Dark Matter Distribution Around Massive Black Holes

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Theory Talk!

Question

What is the effect of the massive black hole on the dark matter distribution at the galactic centre?

First Motivation [1]

Testing black hole no-hair theorem for the Galactic center object

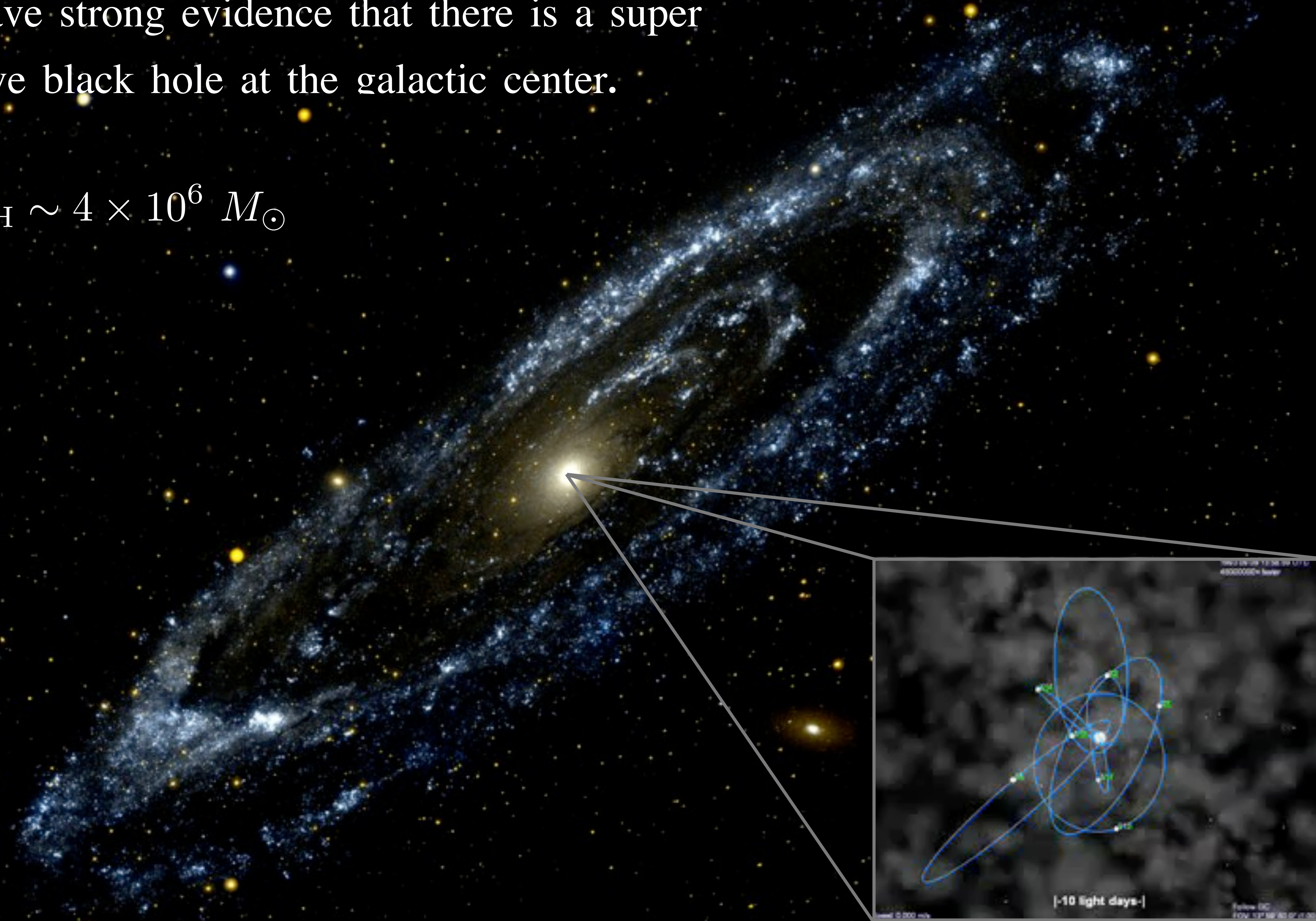
Second Motivation [2]

Indirect detection of dark matter

1. LS and C. M. Will, Class. Quantum Grav. 28 225059, 2011 (arXiv: 1106.5056)
2. LS, F. Ferrer, and C. Will, PRD 88, 063522, 2013 (arXiv: 1305.2619)

We have strong evidence that there is a super massive black hole at the galactic center.

$$m_{\rm BH} \sim 4 \times 10^6 \; M_\odot$$



- The black hole no-hair theorem

All properties of a neutral Black Hole are determined by its **mass (m)** and **spin (J)**.

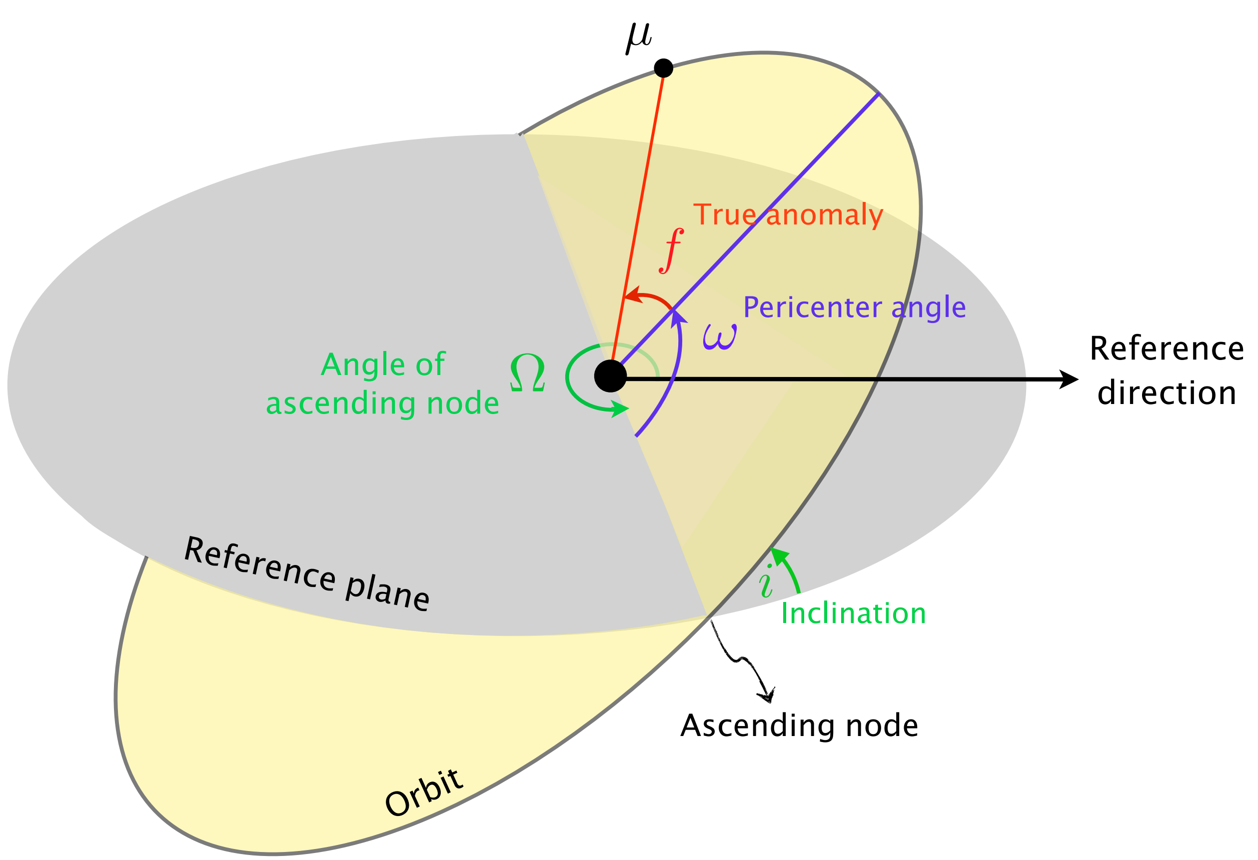
$$Q_2 = -\frac{J^2}{m}$$ if no-hair theorem holds.

quadrupole moment

Question

Does the above equation hold for the central object in our galaxy?

Using the galactic center black hole to test the no-hair theorem [1]



Orbit perturbation in field of a rotating BH:

$$\text{E.O.M.} = \text{Newtonian} + \underbrace{\text{Schwarzschild} + \text{Frame dragging} + \text{Quadrupolar}}_{\text{perturbing terms}}$$

1. C. M. Will, Astrophys. J. Lett. 674 L25, 2008 (arXiv: 0711.1677)

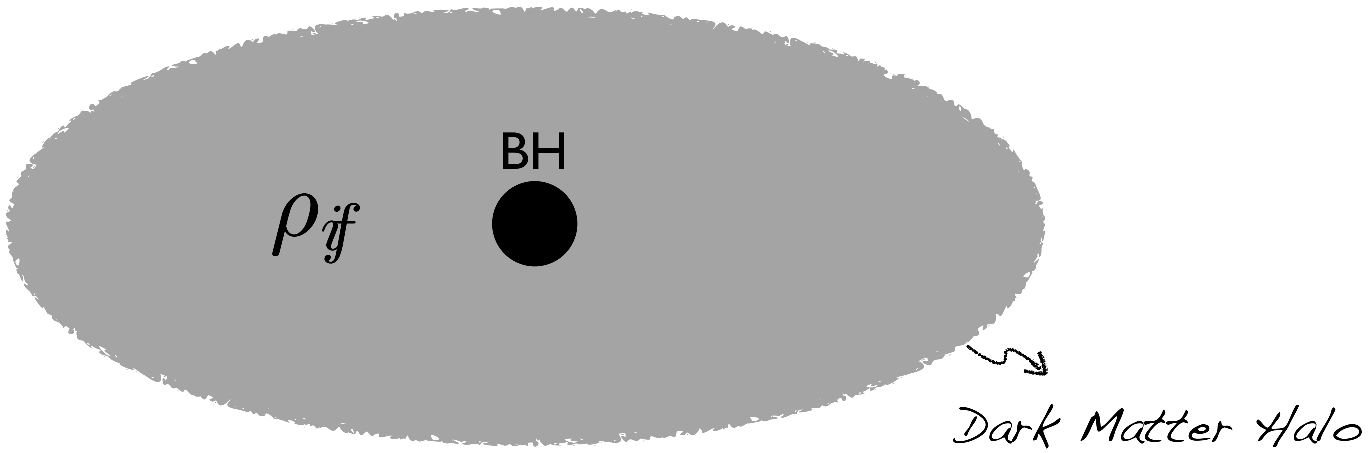
Frame-dragging and **quadrupolar** effects ⇒ precession of stars orbital planes

observations of the orbits precession of at least two stars ⇒ determination of **spin** and **quadrupole** moment

Complications in testing the black hole no-hair theorem using stellar motion:

- Perturbing effects of a **distribution of stars** in the surrounding cluster

- Perturbing effects of **dark matter**



Assumption:

The growth of the massive black hole is adiabatic (slow).

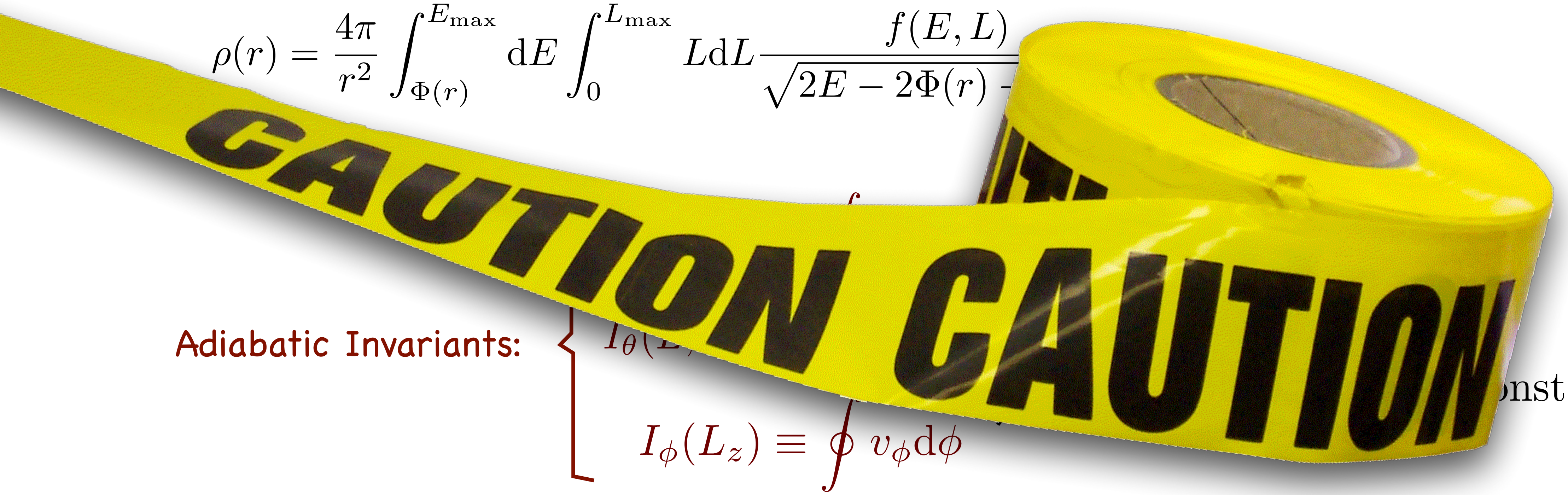
$$\rho_i \xrightarrow{\text{Eddington's Method}} f_i \xrightarrow{\text{Adiabatic Invariants}} f_f \xrightarrow{\text{integration in the phase space}} \rho_f$$

Newtonian Analysis

$$\rho(\mathbf{r}) = \int f(\mathbf{r},\mathbf{v})\mathrm{d}^3\mathbf{v}$$

Change of variables: $\mathbf{v} \;\rightarrow\; E\;,\; L\;,\; L_z$

$$\rho(r) = \frac{4\pi}{r^2}\int_{\Phi(r)}^{E_{\rm max}} \mathrm{d}E \int_0^{L_{\rm max}} L\mathrm{d}L \frac{f(E,L)}{\sqrt{2E - 2\Phi(r) - \cdots}}$$



Adiabatic Invariants: $I_\theta(L$ … onstant.

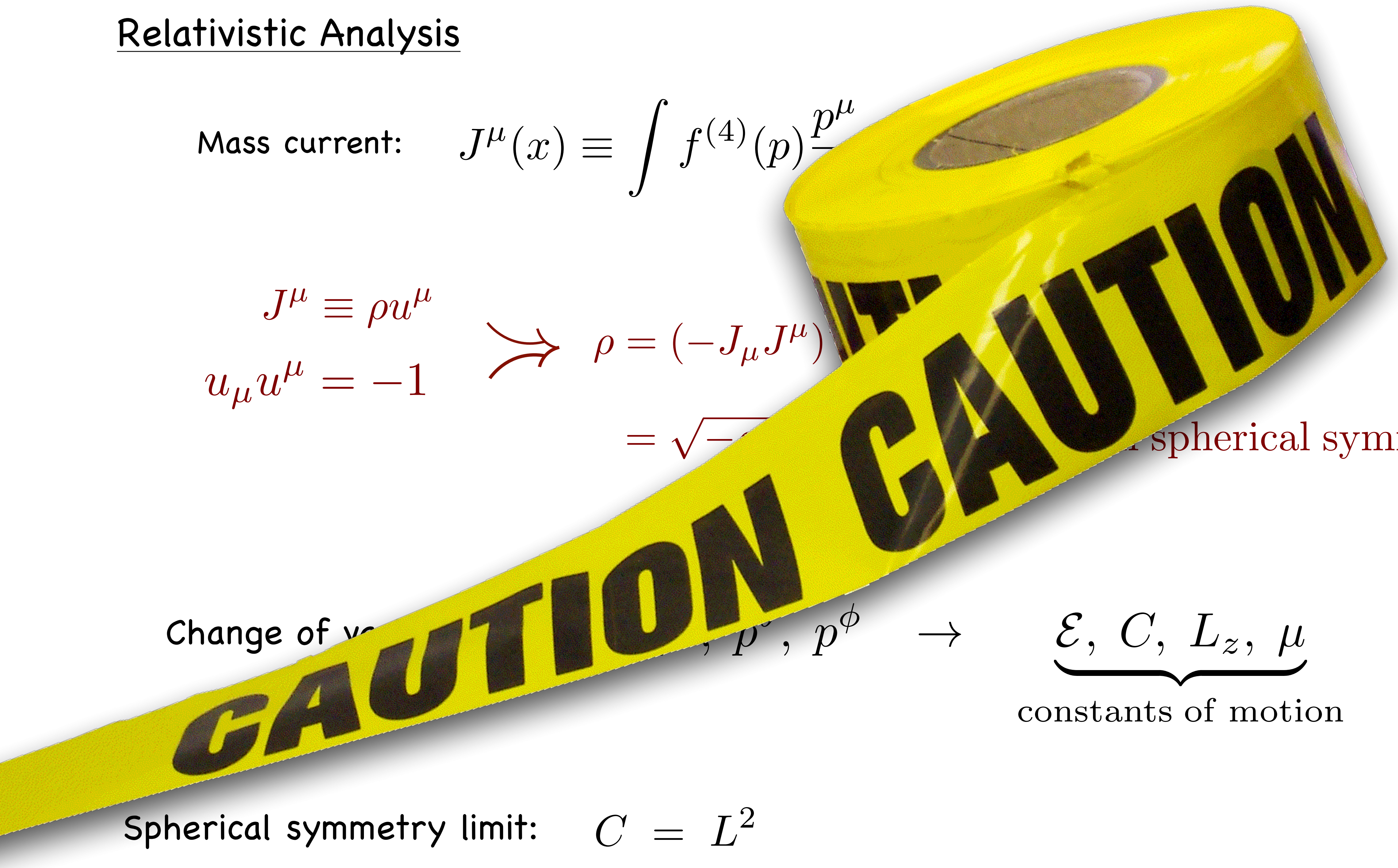
$$I_\phi(L_z) \equiv \oint v_\phi \mathrm{d}\phi$$

$$\underbrace{I_{r,i}(E_i,L)}_{\rm DM} = \underbrace{I_{r,f}(E_f,L)}_{\rm DM+BH} \quad \Rightarrow \quad E_i = E_i(E_f,L)$$

$$\boxed{f_f(E_f,L) = f_i(E_i(E_f,L),L)}$$

Relativistic Analysis

Mass current: $J^{\mu}(x) \equiv \int f^{(4)}(p) \frac{p^{\mu}}{\ldots}$



$J^{\mu} \equiv \rho u^{\mu}$

$u_{\mu} u^{\mu} = -1$

$\Rightarrow \rho = (-J_{\mu} J^{\mu}) \ldots$

$= \sqrt{-\ldots}$ … spherical symmetry)

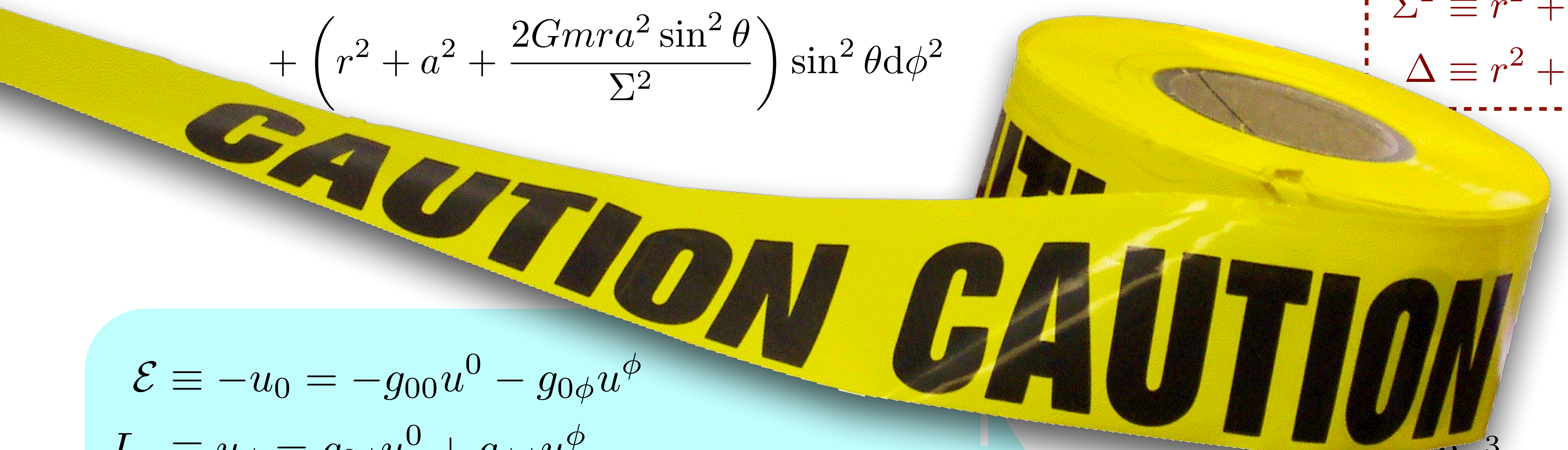
Change of v… $\ldots, p^{\ldots}, p^{\phi} \rightarrow \underbrace{\mathcal{E},\ C,\ L_z,\ \mu}_{\text{constants of motion}}$

Spherical symmetry limit: $C = L^2$

Kerr metric in Boyer-Lindquist coords:

$$\mathrm{d}s^2 = -\left(1 - \frac{2Gmr}{\Sigma^2}\right)\mathrm{d}t^2 + \frac{\Sigma^2}{\Delta}\mathrm{d}r^2 + \Sigma^2\mathrm{d}\theta^2 - \frac{4Gmra}{\Sigma^2}\sin^2\theta\,\mathrm{d}t\mathrm{d}\phi + \left(r^2 + a^2 + \frac{2Gmra^2\sin^2\theta}{\Sigma^2}\right)\sin^2\theta\,\mathrm{d}\phi^2$$

$$a \equiv J/m$$
$$\Sigma^2 \equiv r^2 + a^2\cos^2\theta$$
$$\Delta \equiv r^2 + a^2 - 2Gmr$$



$$\mathcal{E} \equiv -u_0 = -g_{00}u^0 - g_{0\phi}u^\phi$$
$$L_z \equiv u_\phi = g_{0\phi}u^0 + g_{\phi\phi}u^\phi$$
$$C \equiv \Sigma^4\left(u^\theta\right)^2 + \sin^{-2}\theta L_z^2 + a^2\cos^2\theta(1-\mathcal{E}^2)$$
$$\mu^2 = -g_{\mu\nu}p^\mu p^\nu$$

$$\sqrt{-g}\,\mathrm{d}^4p = \frac{2\mu^3}{\Sigma^2\Delta|u_r||u^\theta|\sin\theta}\mathrm{d}\mathcal{E}\mathrm{d}C\mathrm{d}L_z\mathrm{d}\mu$$

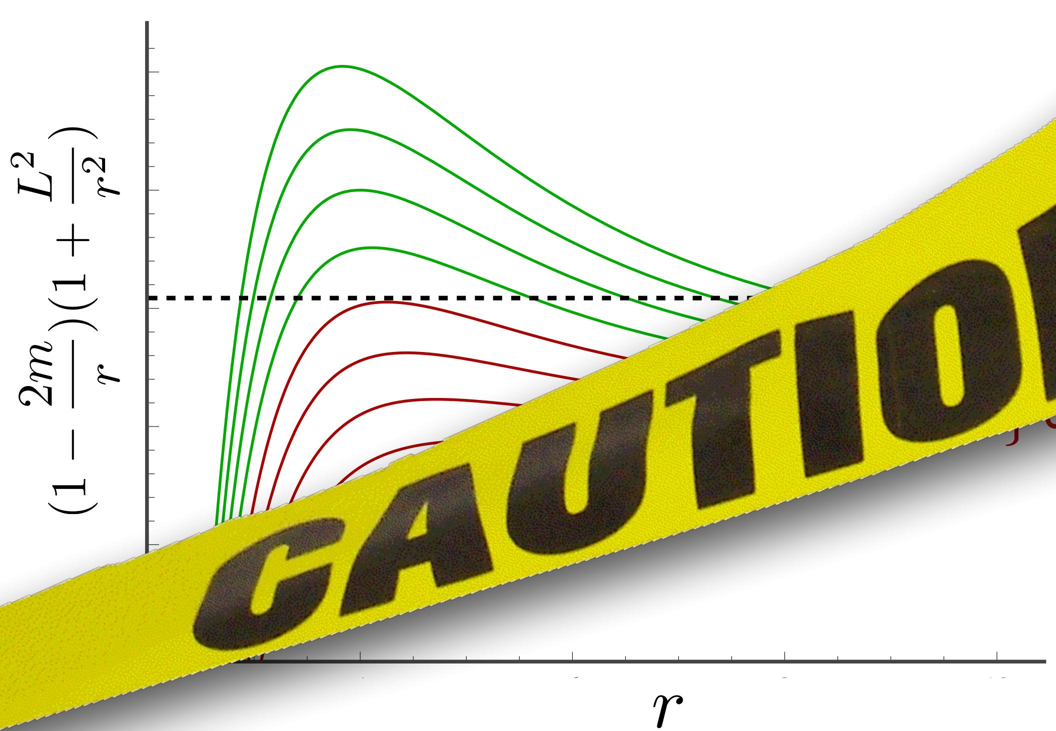
Schwarzschild BH: $$\rho(r) = -\frac{4}{r^4\sqrt{1-2Gm/r}\sin\theta}\int f(\mathcal{E},L)\frac{\mathcal{E}}{u^r u^\theta}\mathrm{d}\mathcal{E}\mathrm{d}L^2\mathrm{d}L_z$$

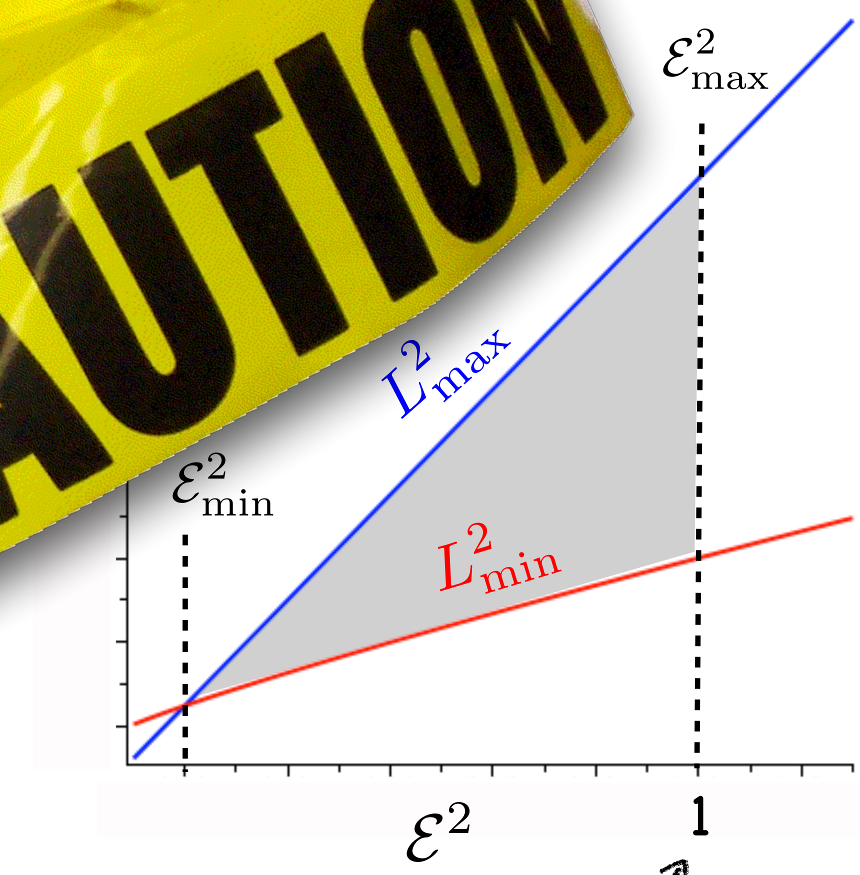
$$u^\theta = r^{-2}\sqrt{L^2-\frac{L_z^2}{\sin^2\theta}} \quad \rightsquigarrow \quad L_{z,min},\ L_{z,max}$$

$$u^r = \sqrt{V(r)} \quad \rightsquigarrow \quad L^2_{max}$$

$$\left.\begin{aligned} V(r) = \mathcal{E}^2 - (1-\frac{2m}{r})(1+\frac{L^2}{r^2}) \\ V(r)=0\,,\ \frac{\mathrm{d}V}{\mathrm{d}r}=0 \end{aligned}\right\} \quad \rightsquigarrow \quad L^2_{min}$$

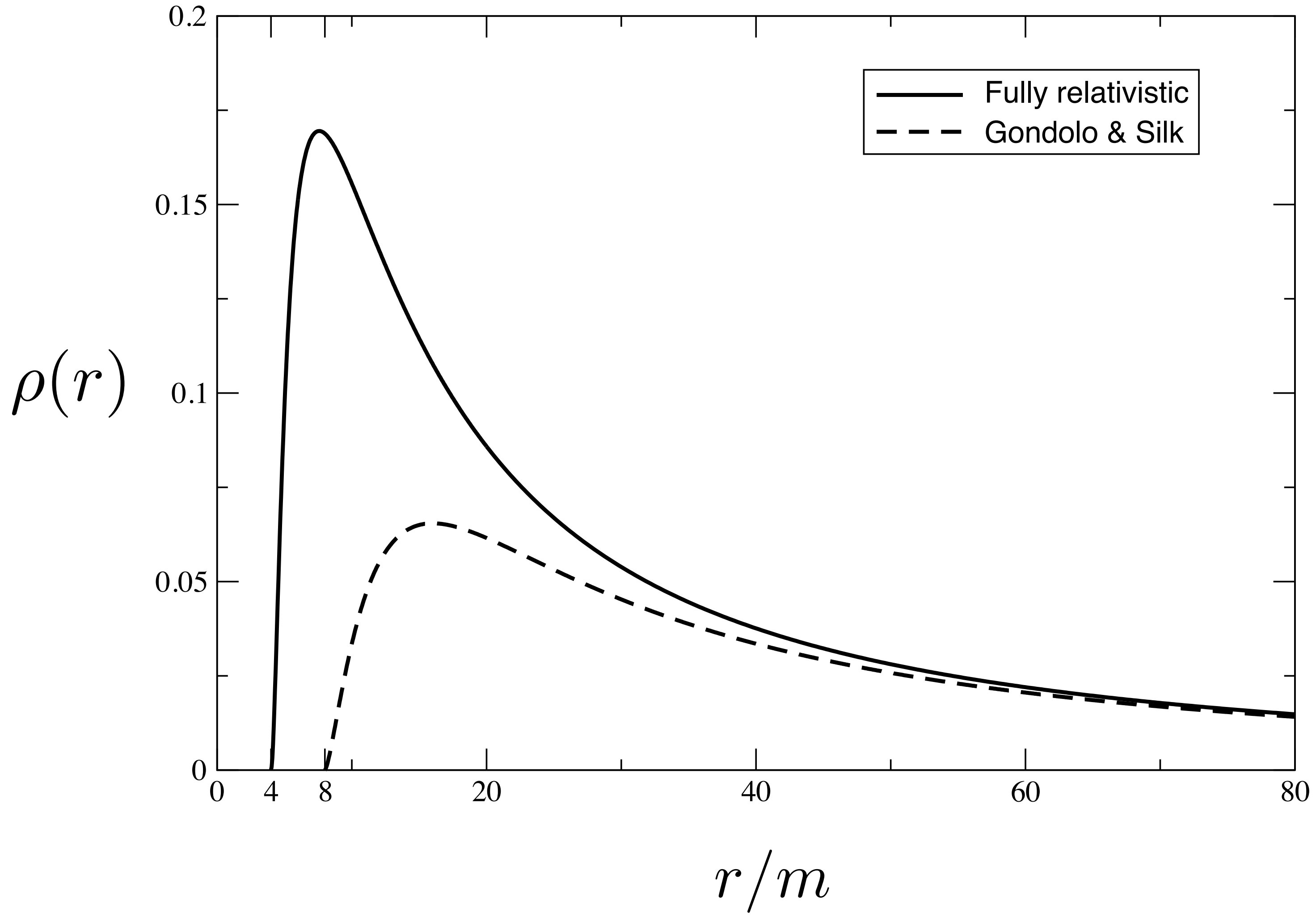
CAUTION CAUTION CAUTION





bounded non-relativistic DM particles

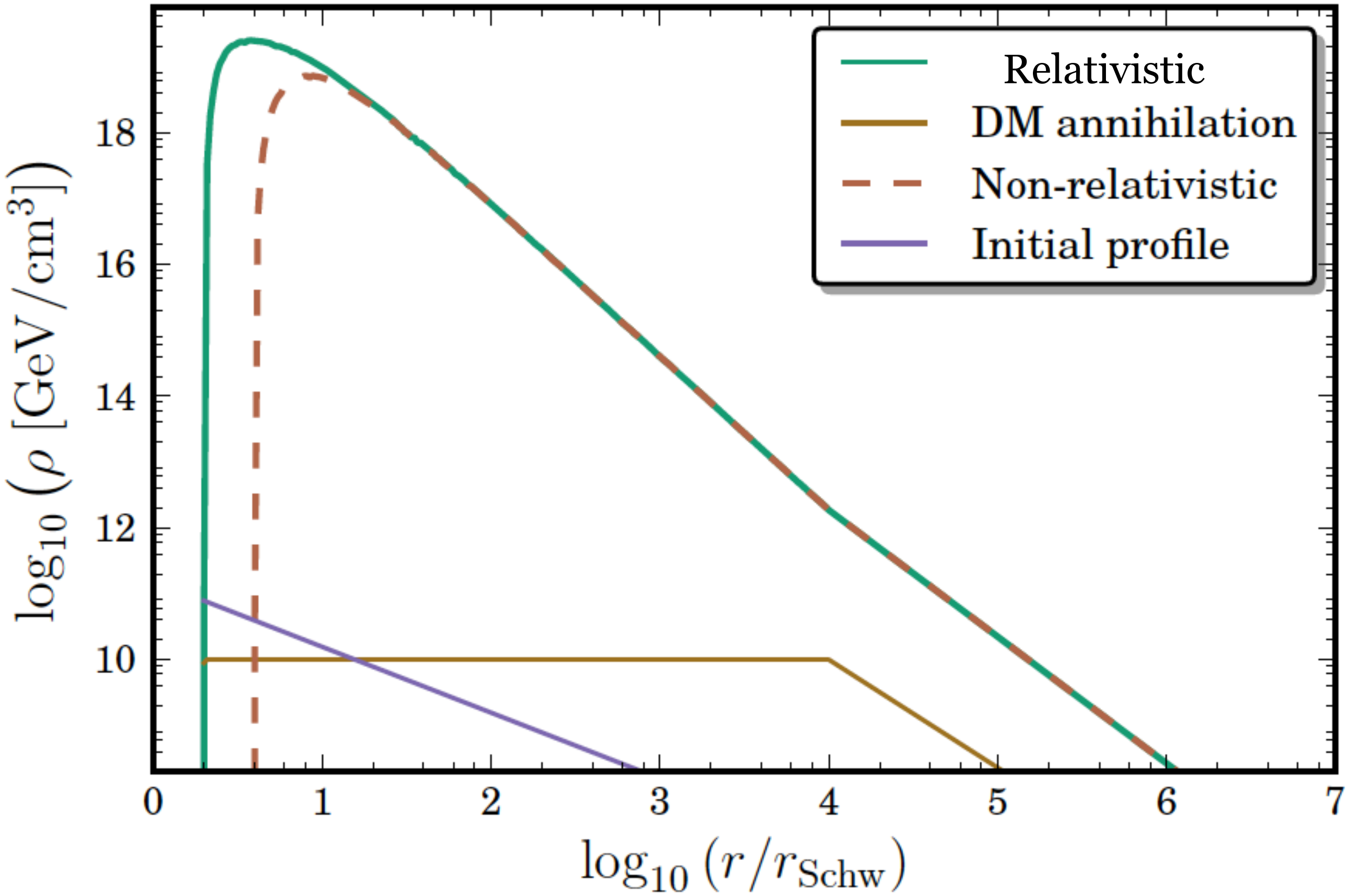
constant distribution function: $f(\mathcal{E}, L) = f_0 = \text{const}$



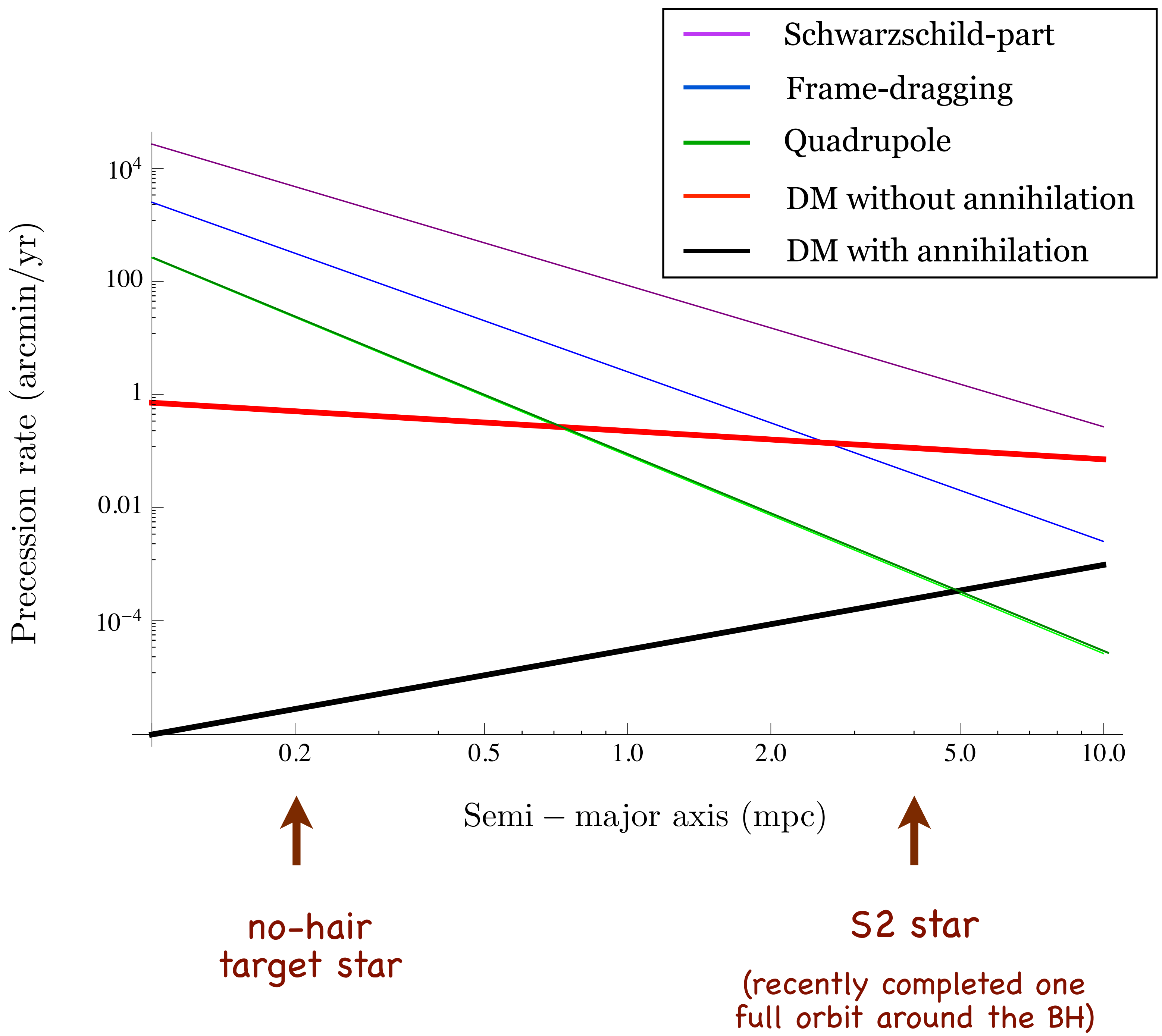
Hernquist profile density: $\rho_i(r) = \rho_H(r) = \dfrac{\rho_0}{(r/a)(1+r/a)^3}$

If DM particles self-annihilate:

$$\rho(r) = \frac{\rho_{\rm core}\rho_f(r)}{\rho_{\rm core} + \rho_f(r)}$$



Precession rates at the source for a target star with $e = 0.95$:



summary:

- We have developed a fully relativistic approach for adiabatic growth of BH in DM distribution.
- Significant differences with results of G&S (1999) have been found: In particular ρ vanishes at r=4m not 8m, and it is substantially larger at small r than what G&S found (The profile is more cuspy).
- The pericenter precession caused by the DM spike is potentially detectable if DM does not self annihilate.

Future work:

- Considering a rotating BH: How non-spherical does the DM distribution become?
- How will the enhancement of the DM density due to relativistic considerations boost the prospect for the indirect detection of DM?