Cosmic Background Radiation
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short version

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1 Cosmic Background Radiation: Energy Distribution in the Universe

1.1 Olber's paradox

The night sky is dark. This is an obvious fact for us (not actually in cities like Tokyo). But, is it true? In the 19th century, a German astronomer Heinrich Wilhelm Olbers (1758–1840) had doubts about this. If the universe is static and infinite and has an infinite number of stars distributed in an infinitely large space, the night sky should be as bright as the solar surface.

The surface brightness of an object is proportional to F/Ω where F is the flux and Ω is the solid angle. Assuming constant luminosity L at a distant d, then $F \propto L/d^2$. Assuming the size of an object r, then $\Omega \propto r^2/d^2$. Therefore, the surface brightness becomes constant.

There is another approach to the paradox. Assume the number density of objects n is homogeneous and isotropic. Then the number of objects within a shell at a distance r is $dN \propto nr^2dr$. As flux from an object is $F \propto L/r^2$, flux from all stars in a shell is $dI = FdN \propto Lndr$. Therefore, the total flux out to radius R will be $I = \int dI \propto Ln[r]_0^R$. By setting R to ∞, the flux from the sky becomes infinite.

However, we know the universe is not infinite from observations. Therefore, the paradox is solved. But, is the sky *completely* dark? Actually not. There is dim but almost isotropic diffuse emission in the sky. This is the so-called cosmic background radiation.

1.2 Composition and Mass/Energy Density in the Universe

The critical density of the universe is

$$\rho_c c^2 = \frac{3H_0^2 c^2}{8\pi G} = 1.69 \times 10^{-8} h^2 \text{ erg cm}^{-3} = 8.28 \times 10^{-9} h_{0.7}^2 \text{ erg cm}^{-3}. \quad (1)$$

Density parameters are defined as $\Omega_X = \rho_X/\rho_c$. The latest Planck data tell us the composition as follows with $h \sim 0.678$:

	Ω_X	Energy Density [erg/cm^3]
Dark Energy	0.69	5.4×10^{-9}
Dark Matter	0.31	2.4×10^{-9}
Baryon	0.048	3.7×10^{-10}
Neutrino	< 0.0055	$< 4.3 \times 10^{-11}$
Radiation	5.4×10^{-5}	4.2×10^{-13}

1.3 Cosmic Background Radiation in various wavelengths

The cosmic background radiation is almost isotropic radiation in the sky. It is the result of integrated emission from its origins over the cosmic history.

1.3.1 Cosmic Microwave Background Radiation

The cosmic microwave background (CMB) is the thermal radiation left over from the Big Bang. The CMB has a thermal black body spectrum at a temperature of 2.725 K. The CMB energy density at the present day is given by

$$\rho_{\rm CMB} = aT^4 \sim 4.2 \times 10^{-13} \text{ erg/cm}^3 \sim 0.26 \text{ eV/cm}^3, \quad (2)$$

where a is the radiation constant ($4\sigma_{\rm SB}/c = 7.5657 \times 10^{-15}$ erg/cm^3/K^4). The CMB energy density is roughly 10^{-4} of the critical density ρ_c.

1.3.2 Cosmic Optical/Infrared Background Radiation

The cosmic optical background (COB) and the cosmic infrared background (CIB), so-called extragalactic background light (EBL), are the diffuse, isotropic background radiation from far-infrared (FIR) to ultraviolet (UV) wavelengths, is believed to be predominantly composed of the light from stars and dust integrated over the entire history of the universe (Dwek & Krennrich 2013). The observed spectrum of the local EBL at $z = 0$ has two peaks of comparable energy density. The first peak in the optical to the near-infrared (NIR) is attributed to direct starlight, while the second peak in the FIR is attributed to emission from dust that absorbs and reprocesses the starlight.

The energy source of stars is nuclear reaction in stars. In the main-sequence phase, some fraction of the rest mass energy ($\eta_Y = 0.0072$ per unit mass) is released in the $4p \rightarrow^4$ He reaction. Most of the energy go in to radiation. The mass ratio against total baryons of the Helium originated in the Big Bang is $Y_{\rm BN} = 0.24$, while that in current galaxies is $Y_0 = 0.28$. $\Delta Y = 0.04$ of Helium are generated in the cosmic star formation history. If the fraction of baryonic gas contained in galaxies is $f_* \sim 0.1$, the total background energy density will be

$$\rho_{\rm COB} \simeq \frac{\eta_Y \Delta Y f_* \Omega_b \rho_c c^2}{1+z_c} \sim 5.2 \times 10^{-15}\ {\rm erg/cm^3} \sim 3.3 \times 10^{-3}\ {\rm eV/cm^3}, \quad (3)$$

where we take the typical star forming redshift as $z_c = 1$. By converting this value into the flux, we can get $I_{\rm COB} \sim 1.2 \times 10^{-5}$ erg/cm^2/s/sr $\sim$ 12 nW/m^2/sr. The nuclear fusion to heavier elements will also contribute to the COB in the similar order of magnitude.

1.3.3 Cosmic X-ray Background Radiation

The cosmic X-ray background (CXB) is believed to be composed of X-ray emission from AGNs. X-ray emission from AGNs is dominated by Comptonization of disk photons in a corona above the accretion disk. Thus, the radiation energy source is matter accretion on to the central supermassive black hole (SMBH). Here, it is well known that the mass of the SMBH correlates strongly with physical properties of the host galaxy. For example, it is known that the SMBH mass $M_{\rm BH}$ appears a fairly constant fraction of the stellar bulge mass M_b as $M_{\rm BH} \sim 10^{-3} M_b$ (Haring & Rix 2004). The mass density of stars in spheroid is $\Omega_{*,\rm sph} = 0.0018h^{-1}$ (Fukugita & Peebles 1998). The radiation efficiency of BH accretion disk is $\eta \sim 0.1$. The flux ratio between optical and X-ray is known to be $F_X/F_{\rm opt} \sim 0.1$. Therefore, the total CXB energy density is

$$\rho_{\rm CXB} \sim \frac{F_X}{F_{\rm opt}} \frac{\eta f_{\rm BH,\ bulge} \Omega_{*,\ \rm sph}}{1+z_c} \rho_c c^2 \sim 6.4 \times 10^{-17}\ {\rm erg/cm^3} \sim 4.0 \times 10^{-8}\ {\rm keV/cm^3} \quad (4)$$

where we take the typical AGN redshift as $z_c = 3.0$ and $h = 0.70$. By converting this value into the flux, we can get $I_{\rm CXB} \sim 1.5 \times 10^{-7}$ erg/cm^2/s/sr $\sim$ 95 keV/cm^2/s/sr.

1.3.4 Cosmic Gamma-ray Background Radiation

The cosmic gamma-ray background (CGB) is expected to be dominated by blazars, since blazars are the most numerous population in known extragalactic gamma-ray emitting sources. Blazars are divided into two categories by their optical spectra. Those are BL Lacertae objects (BL Lacs) and flat-spectrum radio quasars (FSRQs). Gamma-ray emission from blazars is thought to be originated in the inverse-Compton process in sub-pc scale jet in which electrons scatter internal synchrotron radiation, so-called synchrotron self-Compton emission (SSC: Jones et al., 1974) or external radiation, so-called external Compton (EC: Dermer & Schlickeiser, 1993; Sikora et al., 1994). In the case of BL Lacs, SSC dominates the gamma-ray emission, while EC dominates in the case of FSRQs.

Let consider FSRQs. The blazar jet radiation power is $L_{\rm blz} = 4\pi R_b^2 \Gamma^2 c u'_{\rm seed}$ where R_b is the comoving size of the emitting blob, Γ is the bulk Lorentz factor, and $u'_{\rm seed}$ is the seed photon energy density in the jet comoving frame. R_b is approximated as $R_b \approx \Gamma c t_{\rm var}$, where $t_{\rm var}$ is the observed variability time scale and the beaming factor is approximated $\approx \Gamma$. Assuming typical values of $\Gamma \sim 10$ and $t_{\rm var} \sim 1$ day, we have $R_b \sim 3 \times 10^{16}$ cm. As we consider FSRQs, the emission region locates inside the broad line region (BLR) where the disk radiation is reprocessed. The Thomson scattering opacity of the BLR is $\tau_{\rm sc} \approx n_{e,\rm BLR} \sigma_T r_{\rm BLR} \simeq 0.021 n_{e,\rm BLR,4.5} r_{\rm BLR,18}$ where n_e is the electron density in the BLR and $r_{\rm BLR}$ is the distance to the BLR from the central SMBH. The energy density of scattered disk photons in the jet comoving frame is given by $u'_{\rm disk} = \Gamma^2 \tau_{\rm sc} L_{\rm disk} / 4\pi r_{\rm BLR}^2 c$. From multi-wavelength studies of blazars, the radiation is dominated by gamma rays in FSRQs. Thus, $L_{\gamma,\rm blz} \approx \Gamma^4 \tau_{\rm sc} L_{\rm disk} R_b^2 / r_{\rm BLR}^2$.

The fraction of radio galaxies is ~10% of whole AGNs. Considering the beaming effect, the fraction of blazars is approximated as $f_{\rm blz} \approx 0.1 (1/\Gamma)^2 / 4\pi$, which will be $\sim 8 \times 10^{-5}$ with $\Gamma = 10$. Therefore, the total CGB energy density is

$$\rho_{\rm CGB} \approx f_{\rm blz} \frac{L_{\gamma,\rm blz}}{L_{\rm disk}} \frac{\eta f_{\rm BH,\ bulge} \Omega_{*,\ \rm sph}}{1+z_c} \rho_c c^2 \tag{5}$$

$$\approx f_{\rm blz} \Gamma^4 \tau_{\rm sc} \left(\frac{R_b}{r_{\rm BLR}} \right)^2 \frac{\eta f_{\rm BH,\ bulge} \Omega_{*,\ \rm sph}}{1+z_c} \rho_c c^2 \tag{6}$$

$$\sim 4.6 \times 10^{-19}\ {\rm erg/cm^3} \sim 2.9 \times 10^{-16}\ {\rm GeV/cm^3}, \tag{7}$$

where we assume $\tau_{\rm sc} = 0.01$, $\Gamma = 10$, $R_b = 3 \times 10^{16}$ cm, and $r_{\rm BLR} = 10^{17}$ cm. By converting this value to the flux, we can get $I_{\rm CGB} \sim 1.0 \times 10^{-9}$ erg/cm^2/s/sr $\sim 6.8 \times 10^{-7}$ GeV/cm^2/s/sr.

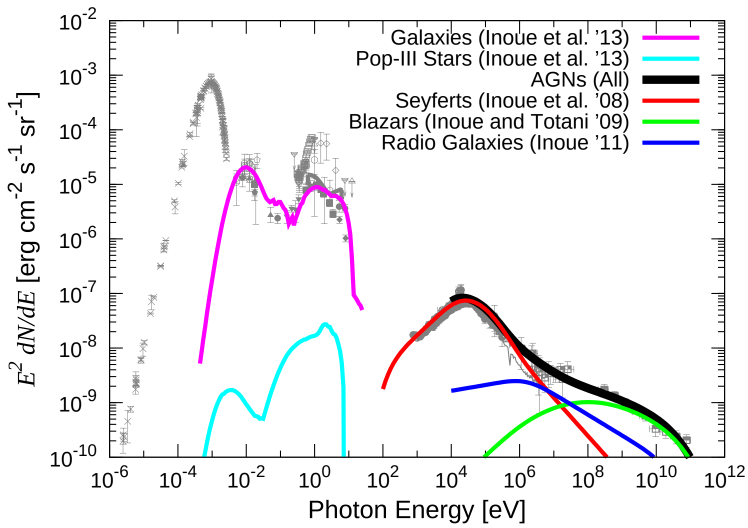

Figure 1: The cosmic background radiation spectrum from microwave to gamma-ray energies. Contribution from galaxies, Pop-III stars, Seyferts, blazars, radio galaxies, and all AGNs is shown by purple, cyan, red, green, blue, and black curve, respectively.

Figure. 1 shows the measured cosmic background radiation spectrum from microwave to gamma rays. Although the estimations above subsections are simplified calculation, the order of estimation is roughly consistent with the measurements.

1.4 Cosmic Background Radiation Spectrum

The intensity of the cosmic background radiation is given by per time, surface area, solid angle, and energy as I_ν, dI/dE, or etc. The unit will be [erg/cm^2/s/Hz/sr], [erg/cm^2/s/eV/sr], or else.

Although the cosmic microwave background is truly diffuse emission, the background radiation in other wavelengths is the integration of light from individual sources. Let assume the comoving number density of a source class, which is called as the luminosity function, is $\phi(L_{\rm ref}, z)$ where $L_{\rm ref}$ is the luminosity at a reference energy and z is the redshift. For example, in the GeV gamma-ray band, 0.1–100 GeV luminosity $L_{0.1-100\ \rm GeV}$ is often used. The total number density of a source class at z is given by $\int dL_{\rm ref}\phi(L_{\rm ref}, z)$.

Flux from a source is given by a function of $L_{\rm ref}$, z, E as

$$F_\nu(L_{\rm ref}, z, \nu) = \frac{1+z}{4\pi d_L(z)^2} L_\nu[L_{\rm ref}, z, (1+z)\nu], \tag{8}$$

where $L_\nu[L_{\rm ref}, z, \nu] = dL[L_{\rm ref}, z, \nu]/d\nu$ is the radiation spectrum of a source (the unit is [erg/s/Hz] or else) and $d_L(z)$ is the luminosity distance. The factor of $(1+z)$ is for the redshift effect correction term for $d\nu$.

The number count of a source class per solid angle and per flux at a flux F_ν is

$$\begin{aligned} \frac{dN}{dF_\nu} &= \frac{d}{dF_\nu} \int dL_{\rm ref} \int dz \frac{d^2V}{dz d\Omega} \phi(L_{\rm ref}, z) &(9)\\ &= \frac{\partial L_{\rm ref}}{\partial F_\nu} \int dz \frac{d^2V}{dz d\Omega} \phi(L_{\rm ref}, z), &(10) \end{aligned}$$

where $dV^2/dz d\Omega$ is the comoving volume element. Thus, the cumulative source counts brighter than $F_{\nu,{\rm min}}$ becomes

$$N(> F_{\nu,{\rm min}}) = \int_{F_{\nu,{\rm min}}} dL_{\rm ref} \int dz \frac{d^2V}{dz d\Omega} \phi(L_{\rm ref}, z). \tag{11}$$

The diffuse intensity from sources brighter than $F_{\nu,{\rm min}}$ is

$$\begin{aligned} I_\nu(\nu, F_{\nu,{\rm min}}) &= \int_{F_{\nu,{\rm min}}} dF_\nu F_\nu \frac{dN}{dF_\nu} &(12)\\ &= \int_{L_{\rm ref,min}} dL_{\rm ref} \int dz \frac{d^2V}{dz d\Omega} \phi(L_{\rm ref}, z) F_\nu &(13)\\ & &(14) \end{aligned}$$

The total background intensity is given by setting $F_{\nu,{\rm min}} \to 0$:

$$\begin{aligned} I_\nu(\nu) &= \int dL_{\rm ref} \int dz \frac{d^2V}{dz d\Omega} \phi(L_{\rm ref}, z) F_\nu &(15)\\ &= \int dL_{\rm ref} \int dz \frac{d^2V}{dz d\Omega} \phi(L_{\rm ref}, z) \frac{1+z}{4\pi d_L(z)^2} L_\nu[L_{\rm ref}, z, (1+z)\nu] &(16) \end{aligned}$$

Since the universe is homogeneous, we can estimate the total background intensity by time integration of radiative energy per comoving volume:

$$I_\nu(\nu) = \int dL_{\rm ref} \int dz \frac{c}{4\pi} \left| \frac{dt}{dz} \right| \phi(L_{\rm ref}, z) L_\nu[L_{\rm ref}, z, (1+z)\nu] \tag{17}$$

The cosmic energy density of the universe in the unit of [erg/cm^3/Hz] be given in the form of

$$\rho_\nu(\nu) = \frac{4\pi}{c} I_\nu(\nu), \tag{18}$$

assuming the particle velocity is equal to the speed of light.

In cosmology,

$$\frac{dt}{dz} = -\frac{1}{H_0(1+z)\sqrt{(1-\Omega_M-\Omega_\Lambda)(1+z)^2+\Omega_M(1+z)^3+\Omega_\Lambda}} \tag{19}$$

$$\frac{d^2V}{dzd\Omega} = \frac{cd_L^2}{4\pi(1+z)}\left|\frac{dt}{dz}\right| \tag{20}$$

2 Attenuation of High Energy Particles Propagating the Universe

2.1 Gamma-ray Attenuation

High energy gamma rays propagating the universe are absorbed by background radiation via electron and positron pair creation. Setting the gamma-ray energy E_γ and the background photon energy ϵ, the inner product of four-vector $E_\gamma\epsilon(1-\cos\theta)$ is the Lorentz invariant and it is equal to $2E_{\rm CM}^2$ where $E_{\rm CM}$ is the individual photon energy in the center-of-mass system. Here, θ is the angle between the gamma ray and the background photon. Then, $E_{\rm CM} = \sqrt{0.5E_\gamma\epsilon(1-\cos\theta)}$. The pair creation occurs when $E_{\rm CM} > m_ec^2$. The cross section peaks at $\sim 1.4m_ec^2$. The background photon energy is

$$\epsilon_{\rm peak} \simeq \frac{2m_e^2c^4}{E_\gamma} \simeq 0.5\left(\frac{E_\gamma}{1\ \rm TeV}\right)^{-1}\ \rm eV. \tag{21}$$

In terms of wavelength, $\lambda_{\rm peak} \simeq 2.4(E_\gamma\ [\rm TeV])\ \mu m$.

The cross section for the pair production process is [**?**]

$$\sigma_{\gamma\gamma}(E_\gamma,\epsilon,\theta) = \frac{3\sigma_T}{16}(1-\beta^2)\left[2\beta(\beta^2-2)+(3-\beta^4)\ln\left(\frac{1+\beta}{1-\beta}\right)\right], \tag{22}$$

where $\sigma_{\rm T} = 8\pi e^4/(3m_e^2) = 6.65\times10^{-25}$ cm^2 is the Thomson cross section, and β is

$$\beta \equiv \sqrt{1-\frac{2m_e^2c^4}{\epsilon E_\gamma(1-\cos\theta)}};\quad \mu\equiv\cos\theta. \tag{23}$$

For a photon emitted by a source at redshift z_s and observed at $z = 0$ with energy E_γ, the contribution to the $\gamma\gamma$ optical depth between z_s and z_0 ($0 < z_0 < z_s$) is

$$\tau_{\gamma\gamma}(E_\gamma, z_0, z_s) = \int_{z_0}^{z_s} dz \int_{-1}^{1} d\mu \int_{\epsilon_{\rm th}}^{\infty} d\epsilon c \left|\frac{dt}{dz}\right| \frac{1-\mu}{2} \frac{dn(\epsilon, z)}{d\epsilon} \sigma_{\gamma\gamma}(E_\gamma(1+z), \epsilon, \theta), \tag{24}$$

where $\epsilon_{\rm th}$ is the pair production threshold energy $\epsilon_{\rm th} = 2m_e^2c^4/E_\gamma/(1+z)/(1-\mu)$ and $dn(\epsilon, z)/d\epsilon$ is the proper photon number density in the unit of cm^{-3} eV^{-1}.

2.1.1 Cascade Emission

Electron–positron pairs created by VHE gamma-rays with EBL scatter the cosmic microwave background (CMB) radiation via the inverse Compton (IC) scattering and generate secondary gamma-ray emission component (the so-called cascade emission) in addition to the absorbed primary emission (e.g. Fan et al. '04, Ichiki et al. '08). At redshift z, the scattered photon energy $E_{\gamma,c}$ appears at lower energy than the intrinsic photon energy $E_{\gamma,i}$, typically

$$E_{\gamma,c} \approx \left(\frac{E_{\gamma,i}}{2m_e c^2}\right)^2 E_{\rm CMB}(z) \simeq 0.6\,(1+z)\left(\frac{E_{\gamma,i}}{1\,{\rm TeV}}\right)^2 \ {\rm GeV}. \tag{25}$$

Given the intrinsic luminosity $dL_{\rm int}/dE_\gamma$, we calculate the cascade emissivity $dL_{\rm cas}/dE_\gamma$ as:

$$\frac{dL_{\rm cas}}{dE_\gamma}(E_\gamma, z) = \int_{\gamma_{e,\rm min}}^{\gamma_{e,\rm max}} d\gamma_e \frac{dN_e}{d\gamma_e} \frac{d^2N_{\gamma_e,\epsilon}}{dt dE_\gamma} t_{IC}(\gamma_e, z), \tag{26}$$

where $t_{IC}(\gamma_e, z)$ is the energy-loss time of an electron with a Lorentz factor γ_e and mass m_e by the inverse Compton (IC) emission in the local rest frame,

$$t_{IC}(\gamma_e, z) = \frac{3m_e c}{4\gamma_e \sigma_T u_{\rm CMB}(z)} \approx 7.7\times10^{13}\left(\frac{\gamma_e}{10^6}\right)^{-1}(1+z)^{-4}\ {\rm s}, \tag{27}$$

σ_T is the Thomson scattering cross section, and $u_{\rm CMB}(z)$ is the CMB energy density at z. We consider the CMB photons only here, since the EBL energy density is two orders magnitudes lower than that of CMB. $dN_e/d\gamma_e$ is the electron injection spectrum:

$$\frac{dN_e}{d\gamma_e} = 2\frac{dE_{\gamma,i}}{d\gamma_e}\frac{dL_{\rm int}(E_{\gamma,i}, z)}{dE_{\gamma,i}}\left[1 - e^{-\tau_{\gamma\gamma}(E_{\gamma,i}/(1+z), z)}\right], \tag{28}$$

and $d^2N_{\gamma_e,\epsilon}/dtdE_\gamma$ is the scattered photon spectrum per unit time by the IC scattering:

$$\frac{d^2N_{\gamma_e,\epsilon}}{dtdE_\gamma} = \frac{3\sigma_T c}{4\gamma_e^2}\int d\epsilon \frac{1}{\epsilon}\frac{dn_{\rm CMB}}{d\epsilon}(\epsilon, z)f(x) \tag{29}$$

with $f(x) = 2x\ln(x) + x + 1 - 2x^2$, $(0 < x < 1)$ and $x = E_\gamma/4\gamma_e^2\epsilon$. Here, $E_{\gamma,i} = 2\gamma_e m_e c^2$ is the energy of intrinsic photons and $dn_{\rm CMB}/d\epsilon$ is the CMB photon density. The integration region over the Lorentz factor, γ_e, is $\gamma_{e,\rm min} < \gamma_e < \gamma_{e,\rm max}$, $\gamma_{e,\rm max} = E_{\rm max}/2m_ec^2$ and $\gamma_{e,\rm min} = (E_\gamma/\epsilon)^{1/2}/2$. Since the cooling time $t_{\rm IC}$ is usually shorter than the comoving time, we assume that pairs generate photons at the pair creation site. Because of this fast cooling, the low energy photon spectrum below 100 MeV becomes $\Gamma_{\rm ph} = 1.5$. We do not take into account this spectral effect since it does not affect the VHE spectrum.

The intergalactic magnetic field (IGMF) effect is not important in the study of the CGB spectrum. Although IGMF bends motion of created charged pairs and some fraction of beamed emission is lost, off-axis sources complement this loss. The synchrotron cooling is also not effective for pairs typically created outside galaxies.

2.1.2 Pair Echo Emission: Effect of Magnetic Field

The secondary emission delays and spreads out with a characteristic timescale depending on the properties of the intergalactic magnetic field (IGMF). Therefore, various time scales should be involved to evaluate the time dependent pair echo emission, although we have considered the net secondary flux only in the subsection above. Those are the duration of primary emission $\Delta t_{\rm var}$, angular spreading time $\Delta t_{\rm ang}$, and the delay time due to magnetic deflections Δt_B.

The direction of the upscattered secondary photons deviates from the directions of the parent electron or positron and the primary gamma ray by angles of $\sim 1/\gamma_e$. so the angular spreading time is

$$\Delta t_{\rm ang}(\gamma_e, z) \approx (1+z)\frac{\lambda_{\gamma\gamma} + \lambda_{\rm IC}}{2\gamma_e^2 c}, \tag{30}$$

where the pair creation mean free path $\lambda_{\gamma\gamma} \approx 0.26\sigma_T n_{\rm CIB} \simeq 20\,{\rm Mpc}(n_{\rm CIB}/0.1\,{\rm cm}^{-3})^{-1}$ and the IC cooling length $\lambda_{\rm IC} = 3m_ec^2/(4\gamma_e\sigma_T u_{\rm CMB}) \simeq 690\,{\rm kpc}(\gamma_e/10^6)^{-1}(1+z)^{-4}$.

IGMFs in the propagation region of the pairs gives further deflection. The magnetic deflection angle is

$$\theta_B = \begin{cases} \sqrt{\lambda_{\rm IC} r_{\rm coh}/r_L^2} & (r_{\rm coh} < \lambda_{\rm IC}) \\ \lambda_{\rm IC}/r_L & (r_{\rm coh} > \lambda_{\rm IC}), \end{cases} \tag{31}$$

where $r_{\rm coh}$ is the coherent length and the Larmor radius $r_L = \gamma_e m_e c^2/eB$. The delay time due to magnetic deflection is

$$\Delta t_B \approx (1+z)(\lambda_{\gamma\gamma} + \lambda_{\rm IC})\frac{\theta_B^2}{2c} \tag{32}$$

Therefore, the pair echo delay time can be $\Delta t = \max[\Delta t_{\rm var}, \Delta t_{\rm ang}, \Delta t_{\rm B}]$.

Reference (Incomplete)

This note is based on

- Peacock, J. A. 1999, Cosmological Physics, by John A. Peacock, pp. 704. ISBN 052141072X. Cambridge, UK: Cambridge University Press, January 1999.,
- Totani, T., Lecture note on High Energy Astrophysics, Kyoto University (Japanese version only)