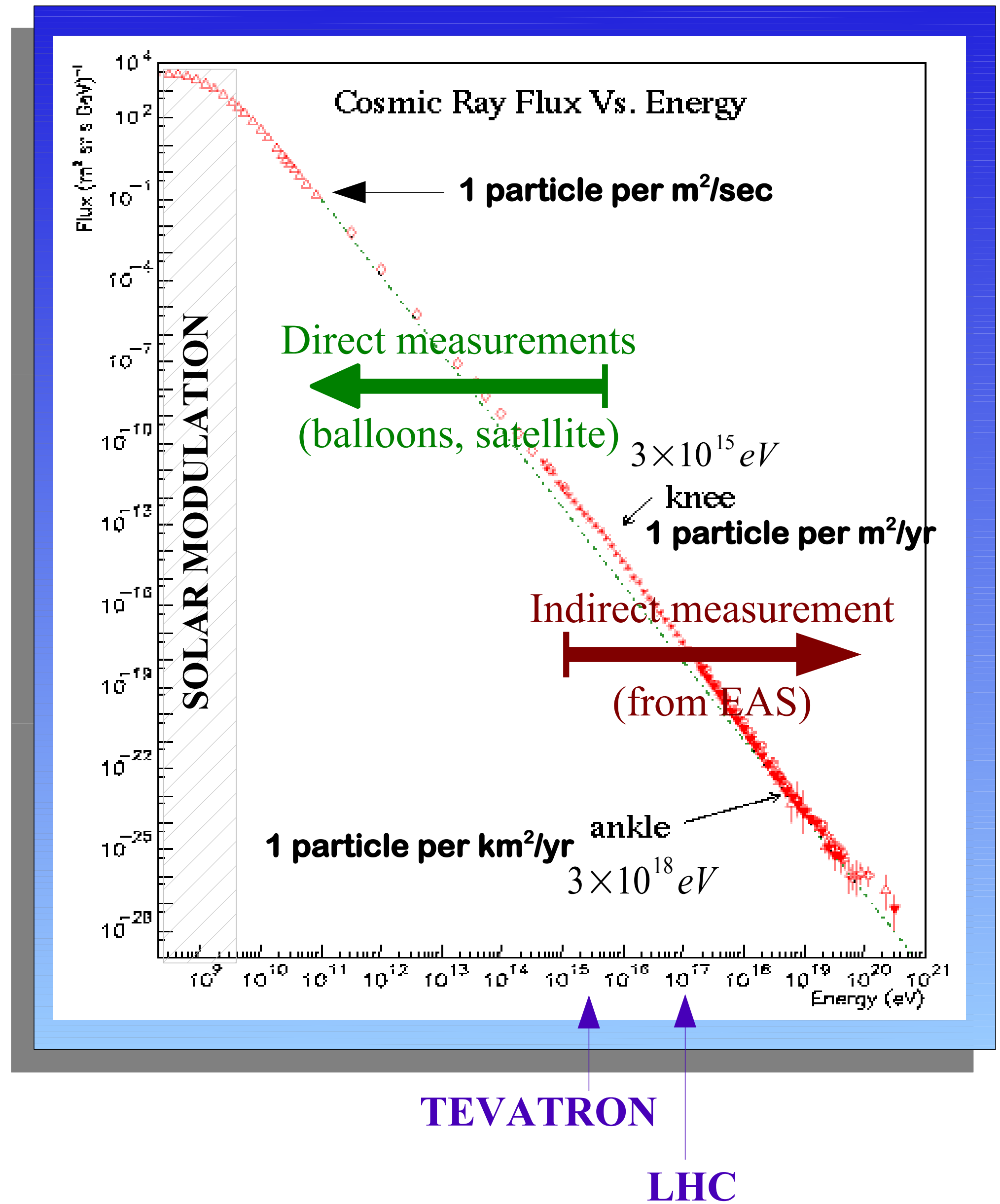
THE SNR-CR CONNECTION

Quick view on CR spectrum

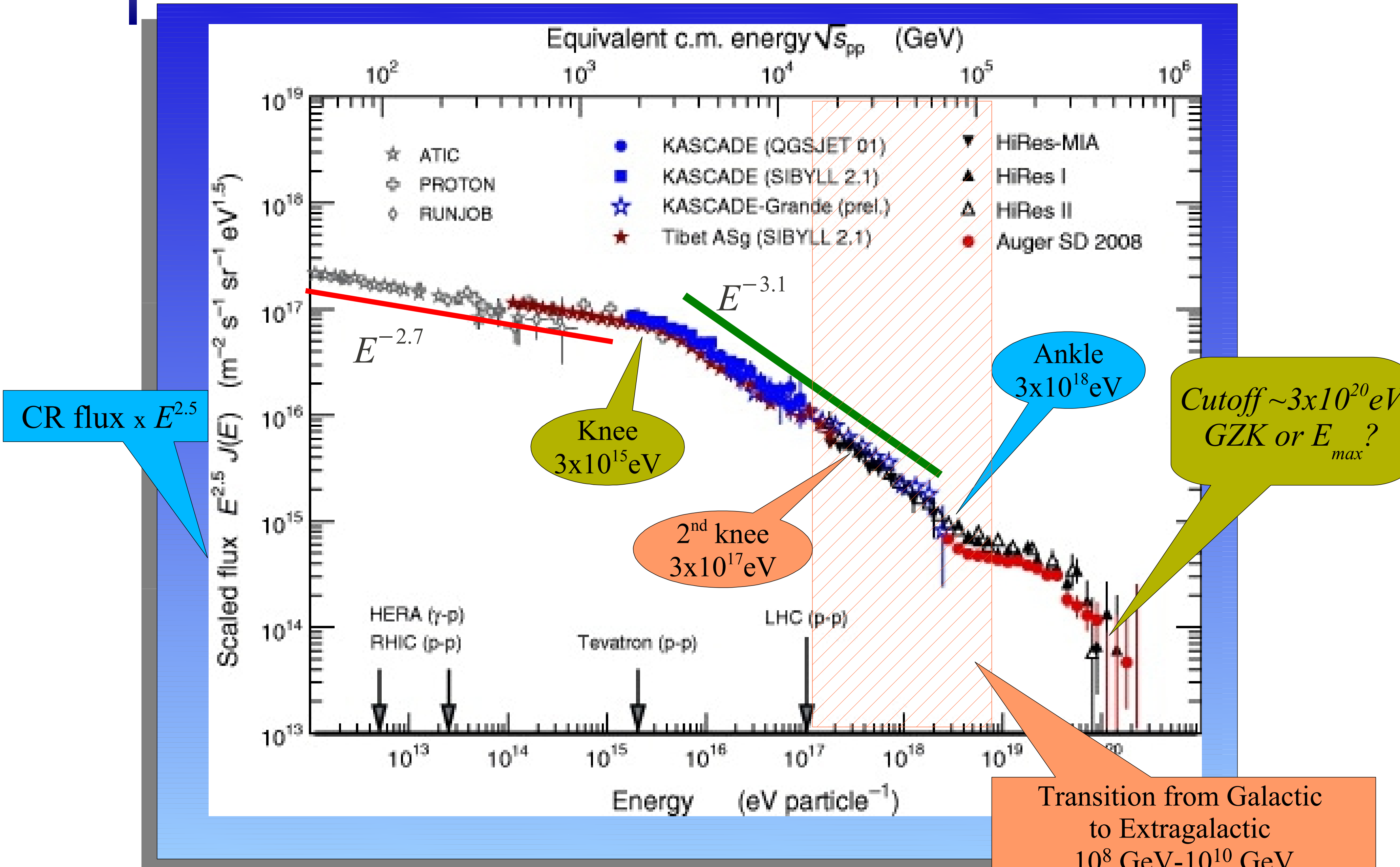
CRs energy density compared with other components:

ENERGY DENSITY IN THE GALAXY	eV/cm^3
Magnetic field ($B^2/8\pi$)	~ 0.5
Gas motion ($Mv^2/2$)	~ 0.5
Starlight	~ 0.5
CMB (2.7 K)	~ 0.5
CRs	~ 0.5

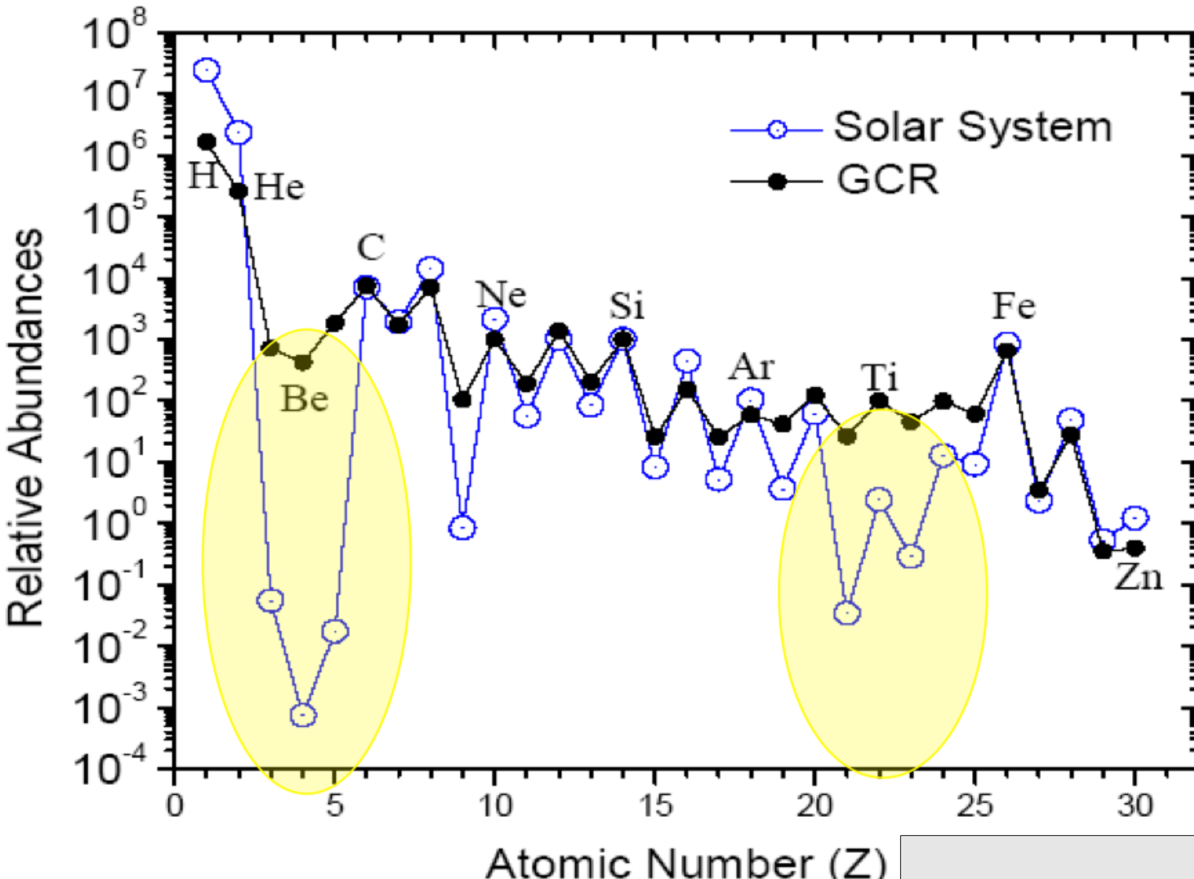
Incredible energy extension: up to $3x10^{20}$ eV !!!



Quick view on CR spectrum



The chemical composition of CRs



The relative aboundances of some elements is larger than in the Solar composition

Those “secondary” elements are produced by primary CRs by spalation

 Primary CRs should propagate in the Galaxy for a time comparable with the interacitio time

$$\tau_{interaction} \approx \frac{1}{n_{gas}\, c\, \sigma_{spal}} \approx few\ Myr$$

Propagation time of CRs

Assuming that cosmic rays propagate simply gyrating along magnetic field lines, than:

$$\tau_{DISC} = \frac{300\ \text{pc}}{(1/3)c} \approx 3000\ \text{years}$$

Propagation time in the vertical direction of the disk

$$\tau_{GAL} = \frac{15\ \text{kpc}}{(1/3)c} \approx 150{,}000\ \text{years}$$

Propagation time in the Galactic plane

$$\tau_{HALO} = \frac{3\ \text{kpc}}{(1/3)c} \approx 30{,}000\ \text{years}$$

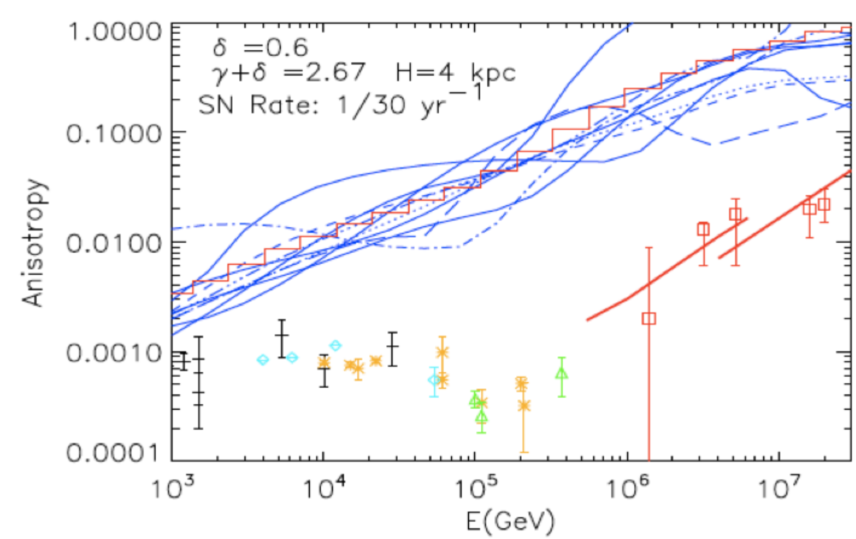
Propagation time in the Galactic magnetic Halo

$$\ll \tau_{interaction}$$

All these time scales are extremely short compared with the residence time

→ CRs have to diffuse in the Galaxy

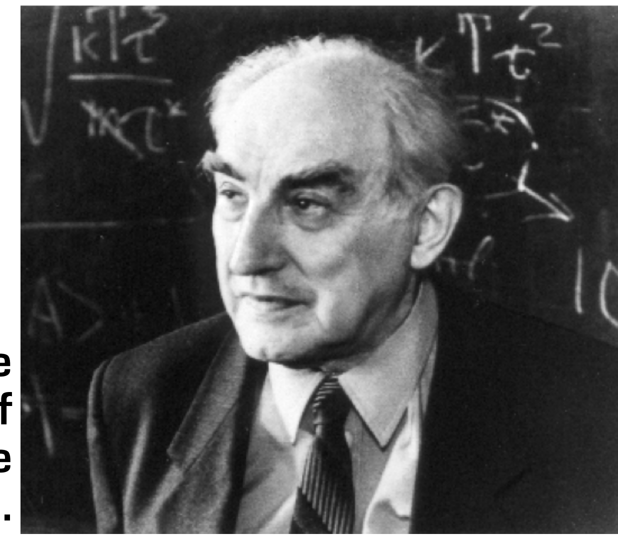
→ The location of sources is lost and cannot be identified measuing the arrival directions
In fact the anisotropy is very small



Origin of Galactic CRs

Zwicky & Baade were the first to postulate that SNR could be plausible sources of CRs (1934)

Vitali Lazarevich Ginzburg made the argument for SNRs as sources of galactic CR in the 60's in a more quantitative form.



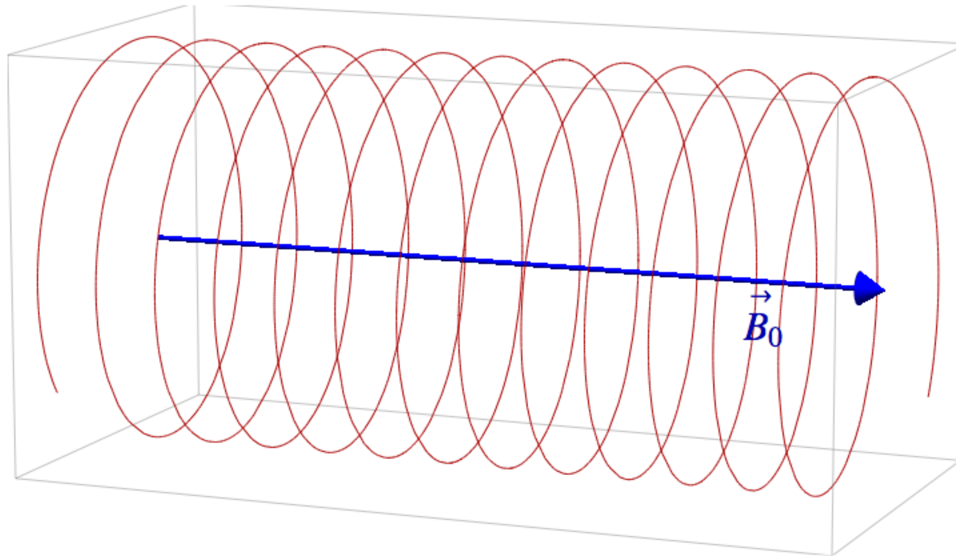
$$W_{CR} \sim U_{CR} Vol_{CR} / \tau_{res} \approx 10^{40}\, erg/s$$
$$W_{SN} \sim R_{SN} \; E_{SN} \approx 3 \cdot 10^{41}\, erg/s$$
$$\Rightarrow \quad \frac{W_{CR}}{W_{SN}} \approx 0.03 \div 0.3$$

In principle ~10% of the SN kinetic energy is enough to explain the CR energy density

But what kind of mechanism can transfer energy to non-thermal particle in a power law spectrum?

DIFFUSIVE MOTION

MOTION IN A REGULAR FIELD



Equation of motion

$$\frac{d\,\boldsymbol{p}}{dt} = q\left[\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}_0\right]$$

Electric field is usually shortcutted in a plasma

Solution

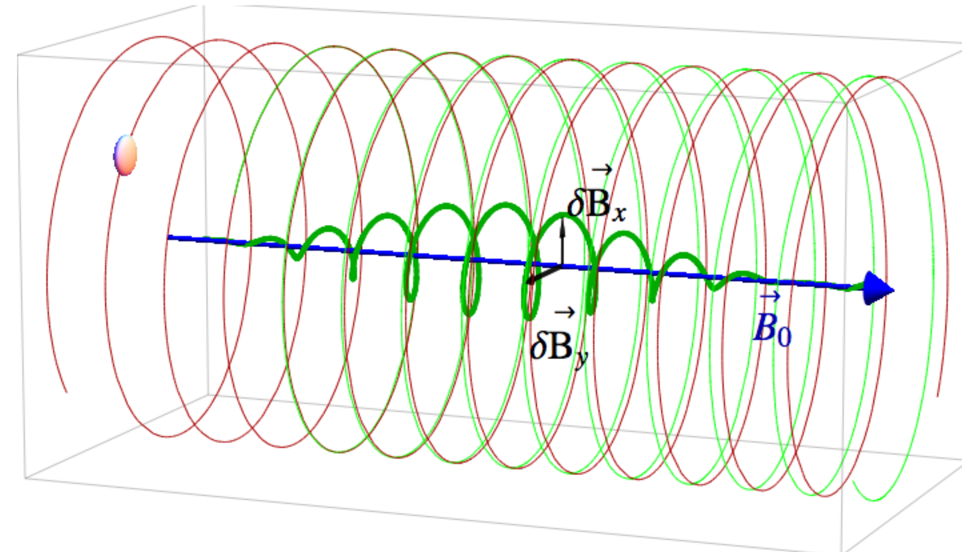
$$\begin{cases} p = const \\ v_x = V_0 \cos(\Omega t) \\ v_y = V_0 \sin(\Omega t) \end{cases}$$

$$\Omega = \frac{q B_0}{m c \gamma}$$

Larmor frequency

- MAGNETIC FIELD DO NOT MAKE WORKS ON PARTICLES PARTICLE'S ENERGY DOES NOT CHANGE
- A RELATIVISTIC PARTICLE MOVES IN THE Z DIRECTION ON AVERAGE AT C/3

MOTION IN PRESENCE OF IRREGULARITIES



Small sinusoidal perturbation

$$\delta B \perp B_0 \quad ; \qquad \delta B \ll B_0$$

$$\delta B_x = B_k \cos(kz + \phi);$$

$$\delta B_y = B_k \sin(kz + \phi)$$

Equation of motion

$$\frac{d\boldsymbol{p}}{dt} = q\frac{\boldsymbol{v}}{c}\left(\boldsymbol{B}_0 + \delta \boldsymbol{B}\right); \qquad p_z = mc\gamma\mu$$

$\boldsymbol{B}_0$ changes only x and y components of the momentum

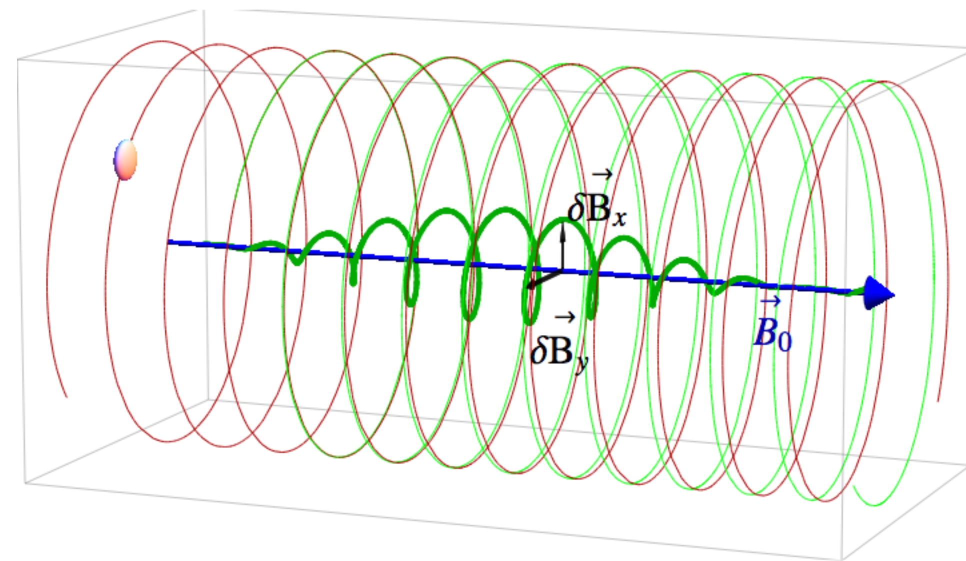
$\boldsymbol{\delta B}$ changes only z component of the momentum

Remember that waves typically move at the Alfvén speed:

$$v_a = \frac{B}{(4\pi\rho)^{1/2}} = 2 \times 10^6 B_\mu n_1^{-1/2} \; cm/s$$

Hence for a relativistic particle these waves, in first approximation, look like static waves.

MOTION IN PRESENCE OF IRREGULARITIES



<u>Equation of motion</u>

$$\frac{d\,\boldsymbol{p}}{dt} = q\,\frac{\boldsymbol{v}}{c}\left(\boldsymbol{B}_0 + \delta\,\boldsymbol{B}\right);$$

$$p_z = mc\,\gamma\,\mu\,; \qquad \mu = \cos(\theta)$$

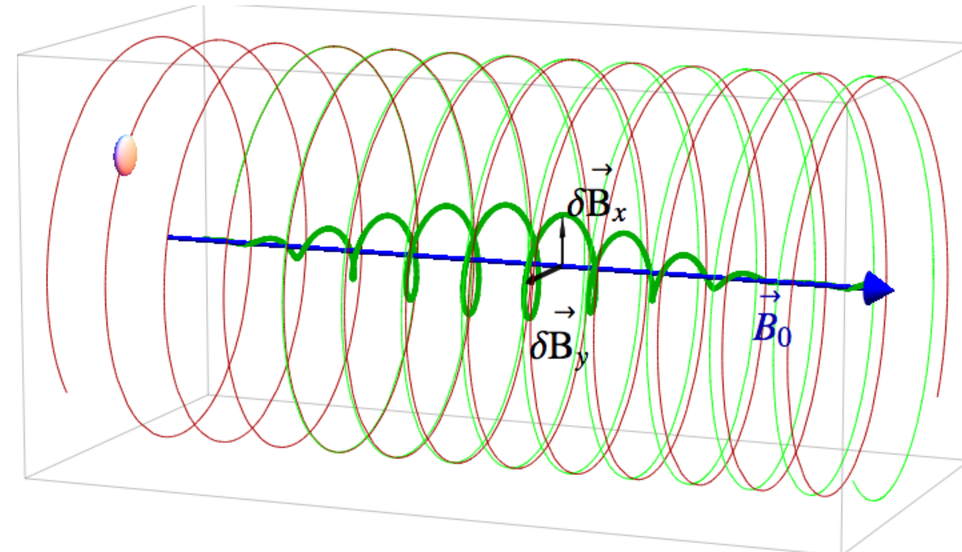
$$m\,c\,\gamma\,\frac{d\,\mu}{dt} = q\,(1-\mu^2)^{1/2}\left[\cos(\Omega\,t)\,\delta\,B_y - \sin(\Omega\,t)\,\delta\,B_x\right]$$

Average displacement over a time Δ*t:*

$$\langle\Delta\,\mu\rangle = \int \frac{d\,\mu}{dt}\,dt = 0\,;$$

The mean value of the pitch angle variation vanishes

MOTION IN PRESENCE OF IRREGULARITIES



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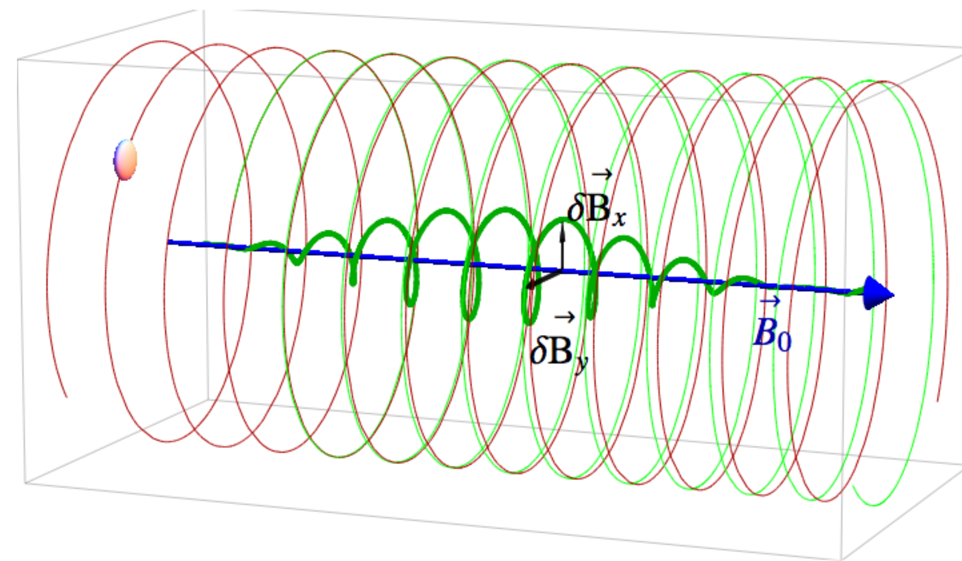
$$mc\gamma\frac{d\mu}{dt} = q\left(1-\mu^2\right)^{1/2}\left[\cos(\Omega t)\,\delta B_y - \sin(\Omega t)\,\delta B_x\right]$$

Average value of the variance over a time Δt:

$$\left\langle \Delta\mu^2 \right\rangle = \frac{q^2\left(1-\mu^2\right)}{\left(mc\gamma\right)^2} B_k^2 \int dt \int dt' \cos\left[\left(\Omega - kv\mu\right)t + \phi\right]\cos\left[\left(\Omega - kv\mu\right)t' + \phi\right];$$

MOTION IN PRESENCE OF IRREGULARITIES



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$$\left\langle \frac{\Delta\mu^2}{\Delta t} \right\rangle_\phi = \frac{q^2(1-\mu^2)\pi B_k^2}{(mc\gamma)^2} \; \frac{1}{v\mu}\,\delta\left(k - \frac{\Omega}{v\mu}\right)$$

← **Resonant condition**

MANY WAVES

IN A GENERAL CASE ONE DOES NOT HAVE A SINGLE WAVE BUT RATHER A POWER SPECTRUM:

$$P(k) = \delta B_k^2 / 8\pi$$

THEREFORE INTEGRATING OVER ALL OF THEM:

$$D_{\mu\mu} = \left\langle \frac{\Delta\mu^2}{\Delta t} \right\rangle = \frac{q^2 (1-\mu^2)\pi}{(mc\gamma)^2} \; \frac{8\pi}{v\mu} \int dk \frac{\delta B_k^2}{8\pi} \delta\left(k - \frac{\Omega}{v\mu} \right) = \; \pi (1-\mu^2) \Omega \; k_{res} \frac{P(k_{res})}{B_0^2/8\pi}$$

OR IN A MORE IMMEDIATE FORMALISM: $D_{\theta\theta} = \pi \Omega k_{res} F(k_{res})$

THE DEFLECTION ANGLE CHANGES BY ORDER UNITY IN A TIME:

$$\tau \approx \frac{1}{\Omega k_{res} F(k_{res})} \;\Rightarrow\; D_{zz} = \frac{1}{3} v (v\tau) \approx \frac{v^2}{\Omega k_{res} F(k_{res})}$$

SPATIAL DIFFUSION COEFFICIENT

PARTICLE SCATTERING

- EACH TIME THAT A RESONANCE OCCURS THE PARTICLE CHANGES PITCH ANGLE BY $\Delta\theta \sim \delta B/B$ WITH A RANDOM SIGN

- THE RESONANCE OCCURS ONLY FOR RIGHT HAND POLARIZED WAVES IF THE PARTICLES MOVES TO THE RIGHT (AND VICEVERSA)

- THE RESONANCE CONDITION TELLS US THAT
  - **1)** IF $k \ll 1/r_L$ PARTICLES SURF ADIABATICALLY AND
  - **2)** IF $k \gg 1/r_L$ PARTICLES HARDLY FEEL THE WAVES

WHERE DO THE WAVES COME FROM?

FROM DIFFUSION TO ENERGY GAIN

Basic concepts

- ALL ACCELERATION MECHANISMS ARE ELECTROMAGNETIC IN NATURE

- MAGNETIC FIELDS DO NOT MAKE WORK ON CHARGED PARTICLES!

- WE NEED ELECTRIC FIELDS

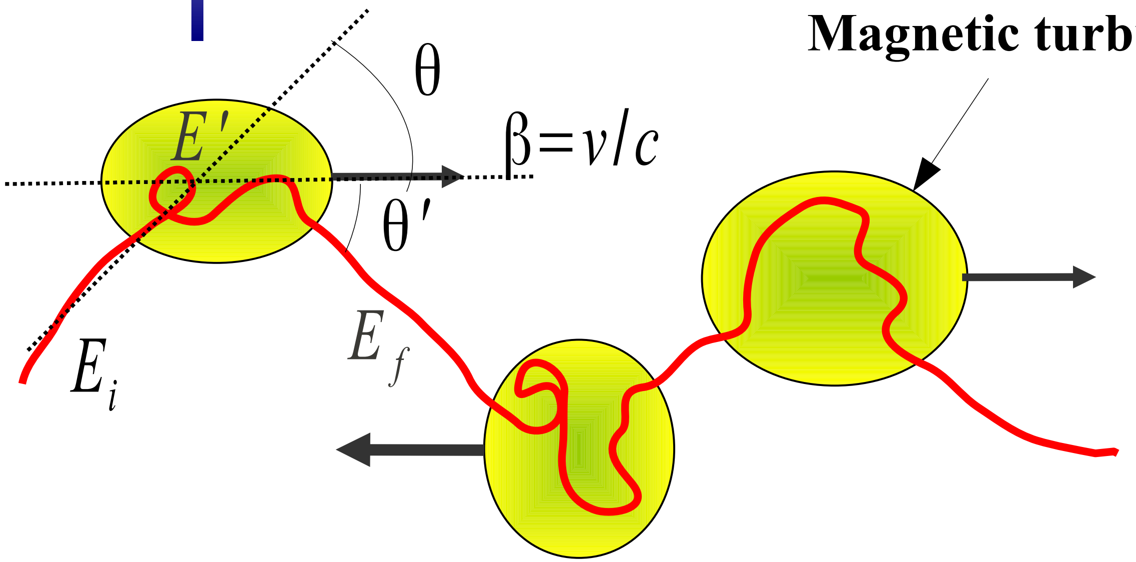
- BUT FOR THE MAJORITY OF ASTROPHYSICAL THE CONDUCTIVITY →∞, HENCE <E> = 0

- THE MAJORITY OF ACCELERATION PROCESS ARE STOCHASTIC

STOCHASTIC ACCELERATION

$$\langle \vec{E} \rangle = 0 \quad \langle \vec{E}^2 \rangle \neq 0$$

A quick look to 2nd order Fermi acceleration (Fermi, 1949)



$$E' = \gamma E_i (1 - \beta\mu)$$

$$E_f' = E_i' = E'$$

$$E_f = \gamma E' (1 + \beta\mu')$$

$$\rightarrow E_f = \gamma^2 E_i (1 - \beta\mu)(1 + \beta\mu')$$

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\mu'} = \int d\mu' \frac{E_f - E_i}{E_i} = 2\left[\gamma^2 (1 - \beta\mu) - 1\right]$$

Assuming isotropy in the cloud's reference frame

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\mu'\mu} = \int_{-1}^{1} d\mu \frac{1}{2} (1 - \beta\mu) 2\left[\gamma^2 (1 - \beta\mu) - 1\right] \propto \beta^2$$

LOSSES AND GAINS ARE PRESENT BUT DO NOT COMPENSATE EXACTLY

A quick look to 2nd order Fermi acceleration (Fermi, 1949)

- IF MAGNETIC FIELD DOES NOT MAKE WORK, WHO ENERGIZE PARTICLES?

MOVING MAGNETIC FIELD → ELECTRIC FIELD

- THE INDUCED ELECTRIC FIELD ENERGIEZES THE PARTICLES

- THE SCATTERING PRODUCES A MOMENTUM TRANSFER, BUT TO WHAT?

A quick look to 2nd order Fermi acceleration (Fermi, 1949)

$$\left\langle \frac{\Delta E}{E} \right\rangle \propto \left(\frac{v}{c} \right)^2$$

- THE ENERGY GAIN IS ONLY PROPORTIONAL TO $(v/c)^2$ AND TYPICALLY $v \sim 10^{-4}\, c$

- THE PREDICTED SPECTRUM STRONGLY DEPENDS ON DETAILS LIKE THE CLOUDS DISTRIBUTION IN THE GALAXY AND THEIR VOLUME FILLING FACTOR
  - IT IS DIFFICULT TO EXPLAIN THE OBSERVED SPECTRUM $E^{-2.7}$
  - THE MAXIMUM ENERGY IS AT MOST ~10 GeV

From 2nd order to 1st order Fermi acceleration

In the '70s many people realized that the Fermi mechanism give a totally different result if applied to shocks (Skilling, 1975; Axford et al., 1977; Krymskii, 1977; Bell, 1978; Blandford and Ostriker, 1978)

WHAT IS A SHOCK?

THE NATURE OF COLLISIONLESS SHOCKS

What is a shock?

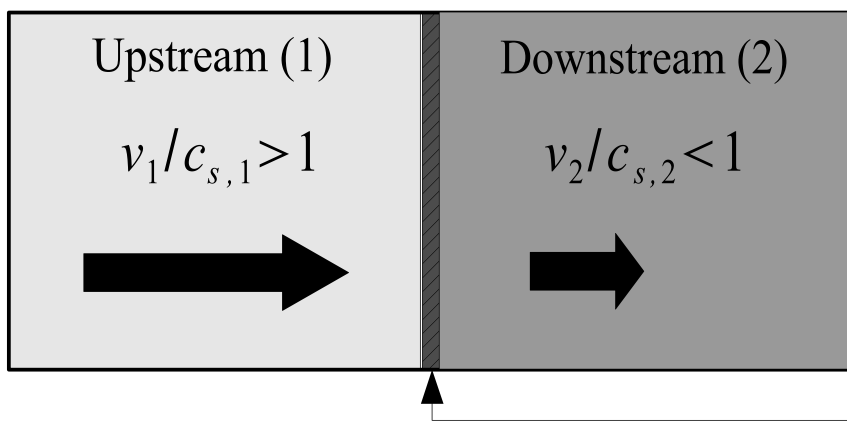
A shock is a discontinuity solution of the fluid equations where a supersonic fluid becomes subsonic (i.e. the entropy increases)

$$[\rho u]_1 = [\rho u]_2$$
$$[\rho u^2 + P]_1 = [\rho u^2 + P]_2$$
$$\left[\frac{1}{2}\rho u^3 + \frac{\gamma}{\gamma-1} P u\right]_1 = \left[\frac{1}{2}\rho u^3 + \frac{\gamma}{\gamma-1} P u\right]_2$$

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right) \rightarrow \frac{2\gamma M_1^2}{\gamma+1}$$
$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2} \rightarrow \frac{\gamma+1}{\gamma-1} \stackrel{\text{def}}{=} r$$
$$\frac{T_2}{T_1} = \frac{P_2}{P_1} \frac{\rho_1}{\rho_2}$$

$M_1 \gg 1$

$M \stackrel{\text{def}}{=} v/c_s$



Shock transition ~ λ

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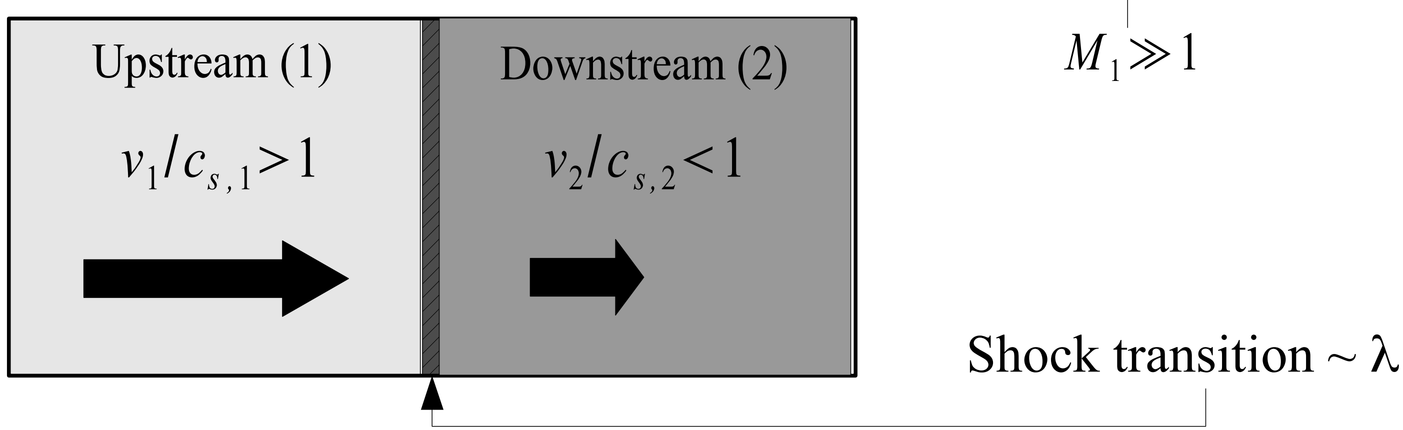

Caveats:

1) What produces the transition?
2) Does the fluid equations describe correctly astrophysical plasmas?

Collisionless Shocks Physics

What produce the shock transition?

$$\lambda_{mfp} \sim \frac{1}{n\sigma} = \begin{cases} \dfrac{1}{N_A \rho_{air} (2\pi a_0^2)} \sim 10^{-7}\, cm & \text{Collisions in air} \\[2ex] \dfrac{1}{n_{ISM}\, \sigma_{Coul}} > 1\, pc & \text{Collisions in the ISM} \end{cases}$$

But observationally
(from Balmer emission):

$$\lambda_{sh} \ll 10^{15}\, cm = 3 \times 10^{-4}\, pc$$

Collisionless Shocks Physics

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Length-scale for EM processes:

Electron skin depth $\quad \sigma_{pe} = \left(\dfrac{4\pi n_e e^2}{m_e} \right)^{1/2} = 5.3\, 10^5\, n_e^{-1/2}\, cm$

Ion skin depth $\quad \sigma_{pi} = \left(\dfrac{4\pi n_i e^2}{m_i} \right)^{1/2} = 2.3\, 10^7\, n_i^{-1/2}\, cm$

p's Larmor radius $\quad r_L(v_{sh}) = \dfrac{m_p v_{sh} c}{eB} = 10^{10} \left(\dfrac{v_{sh}}{3000\, km/s} \right) \left(\dfrac{B}{3\mu G} \right)^{-1} cm$

Shock thickness between these two lengthscale

Electro-magnetic instability

The shock transition is mediated by electromagnetic interactions.
Collisions have no role → the Mach number does not properly describe the shock properties

Alvénic Mach number is more appropriate:

$$M_A = \frac{v_{sh}}{v_A}; \qquad v_A = \frac{B}{\sqrt{4\pi\rho}} \approx 2 B_{\mu G} \left(\frac{n}{cm^{-3}}\right)^{-1/2} km/s$$

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Alfvén waves are a combination of electromagnetic-hydromagnetic waves

Analogy with waves on a string: $v = \sqrt{T/\mu}; \quad T \rightarrow B^2/4\pi, \quad \mu \rightarrow \rho$

Collisionless shocks require $M_A > 1$

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Collisionless shocks require $M_A > 1$

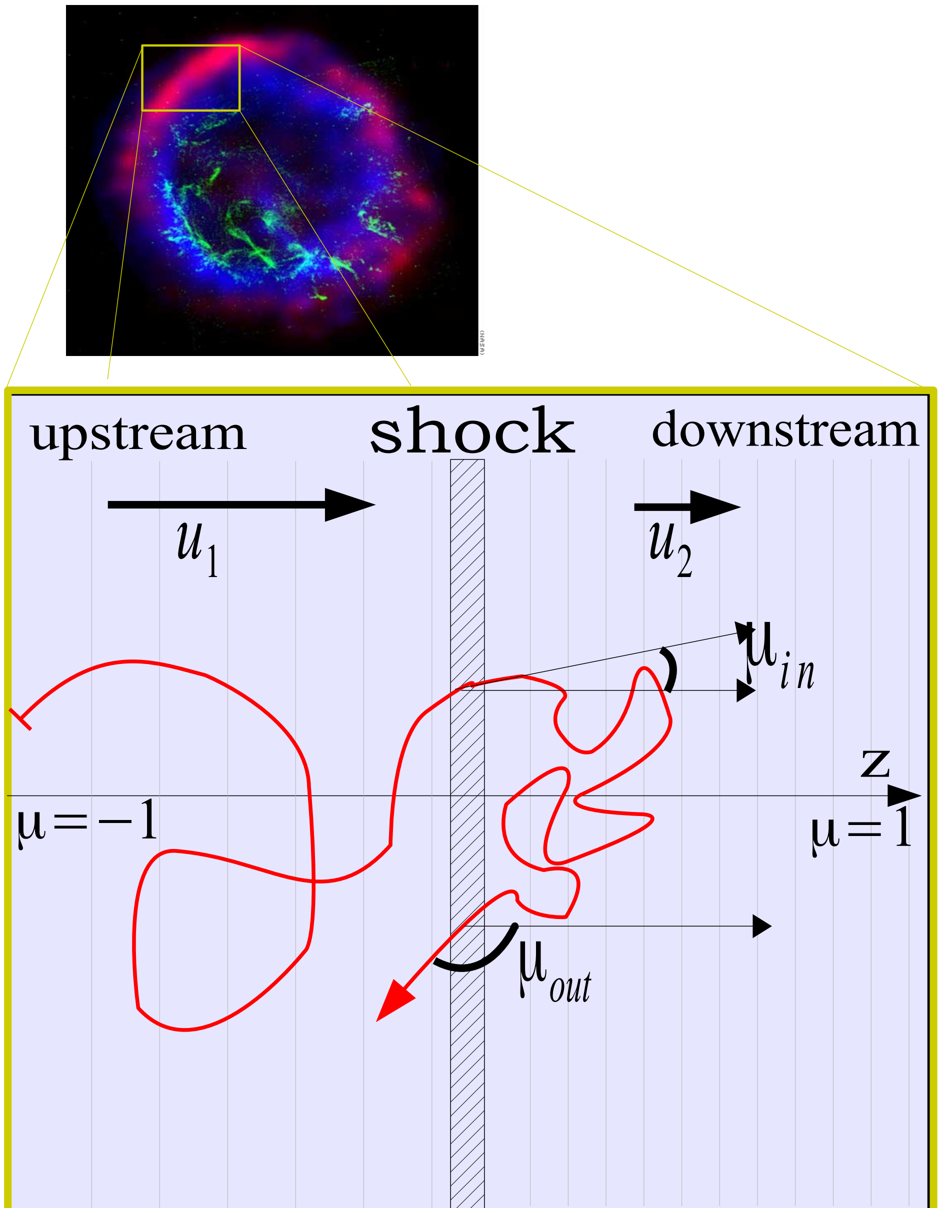
Which instability is responsible for the shock transition?

- Two stream instability
- Weibel instability
- Oblique instability
- Filamentation
- ...

→ **The relative importance depends on the initial conditions of the plasma**

HOW DO SHOCKS ACCELERATE PARTICLES?

ACCELERATION AT SHOCK WAVES: THE TEST-PARTICLE APPROACH



ENERGY GAIN

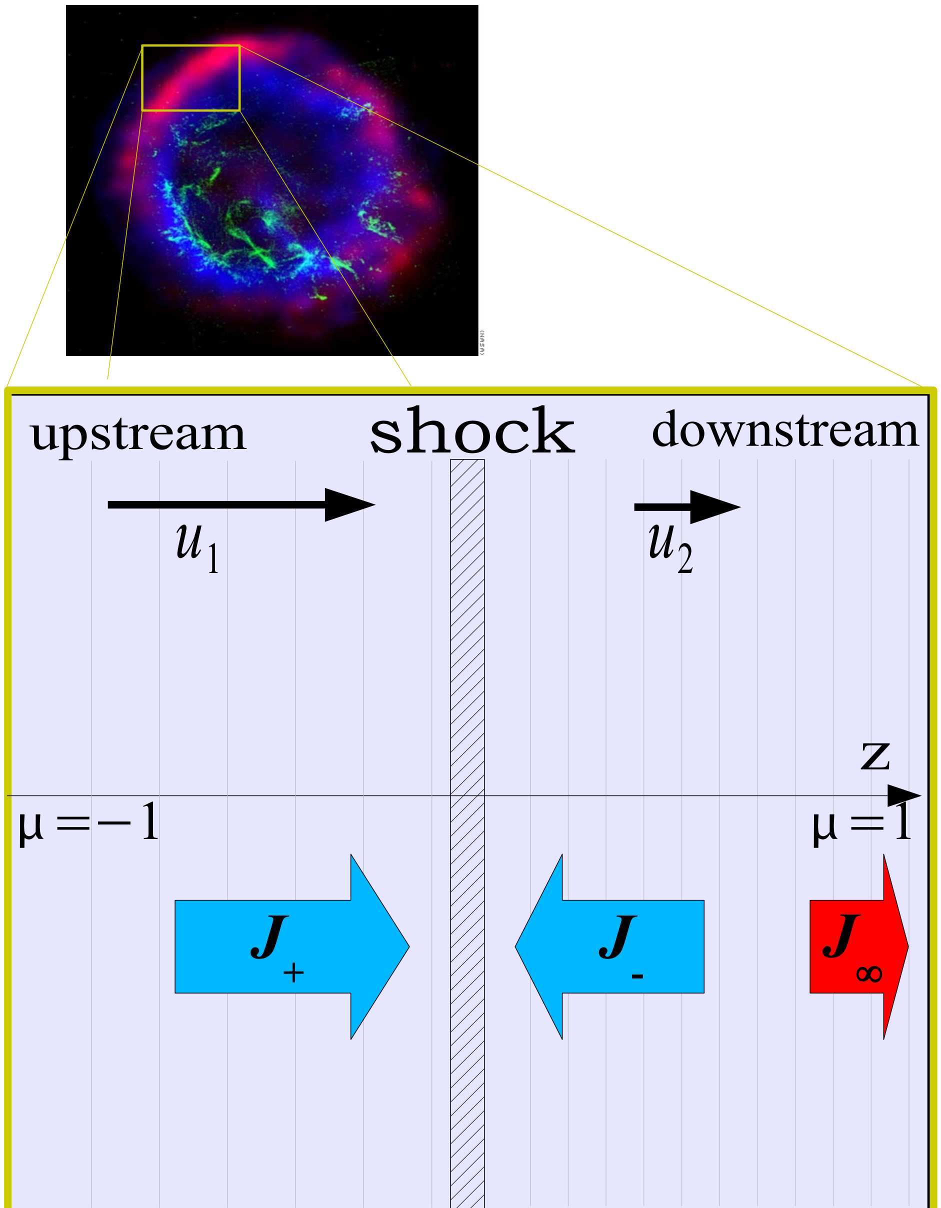
$$E_2 = \frac{(1-\beta_{rel}\mu_{in})(1+\beta_{rel}\mu'_{out})}{1-\beta_{rel}^2} \quad E_1$$

Averaging over $0 < \mu_{\text{£C}} < 1$ and $-1 < \mu_{\varepsilon\text{IN}\ell} < 0$:

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{1+\frac{4}{3}\beta_{rel}+\frac{4}{9}\beta_{rel}^2}{1-\beta_{rel}^2} \; -1 \; \simeq \frac{4}{3}\beta_{rel}$$

The energy gain is now 1[st] order in V_{sh} because in each cycle upstream → downstream → upstream the particle can only gain energy

ACCELERATION AT SHOCK WAVES: PARTICLE SPECTRUM



$$J_\infty = n u_2$$
$$J_- = \int \frac{d\Omega}{4\pi} n c \cos(\theta) = \frac{nc}{4}$$
$$P_{esc} = \frac{J_\infty}{J_+} = \frac{J_\infty}{J_\infty + J_-} \approx 4\frac{u_2}{c}$$

Escaping probability

Energy after k interactions:

$$E_k = E_0(1+\xi)^k \rightarrow k = \frac{\ln(E/E_0)}{\ln(1+\xi)}$$

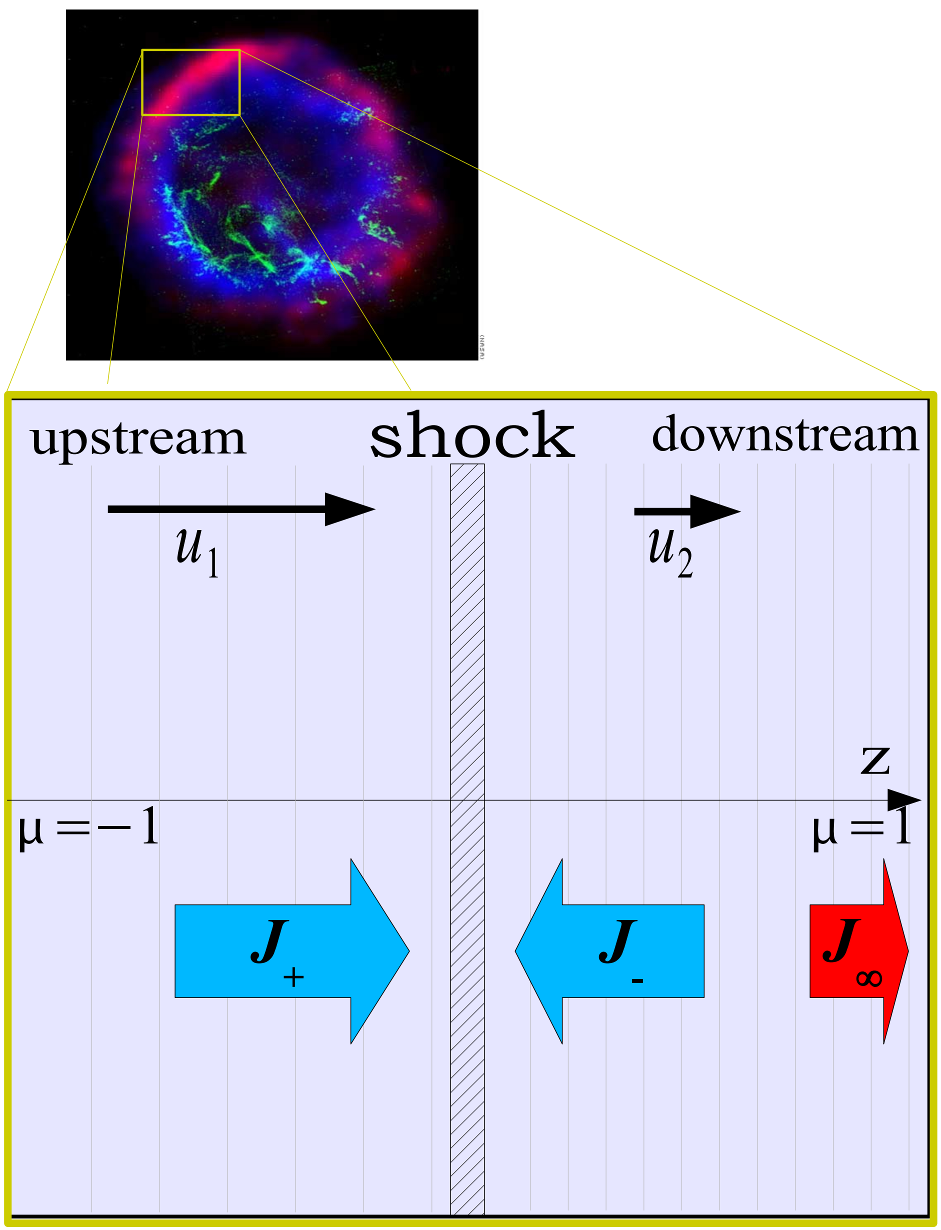
The number of particles with energy > E is:

$$N(>E) \propto \sum_{i=k}^{\infty}(1-P_{esc})^i = \frac{(1-P_{esc})^k}{P_{esc}} = \frac{1}{P_{esc}}\left(\frac{E}{E_0}\right)^{-\delta}$$

$$\delta = -\frac{\ln(1+P_{esc})}{\ln(1+\xi)} \simeq \frac{P_{esc}}{\xi}$$

Both independent on energy

ACCELERATION AT SHOCK WAVES: PARTICLE SPECTRUM



Differential energy spectrum:

$$f(E) \equiv \frac{dN}{dE} \propto E^{-\alpha}$$

Slope:

$$\alpha = 1+\delta \simeq 1+\frac{P_{esc}}{\xi} = 1+\frac{4u_2/c}{4(u_1-u_2)/3c} = \frac{r+2}{r-1} \rightarrow 2$$

For strong shocks and monoatomic gas: $r \equiv \frac{u_1}{u_2} \rightarrow 4$

Spectrum in momentum *p:*

$$4\pi p^2 dp\, f(p)(E) = f(E)\, dE$$

$$f(E) \propto E^{-2} \rightarrow f(p) \propto p^{-4}$$

FINAL REMARKS

Important points:

1) The particle spectrum obtained from the 1st order Fermi acceleration is independent from the scattering properties

2) A power low spectrum is the consequence of P_{esc} and $\Delta E/E$ being independent on the initial energy

3) The slope E^{-2} is valid for strong shocks (r->4)

What depends on the scattering properties is the maximum achievable energy