Particle Acceleration in High-Energy Astrophysics

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Outline

A very basic introduction to the selected aspects of particle acceleration in high-energy astrophysics

- General Remarks
- Fermi Process
- Stochastic (Turbulent) Acceleration
- Diffusive Shock Acceleration
- Evolution of Particle Spectra

I. General Remarks

Astrophysical plasma is collisionless; mean free path for Coulomb (binary) scattering is much larger than scales of the systems: $\boldsymbol{\lambda_{Coul} \gg L}$

Continuous character of the plasma is assured by the magnetic field, without action of the Coulomb scatterings; magnetic field is always strong enough to determine the effective mean free path of particles: $\boldsymbol{L \gg r_g}$

BUT: Astrophysical plasma is highly conductive: no large-scale stable E field, only B field; and B fields alone cannot accelerate particles…

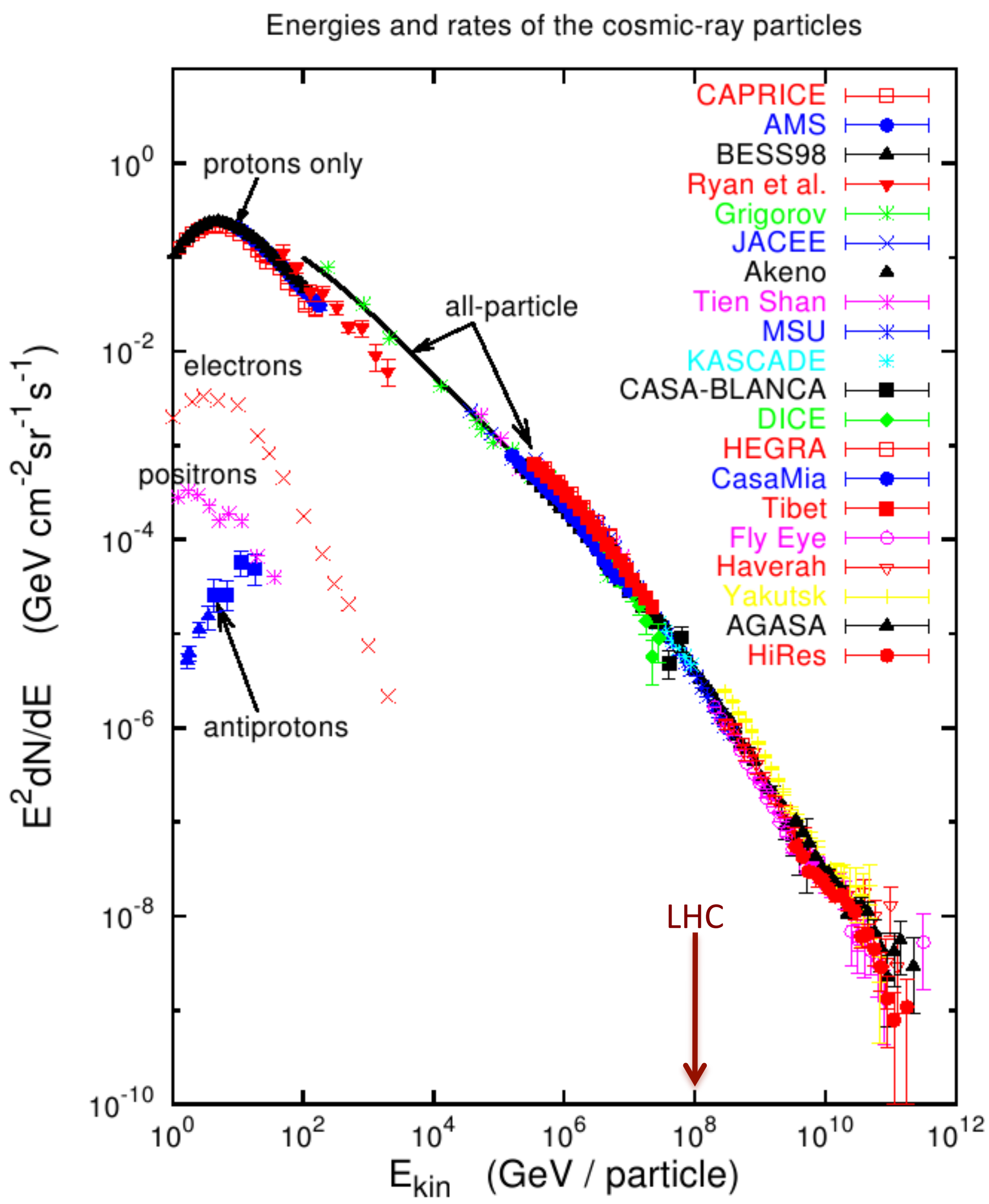
One shot electrostatic accelerators due to the induced electric field (rotating B field; magnetic reconnection regions)

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{1}$$

Frictional acceleration related to moving magnetized regions

$$\vec{E} = -\vec{\beta} \times \vec{B} \tag{2}$$

I. Cosmic Rays

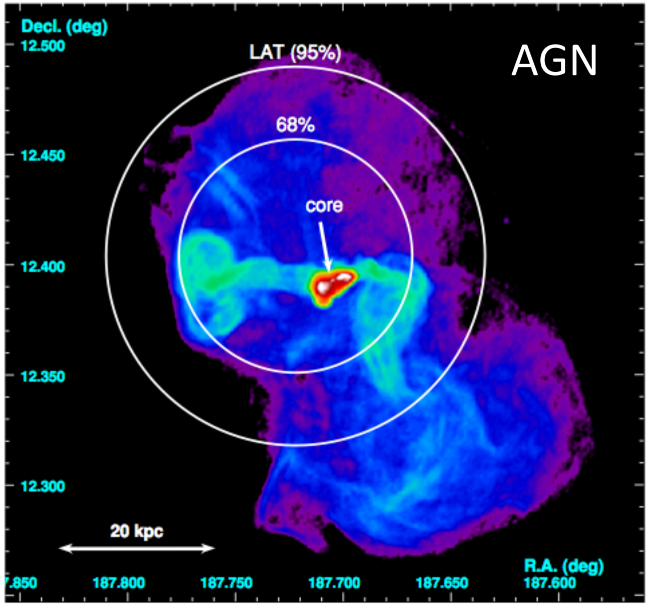


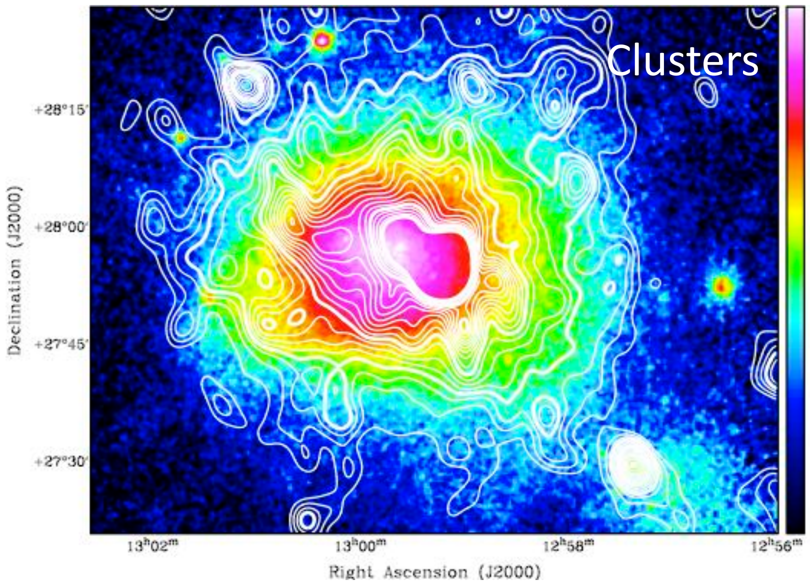
- power-law energy distribution;
- huge energy range!
- energetically dominated by protons
- interesting spectral features ("angle", "knee", etc.);
- energies much above the range of currently operating laboratories.

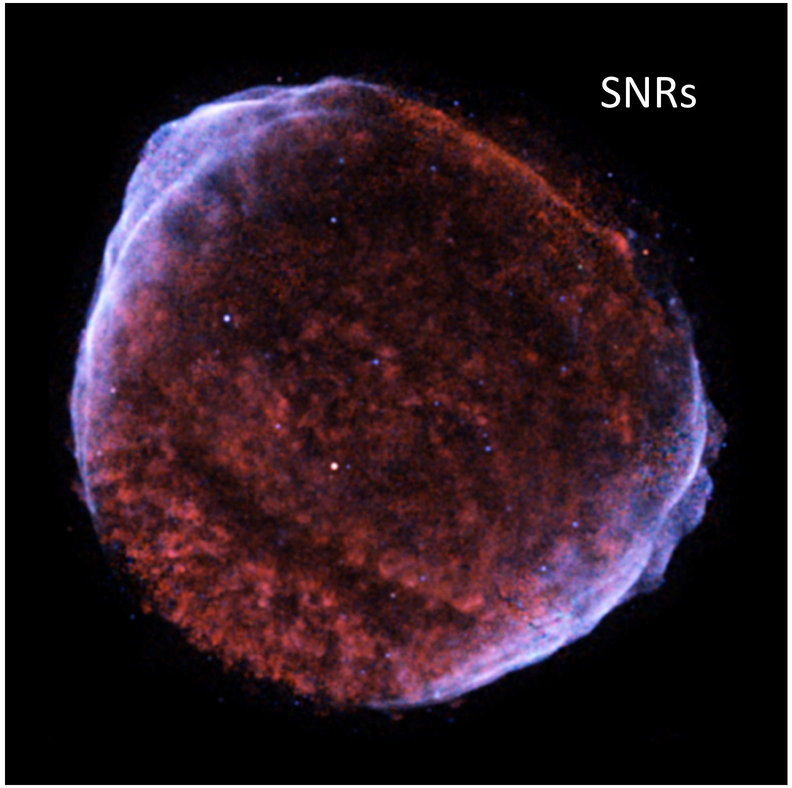
So, how are cosmic rays accelerated?

Astrophysical plasma is very different from laboratory plasma on Earth…

I. Gamma-ray Sources

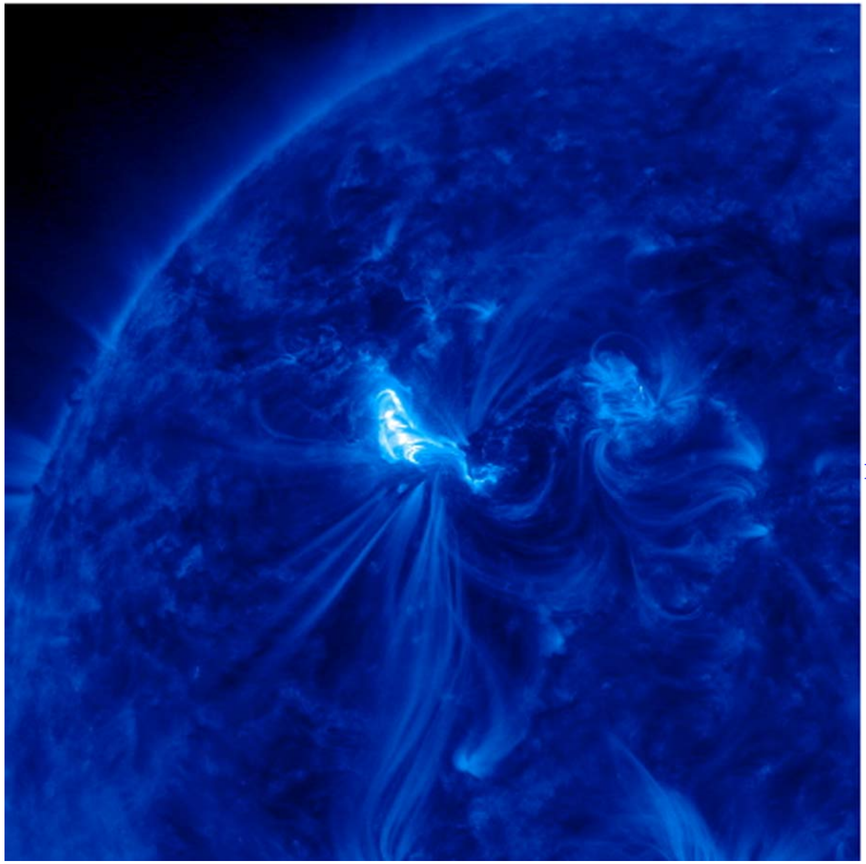




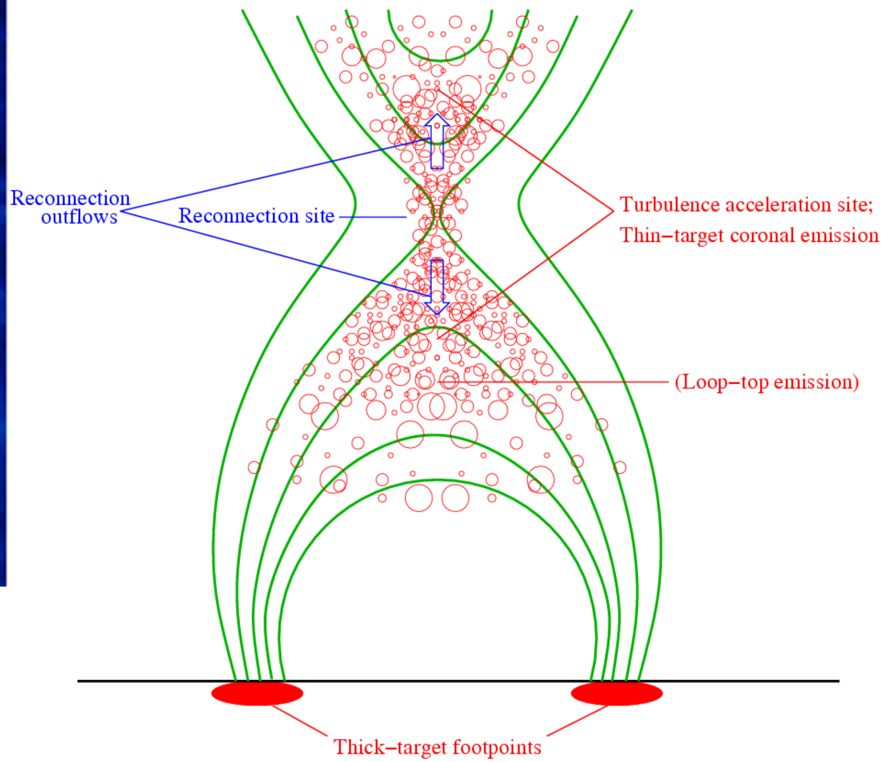


- Same acceleration mechanism operating in different astrophysical sources of high-energy radiation?
- Are SNRs the dominant sources of Galactic CRs (E_{CR} < PeV) indeed?
- What about UHECRs (E_{CR} > EeV)?

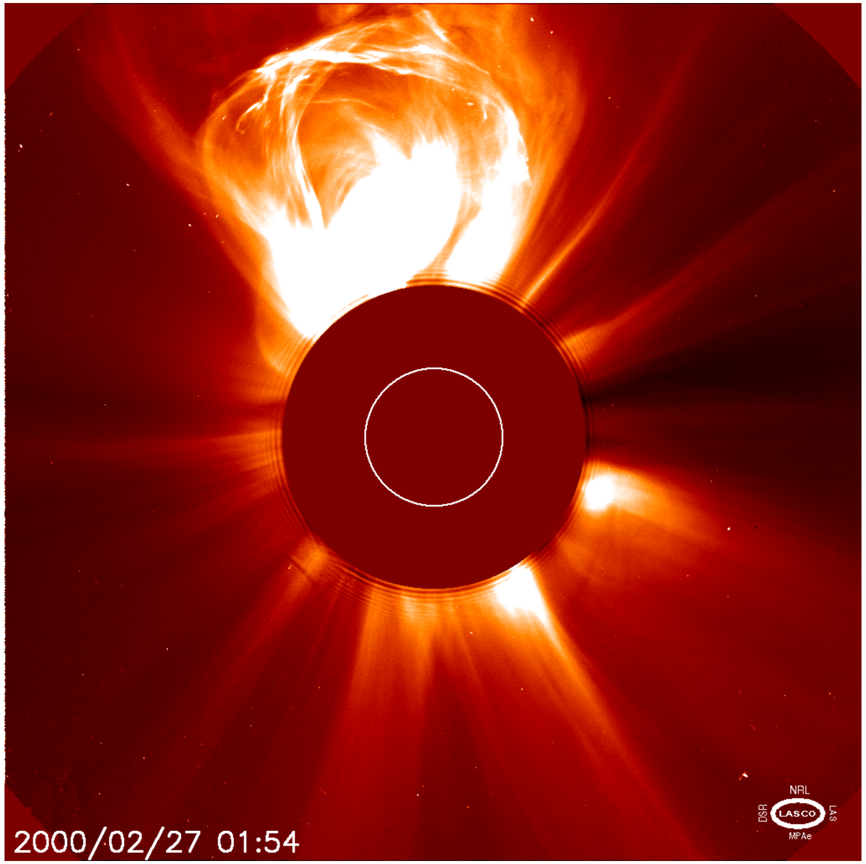
I. Solar Flares



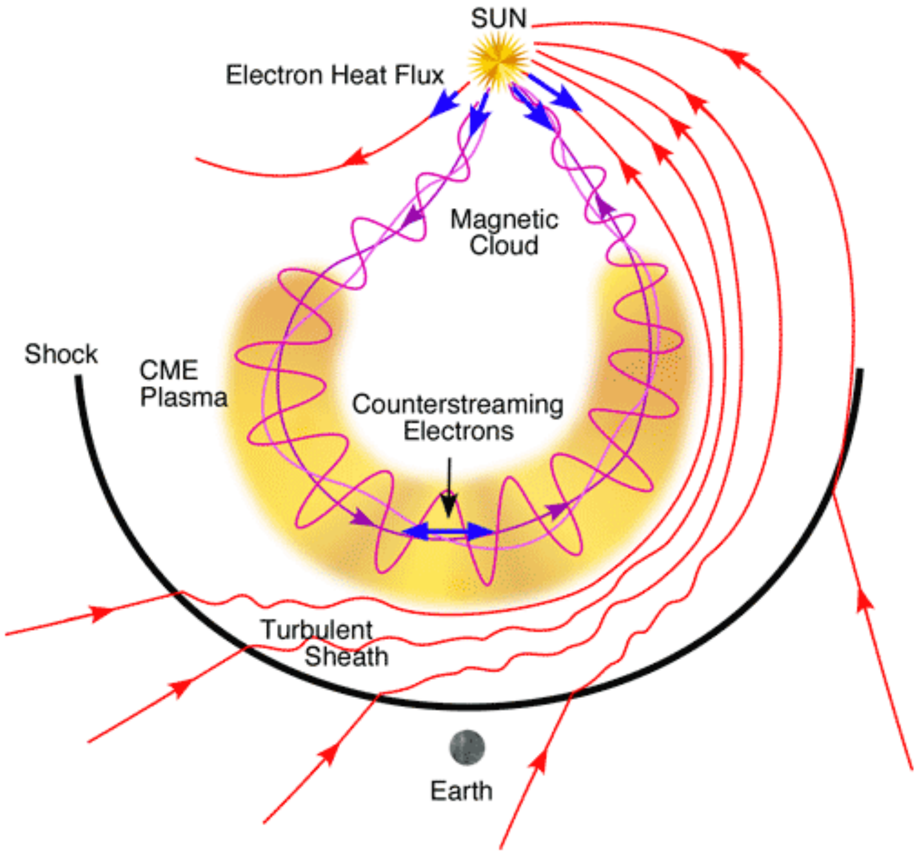
Magnetic reconnection and turbulence
(Liu & Petrosian)



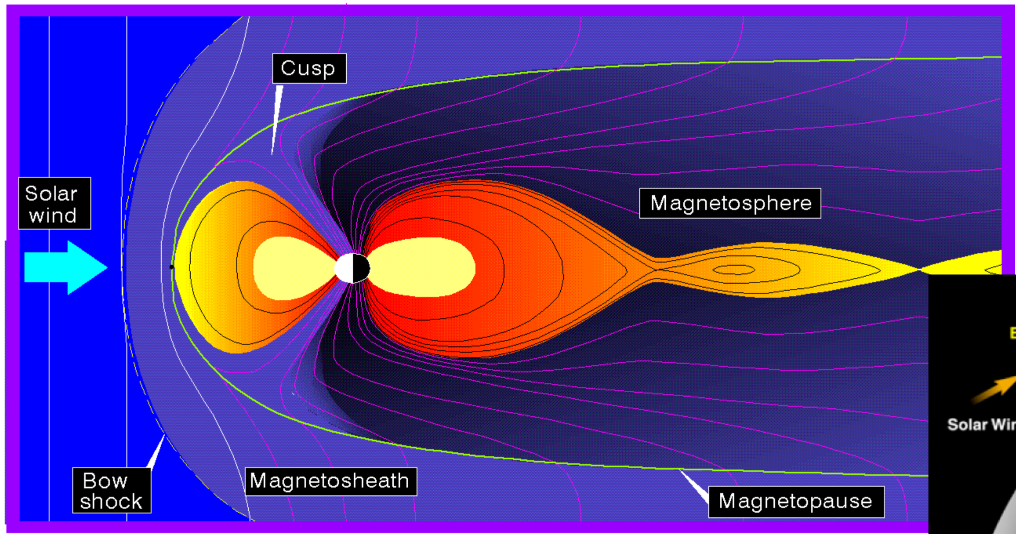
I. Coronal Mass Ejections

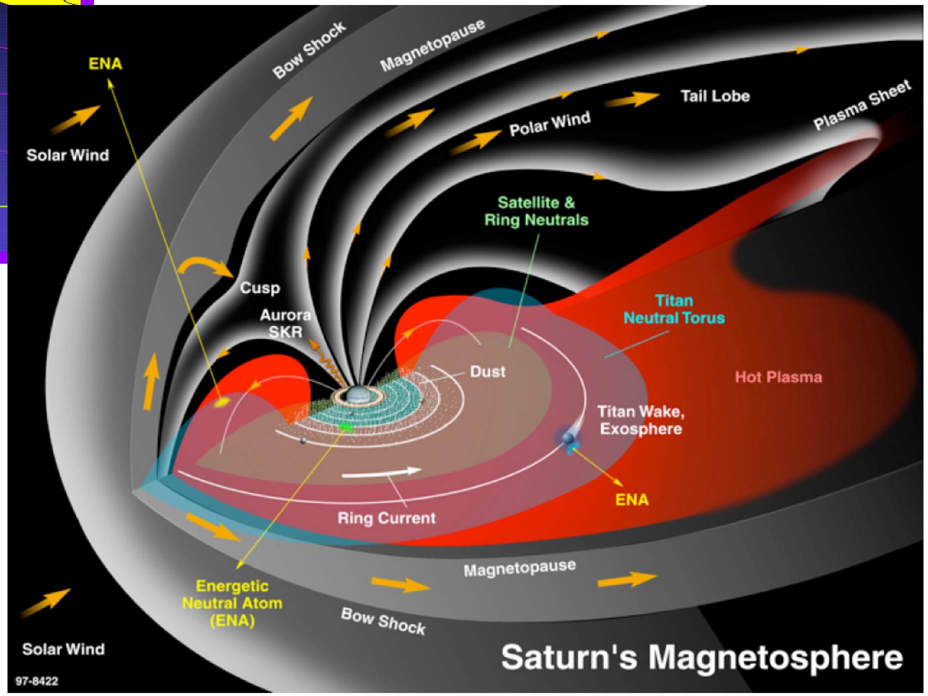


Interplanetary shocks
(Eddy & Zurbuchen)



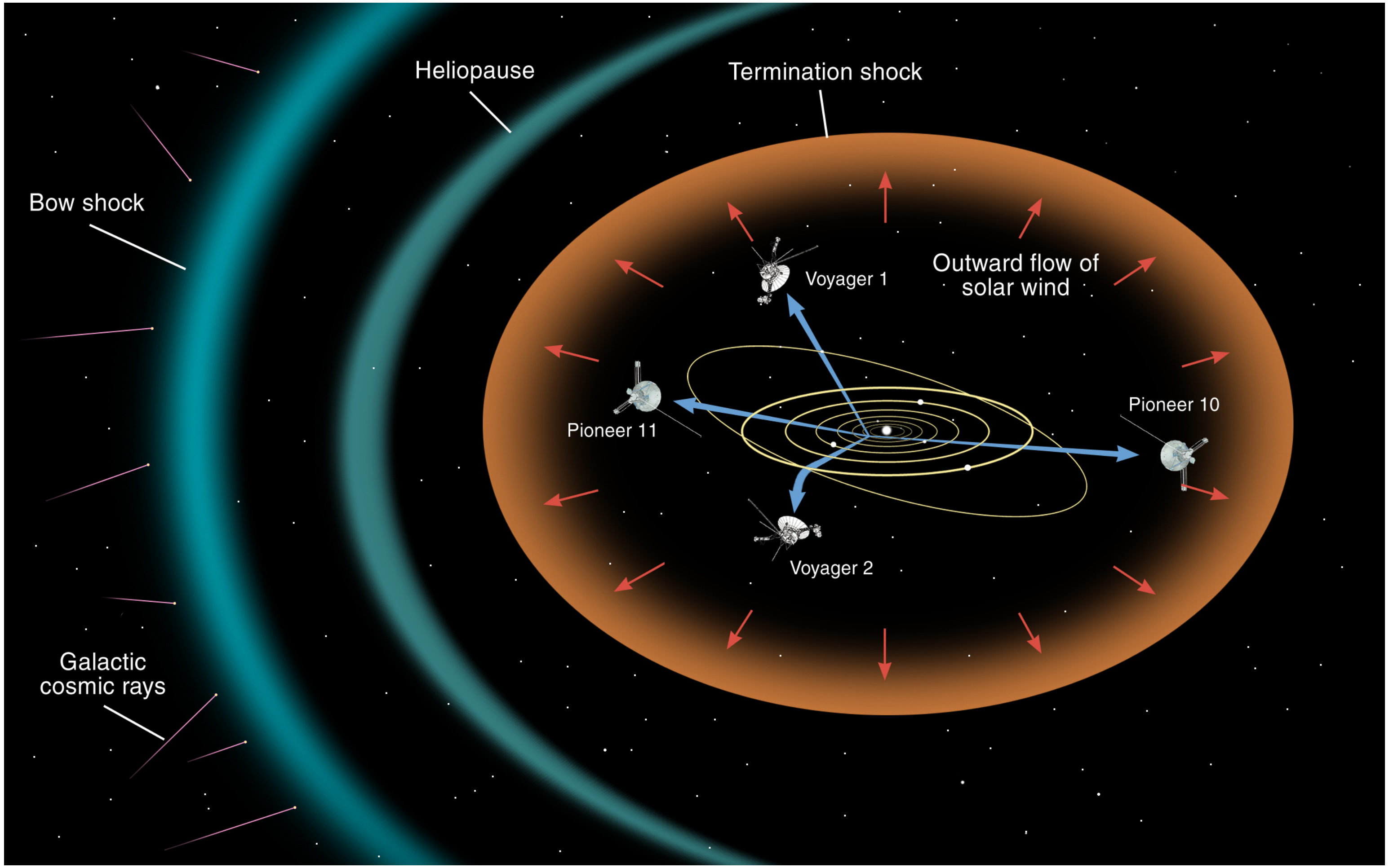
I. Planetary Magnetospheres





Shocks, turbulence,
and magnetic reconnection

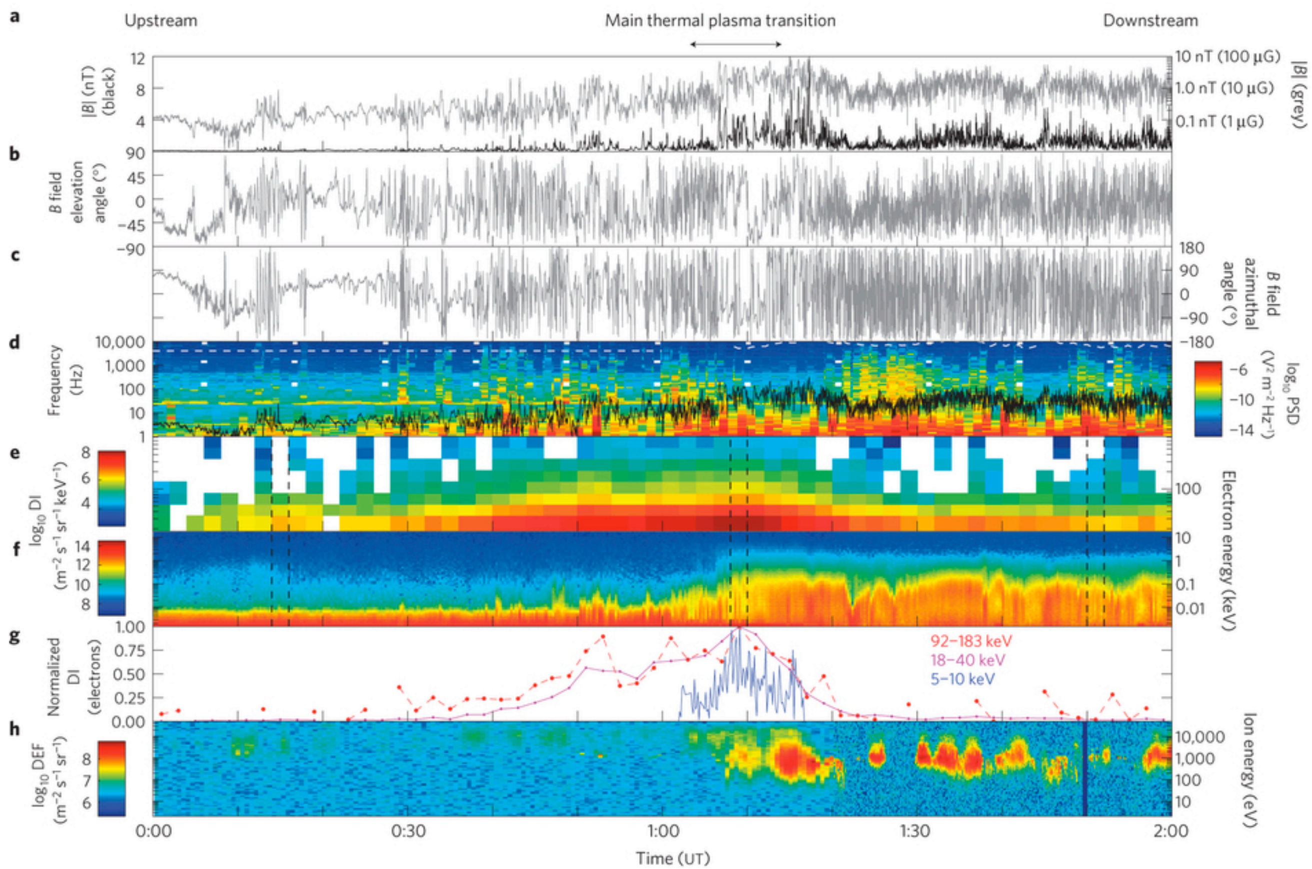
I. Termination Shock



I. Solar System Laboratory

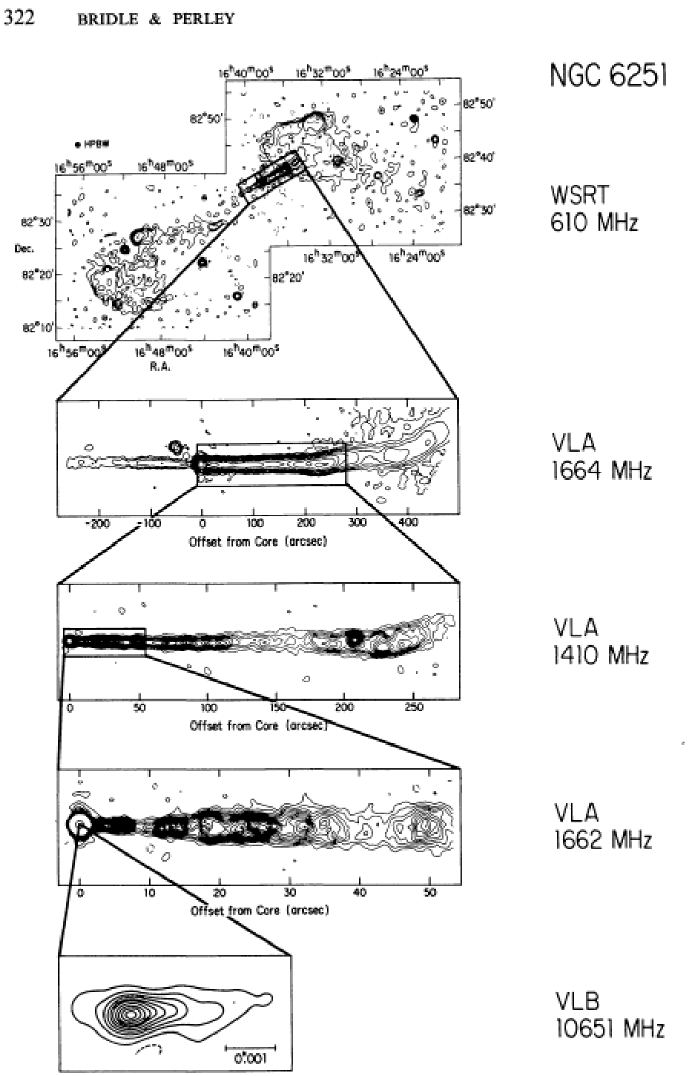
These are all in-situ measurements (particles and magnetic field) – great!
BUT: a very different energy range (electrons up to GeV, ions up to 10s GeV), spatial scales involved (termination shock: ~10^{15} cm), and plasma conditions (low mach number, non-relativistic shocks).

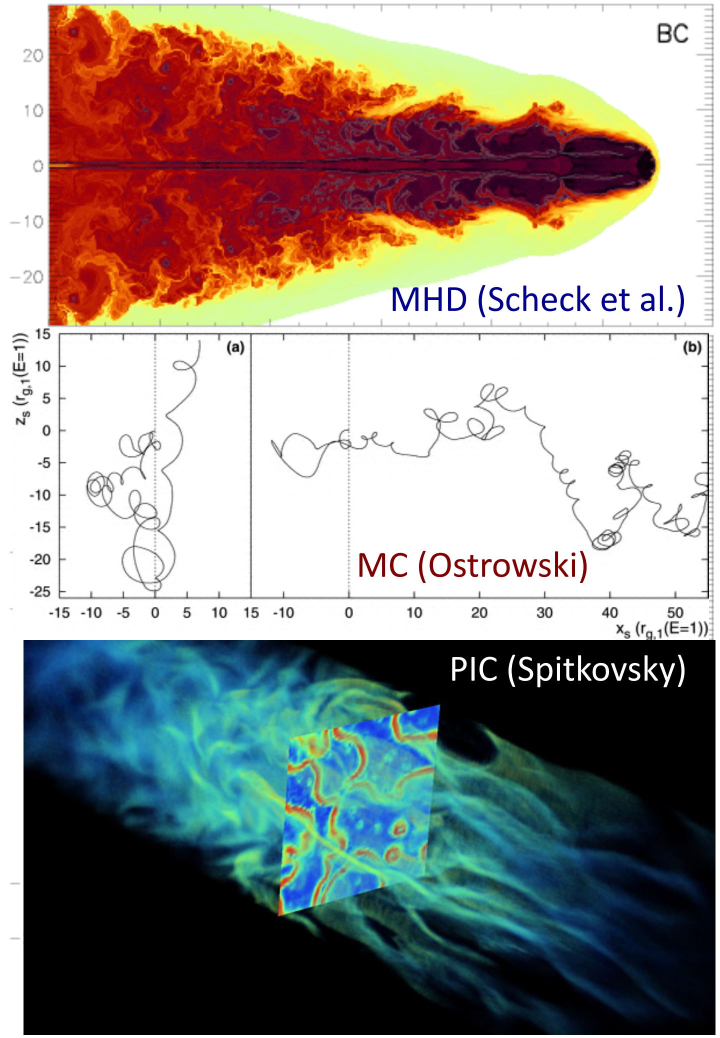
In contrast, in high-energy astrophysics we are talking about 100 TeV electrons and 100 EeV ions, spatial scales up to 10^{25} cm, and high Mach-number or relativistic shocks…



Crossing Saturn bow shock with Cassini (Masters et al. 2013)

I. Scales…

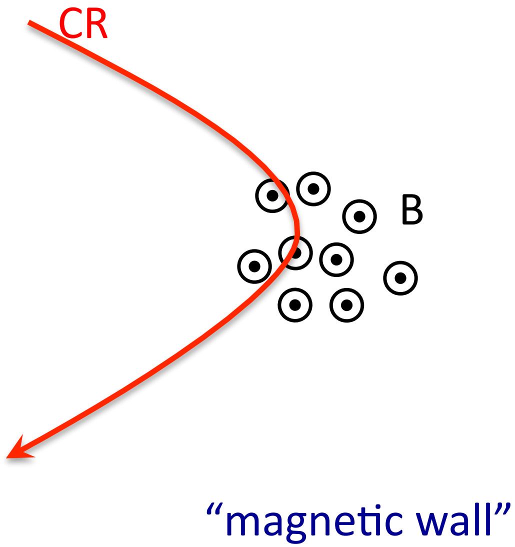




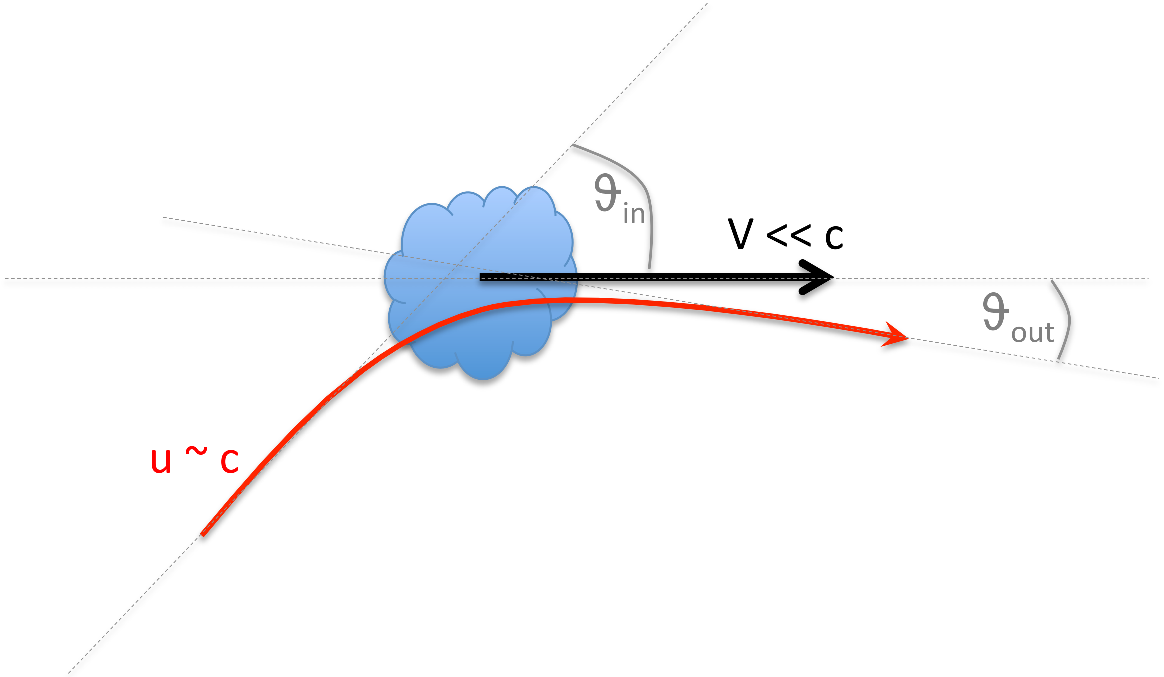
Combination of different numerical techniques, each with its own limitations and problems, rather than one "end-to-end" numerical tool.

II. Fermi Process

Galaxy is filled with randomly moving clouds of gas
with frozen-in magnetic field.
High-energy charged particles can “scatter off” these magnetized clouds.



II. Scattering



$\beta = V/c$
$\gamma = (1-\beta^2)^{-1/2}$

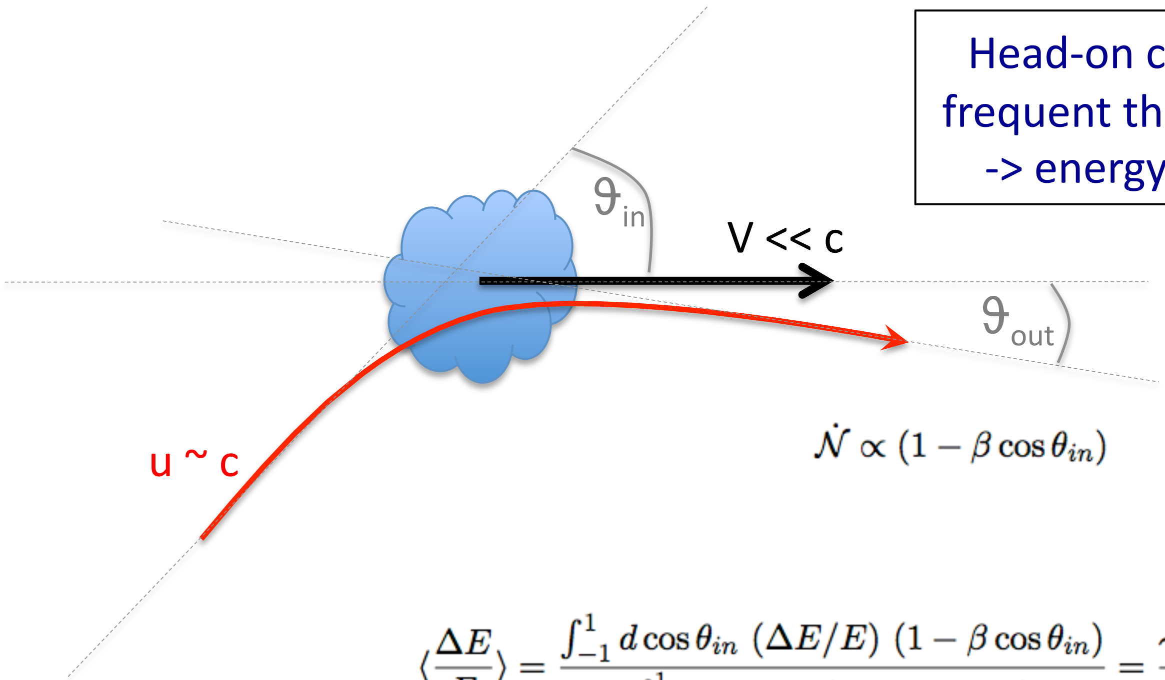
Transformation to the cloud frame $$E'_0 = \gamma\, E_0\, (1 - \beta \cos\theta_{in}) \quad (3)$$

Elastic scattering $$E' = E'_0 \quad (4)$$

Transformation to the observer frame $$E = \gamma\, E'\, (1 + \beta \cos\theta'_{out}) \quad (5)$$

Energy gains or losses depending on ϑ_{in}

II. Acceleration!



Head-on collisions are more frequent than tail-on collisions -> energy gain ***on average***

$$\dot{\mathcal{N}} \propto (1 - \beta \cos\theta_{in}) \tag{6}$$

$$\langle \frac{\Delta E}{E} \rangle = \frac{\int_{-1}^{1} d\cos\theta_{in}\ (\Delta E/E)\ (1-\beta\cos\theta_{in})}{\int_{-1}^{1} d\cos\theta_{in}\ (1-\beta\cos\theta_{in})} = \frac{\gamma^2\beta^2}{3} + \mathcal{O}(\beta^3) \simeq \frac{\beta^2}{3} \tag{7}$$

assuming $\langle \cos\theta'_{out} \rangle = 0$.

2^{nd} order in β, so slow in the case of non-relativistic velocities of scattering centers;

Process ***may*** results in a power-law distribution of the accelerated particles, but with energy index depending on the τ_{acc}/τ_{esc} ratio.

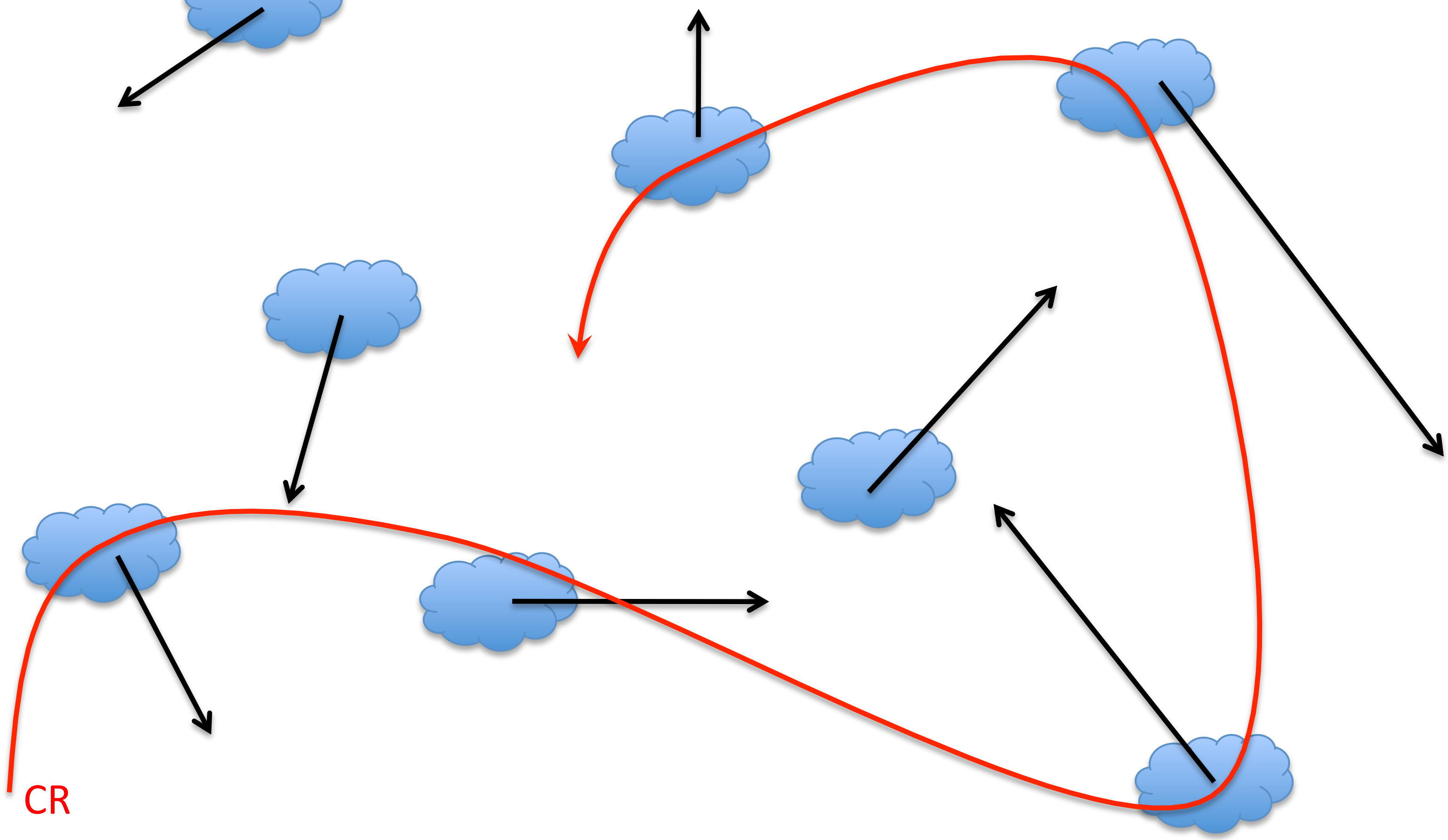
$$\tau_{coll} \simeq \lambda/c \tag{8}$$

$$\frac{dE}{dt} \simeq \langle \frac{\Delta E}{E} \rangle \frac{E}{\tau_{coll}} = \frac{c\beta^2 E}{3\lambda} \equiv \frac{E}{\tau_{acc}} \tag{9}$$

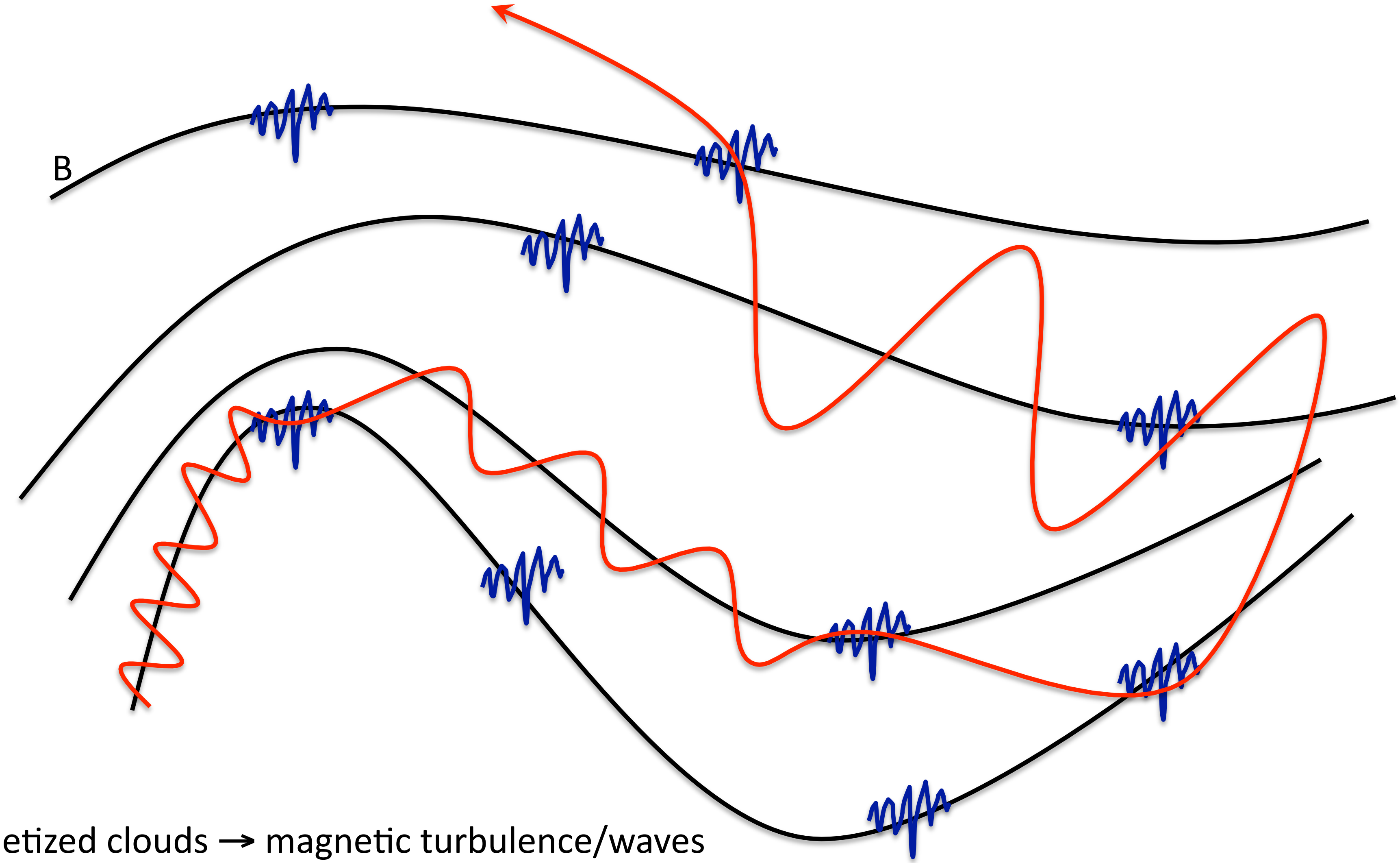
$$\tau_{acc} = 3\ (\lambda/c)\ \beta^2 \tag{10}$$

II. Stochastic Process

Fermi didn’t recognize stochastic nature of the process, and assumed scattering mean free path λ, as well as the escape timescale τ_{esc}, independent on particle energy.



III. Turbulent Acceleration



magnetized clouds → magnetic turbulence/waves

regular acceleration → diffusion in momentum space

$\lambda=\lambda(E)$ and $\tau_{esc}=\tau_{esc}(E)$ related to particle-waves interactions

III. Ideal MHD

$$\frac{\partial}{\partial t}\vec{B} = \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right) \tag{11}$$

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\rho\vec{v}) = 0 \tag{12}$$

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}\right)\left(\frac{p}{\rho^{\hat{\gamma}}}\right) = 0 \tag{13}$$

$$\rho\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}\right)\vec{v} = -\vec{\nabla}p + \frac{1}{4\pi}\left(\vec{\nabla} \times \vec{B}\right) \times \vec{B} \tag{14}$$

Set of MHD equations in the non-relativistic regime for a polytropic fluid, ignoring the effects due to the electric field (displacement current, convective current, etc.), magnetic diffusivity, and energy dissipation.

III. MHD Waves

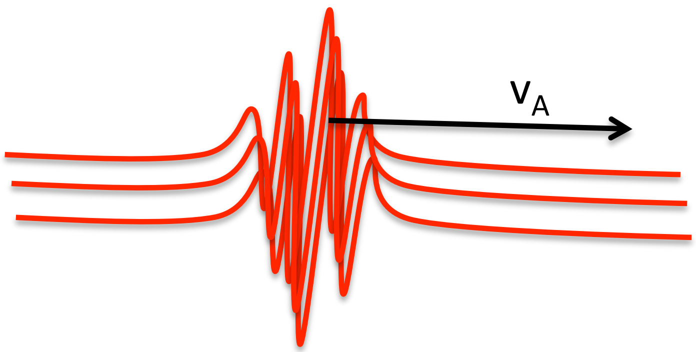
Turbulence is not a superposition of MHD waves, but we often think so…

angle between magnetic field and the direction of propagation

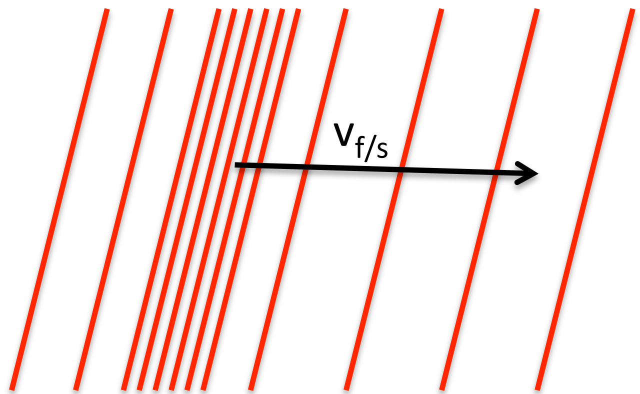
$$v^2_{\mathrm{ph,\,A}} = v^2_{\mathrm{A}}\ \cos^2\phi \tag{15}$$

$$v^2_{\mathrm{ph,\,f/s}} = \frac{1}{2}\left(c^2_{\mathrm{s}} + v^2_{\mathrm{A}}\right) \pm \frac{1}{2}\sqrt{\left(c^2_{\mathrm{s}} + v^2_{\mathrm{A}}\right)^2 - 4\,c^2_{\mathrm{s}}\,v^2_{\mathrm{A}}\ \cos^2\phi} \tag{16}$$

where $v_{\mathrm{A}} = B/\sqrt{4\pi\rho}$ and $c_{\mathrm{s}} = (\hat{\gamma}\,p/\rho)^{1/2}$.



Alfen waves: fluid velocity perturbations and the magnetic field perturbations are transverse to the direction of propagation



Fast and slow magnetosonic waves: compression and rarefaction of the plasma and frozen-in magnetic field, with the perturbed fluid velocity parallel to the propagation

III. Diffusion Approximation

Phase space density of ultrarelativistic particles: $f(x, p; t)$
Total number of particles: $N = \int d^3x \int d^3p \; f(x, p; t)$

For collisionless plasma, the function $f(x, p; t)$ satisfies the relativistic Vlasov equation with the acceleration determined by the Lorentz force due to the average EM field.

Approx. 1: Presence of only a small-amplitude turbulence $(\delta E, \delta B)$ in addition to the large-scale magnetic field $B_0 \gg \delta B$, such that the total plasma fields are $B = B_0 + \delta B$, and $E = \delta E$ (due to high conductivity we expect $E_0 = 0$)

Approx. 2 ("test-particle approach"): Configuration of the electromagnetic field is given, and is independent on the considered particles (their energy spectra).

Ensemble-averaging, $f(x, p; t) = \langle f \rangle + \delta f$, $\langle \delta B \rangle = \langle \delta E \rangle = 0$, and linearization of the Vlasov equation → Fokker-Planck equation for $\langle f \rangle$

Approx. 3 (diffusion approximation): $\langle f \rangle$ is spatially uniform and isotropic in p

Fokker-Planck euation for $\langle f \rangle$ → momentum diffusion equation for one-dimensional momentum distribution $n(p,t) \equiv 4\pi \, p^2 \langle f \rangle$

III. Diffusive Acceleration

$$D_{\rm p} = \frac{1}{3}\left(\frac{v_{\rm A}}{c}\right)^2 \left(\frac{c}{\lambda(p)}\right) p^2 \quad \text{momentum diffusion} \quad (17)$$

$$\kappa = \frac{1}{3}\, c\, \lambda(p) \quad \text{spatial diffusion} \quad (18)$$

$$\tau_{\rm acc} \equiv \frac{p^2}{D_{\rm p}} = 3\left(\frac{c}{v_{\rm A}}\right)^2 \left(\frac{\lambda(p)}{c}\right) \quad \text{acceleration timescale} \quad (19)$$

$$\tau_{\rm esc} \equiv \frac{L^2}{\kappa} = 3\left(\frac{L}{c}\right)^2 \left(\frac{c}{\lambda(p)}\right) \quad \text{escape timescale} \quad (20)$$

λ(p) – mean free path for particle-wave interactions

L – spatial scale of a system

III. Particle-Wave Interactions

$$\int_{k_{min}}^{k_{max}} dk\, k^2\, \mathcal{W}(k) = \frac{(\delta B)^2}{8\pi} \quad , \quad \mathcal{W}(k) \propto k^{-q} \qquad (21)$$

turbulence spectrum

$$\omega - n\,\Omega - k_{||}\, c\, \mu = 0 \qquad (22)$$

resonant wave-particle interactions

$$n = -1 \quad \rightarrow \quad \lambda_{||}(p) \approx r_g \left(\frac{B_0}{\delta B}\right)^2 \left(\frac{L}{r_g}\right)^{q-1} \quad . \qquad (23)$$

"gyroresonance"
for Alfen waves interacting with ultrarelativistic particles
(meaning $\lambda_{A,\, res} \approx r_g$)

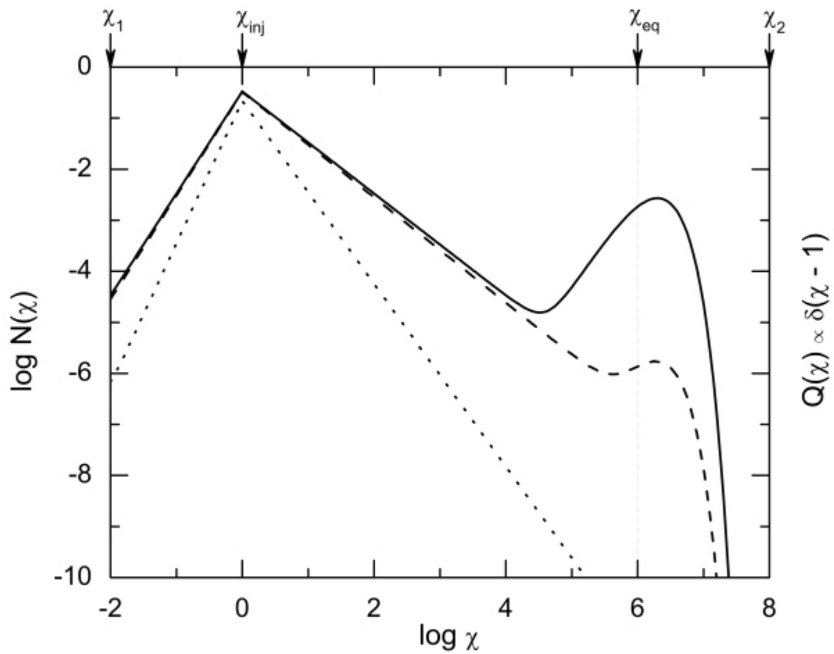
particle gyroradius r_g=pc/eB$_0$
particle gyrofrequency $\Omega = c/r_g$
particle pitch angle $\cos^{-1}\mu$
wavevector parallel to magnetic field $k_{||}$
wave frequency ω
n=0, ±1, ±2, …
q=1 (Bohm), 3/2 (Kreichnan), 5/3 (Kolmogorov), 2 (Fermi)

III. Final Equation

$$\frac{\partial n(p,t)}{\partial t} = \frac{\partial}{\partial p}\left[D_{\mathrm{p}}\,\frac{\partial n(p,t)}{\partial p}\right] - \\ -\frac{\partial}{\partial p}\left[\left(\frac{2\,D_{\mathrm{p}}}{p} + \langle \dot{p}\rangle_{\mathrm{reg}}\right)\,n(p,t)\right] - \frac{n(p,t)}{\tau_{\mathrm{esc}}} + Q(p,t) \quad (24)$$

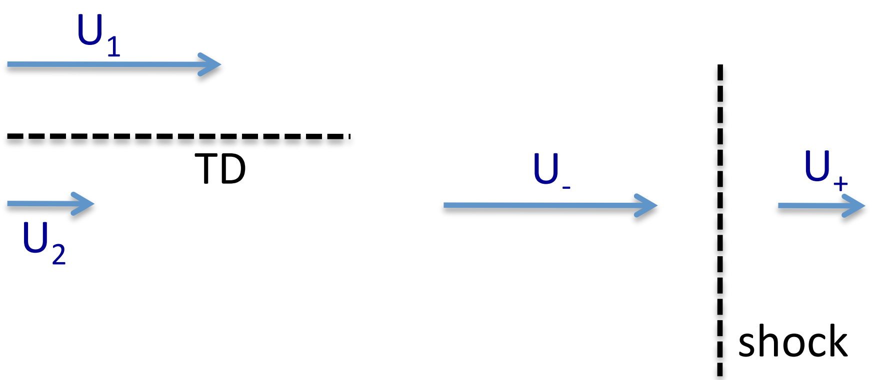
where, e.g., $\langle \dot{p}\rangle_{\mathrm{reg}} = -p/\tau_{\mathrm{rad}}$.

Variety of resulting particle spectra:
power-laws, ultrarelativistic Maxwellians, etc...



IV. Flow Discontinuities

In the case of a moving fluid, surfaces of discontinuity of the fluid parameters may arise. In general, such surfaces may travel in the gas with velocity different from the fluid bulk velocity. However, in the rest-frame of the discontinuity the momentum flux of the fluid, the energy flux of the fluid and the particle flux of the fluid through a surface have to be conserved. There are two possible types of the fluid discontinuities: **tangential discontinuities and shock waves.**



Rankine-Hugoniot conditions for non-relativistic hydro shock

$$\rho_- v_-^2 + p_- = \rho_+ v_+^2 + p_+ \tag{25}$$

$$\frac{\varepsilon_- + p_-}{\rho_-} + \frac{1}{2} v_-^2 = \frac{\varepsilon_+ + p_+}{\rho_+} + \frac{1}{2} v_+^2 \tag{26}$$

$$\rho_- v_- = \rho_+ v_+ \tag{27}$$

$$R \equiv \frac{\rho_+}{\rho_-} = \frac{v_-}{v_+} = \frac{(\hat\gamma + 1)\, \mathcal{M}_-^2}{(\hat\gamma - 1)\, \mathcal{M}_-^2 + 2} \tag{28}$$

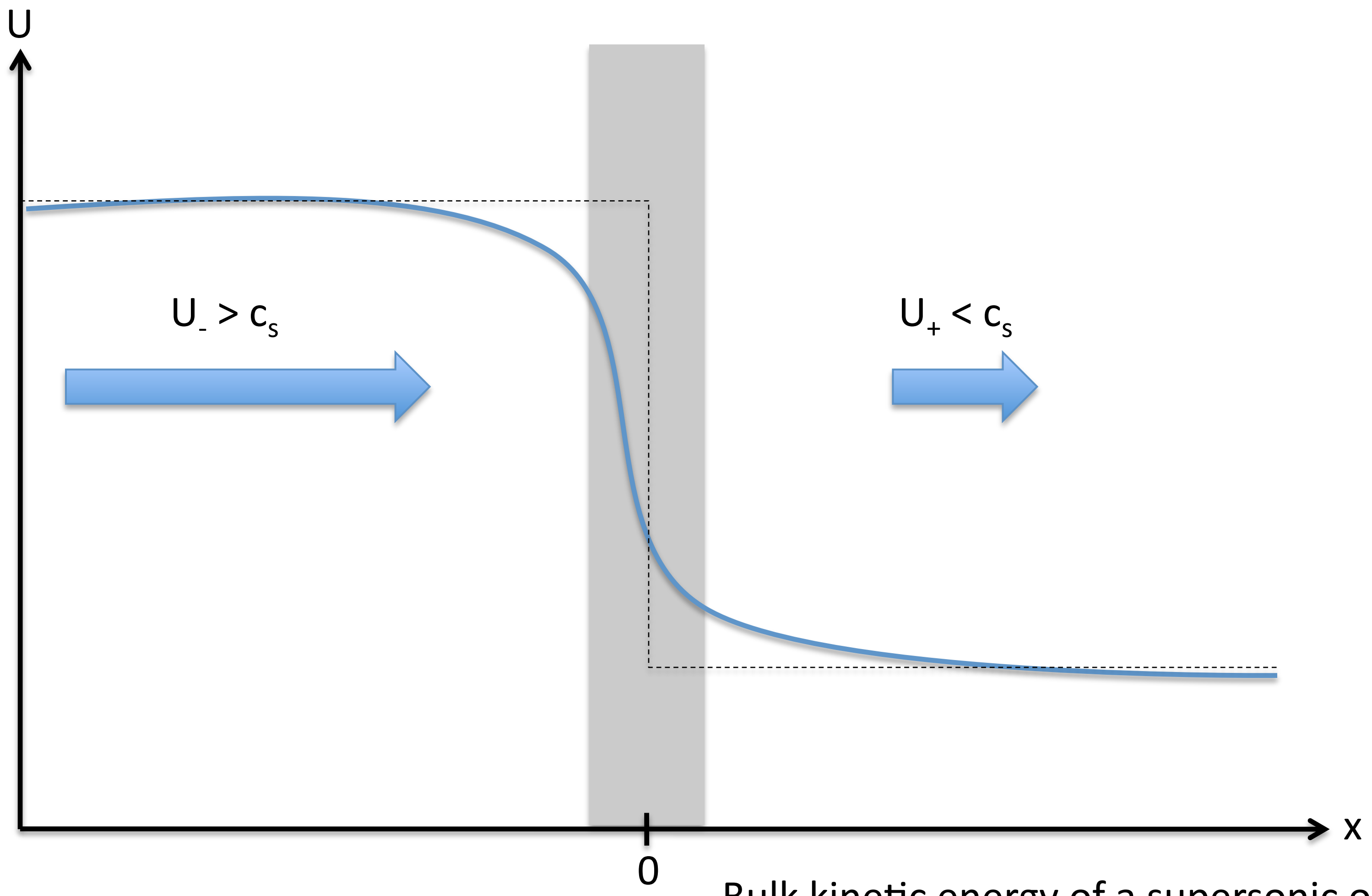
$$\frac{p_+}{p_-} = \frac{2\,\hat\gamma\, \mathcal{M}_-^2}{\hat\gamma + 1} - \frac{\hat\gamma - 1}{\hat\gamma + 1} \tag{29}$$

$$\frac{T_+}{T_-} = \frac{\left[2\,\hat\gamma\, \mathcal{M}_-^2 - (\hat\gamma - 1)\right]\,\left[(\hat\gamma - 1)\, \mathcal{M}_-^2 + 2\right]}{(\hat\gamma + 1)^2\, \mathcal{M}_-^2} \tag{30}$$

$$\mathcal{M}_- \equiv \frac{v_-}{c_{s,-}} \tag{31}$$

(think of a non-linear wave steepening)

IV. Shocks



Bulk kinetic energy of a supersonic outflow is transferred to the plasma internal energy

IV. Advection and Diffusion

Convective derivative

$$\frac{\partial f}{\partial t} + \vec{U} \cdot \vec{\nabla} f = \tag{32}$$

$$= \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_p \frac{\partial f}{\partial p}\right) + \vec{\nabla} \cdot \left(\kappa \vec{\nabla} f\right) - \frac{1}{3}\left(\vec{\nabla} \cdot \vec{U}\right) p \frac{\partial f}{\partial p} + Q - \frac{f}{\tau_{\rm esc}}$$

Momentum diffusion — Spatial diffusion — Adiabatic compression — Source & sink

Advection in momentum space
with velocity

$$-\frac{1}{3}\left(\vec{\nabla} \cdot \vec{U}\right) p$$

IV. At the Shock Front

$$\frac{\partial f}{\partial t} + \vec{U} \cdot \vec{\nabla} f = \vec{\nabla} \cdot \left(\kappa \vec{\nabla} f \right) - \frac{1}{3} \left(\vec{\nabla} \cdot \vec{U} \right) p \frac{\partial f}{\partial p} \tag{33}$$

$$U(x) = \begin{cases} U_- & \text{for} \quad x < 0 \\ U_+ & \text{for} \quad x \geq 0 \end{cases} \tag{34}$$

$$-\vec{\nabla} \cdot \vec{U} = (U_- - U_+)\, \delta(x) \tag{35}$$

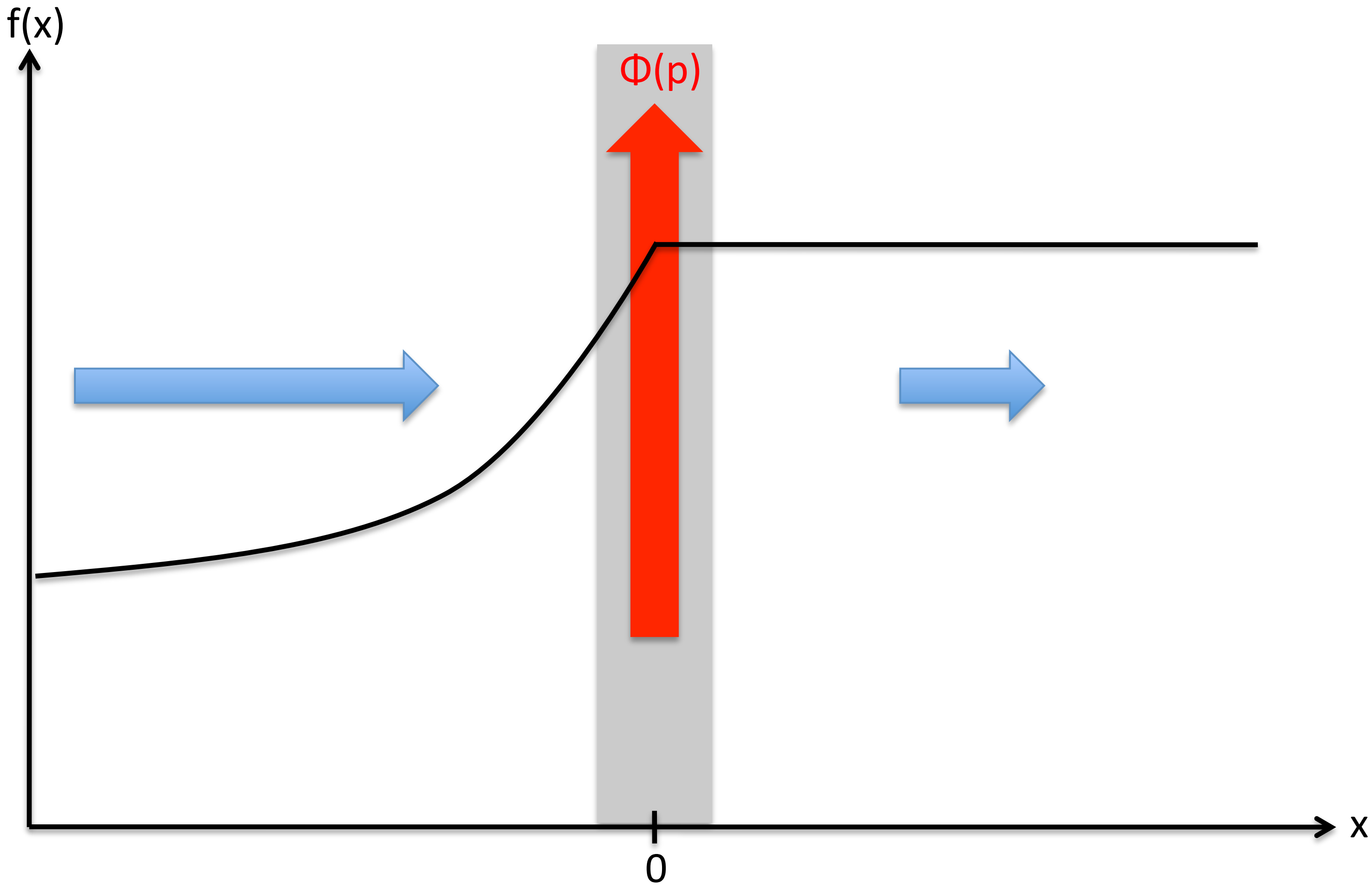
Compression only at the shock front

Momentum flux

$$\Phi(p) = \int d^3x \, 4\pi \, p^2 \left[-\frac{1}{3} (\vec{\nabla} \cdot \vec{U})\, p \right] f(p) = \frac{4\pi \, p^3}{3} (U_- - U_+)\, f_0(p) \tag{36}$$

at the shock front ($f_0(p)$)

IV. Shock Acceleration



reviews: Drury 1983, Blandford & Eichler 1987, Jones & Ellison 1991

IV. Diffusive Shock Acceleration

$$\left(\frac{\kappa_-}{U_-} + \frac{\kappa_+}{U_+}\right)\frac{\partial f}{\partial t} + \frac{U_- - U_+}{3}\,p\,\frac{\partial f}{\partial p} = -U_-\,f \qquad (37)$$

at the shock front

$$\frac{dp}{dt} = \frac{U_- - U_+}{3}\,p\left(\frac{\kappa_-}{U_-} + \frac{\kappa_+}{U_+}\right)^{-1} \qquad (38)$$

$$\frac{df}{dt} = -3\frac{U_-}{U_- - U_+}\,p^{-1}\,f \qquad (39)$$

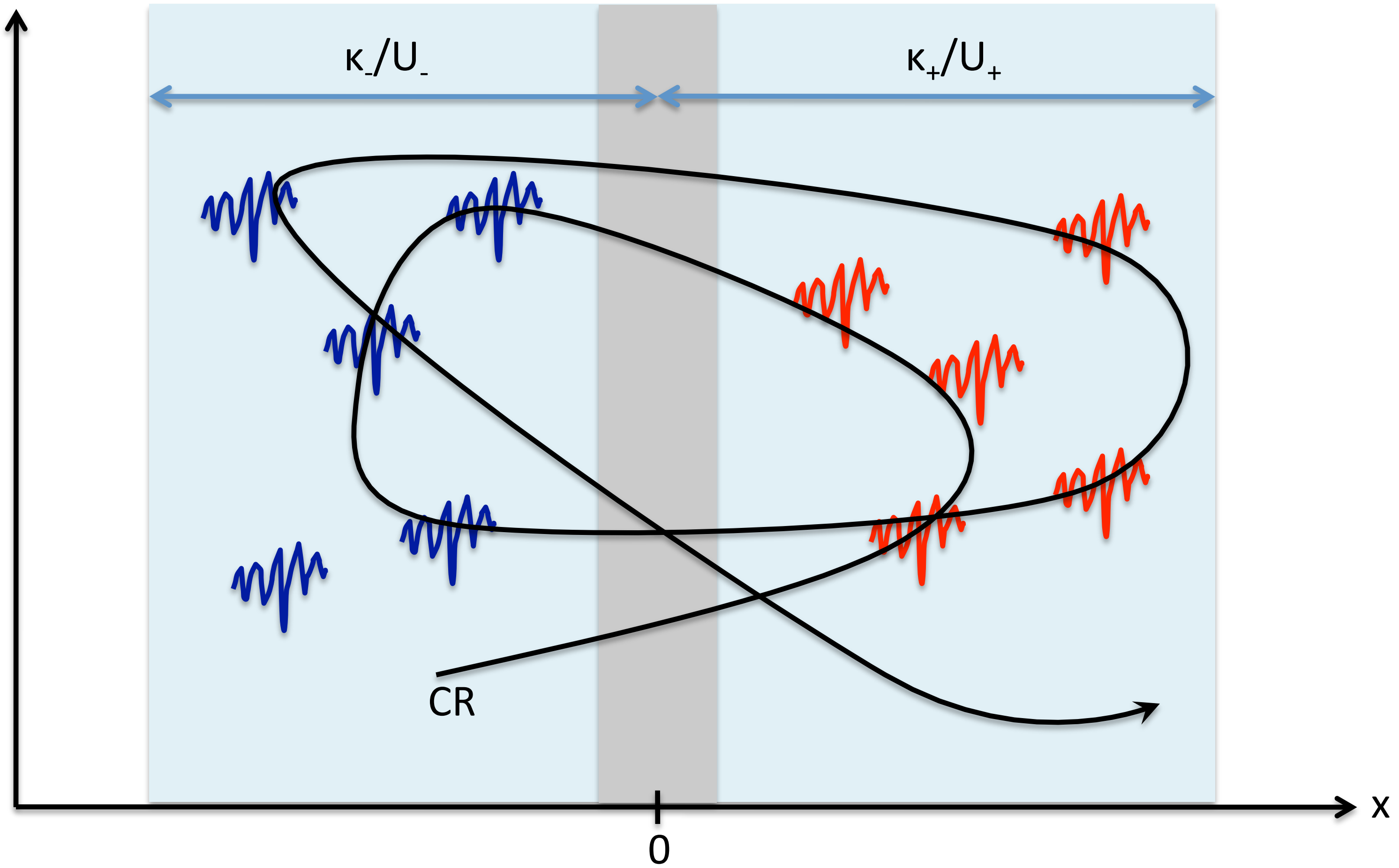
$$\tau_{\rm acc} = \frac{p}{dp/dt} = 3\left(\frac{\kappa_-}{U_-} + \frac{\kappa_+}{U_+}\right)(U_- - U_+)^{-1} \qquad (40)$$

1^{st} order in U/c

$$f(p) \propto p^{-\sigma} \quad \text{where} \quad \sigma = \frac{3R}{R-1} \qquad (41)$$

universal spectrum

IV. 1st-Order Fermi Process



Acceleration due to converging scattering centers which are advected with the flow; particles are isotropized at both sides of a shock, and each transformation upstream -> downstream -> upstream results in the energy gain.

IV. Universal Spectrum

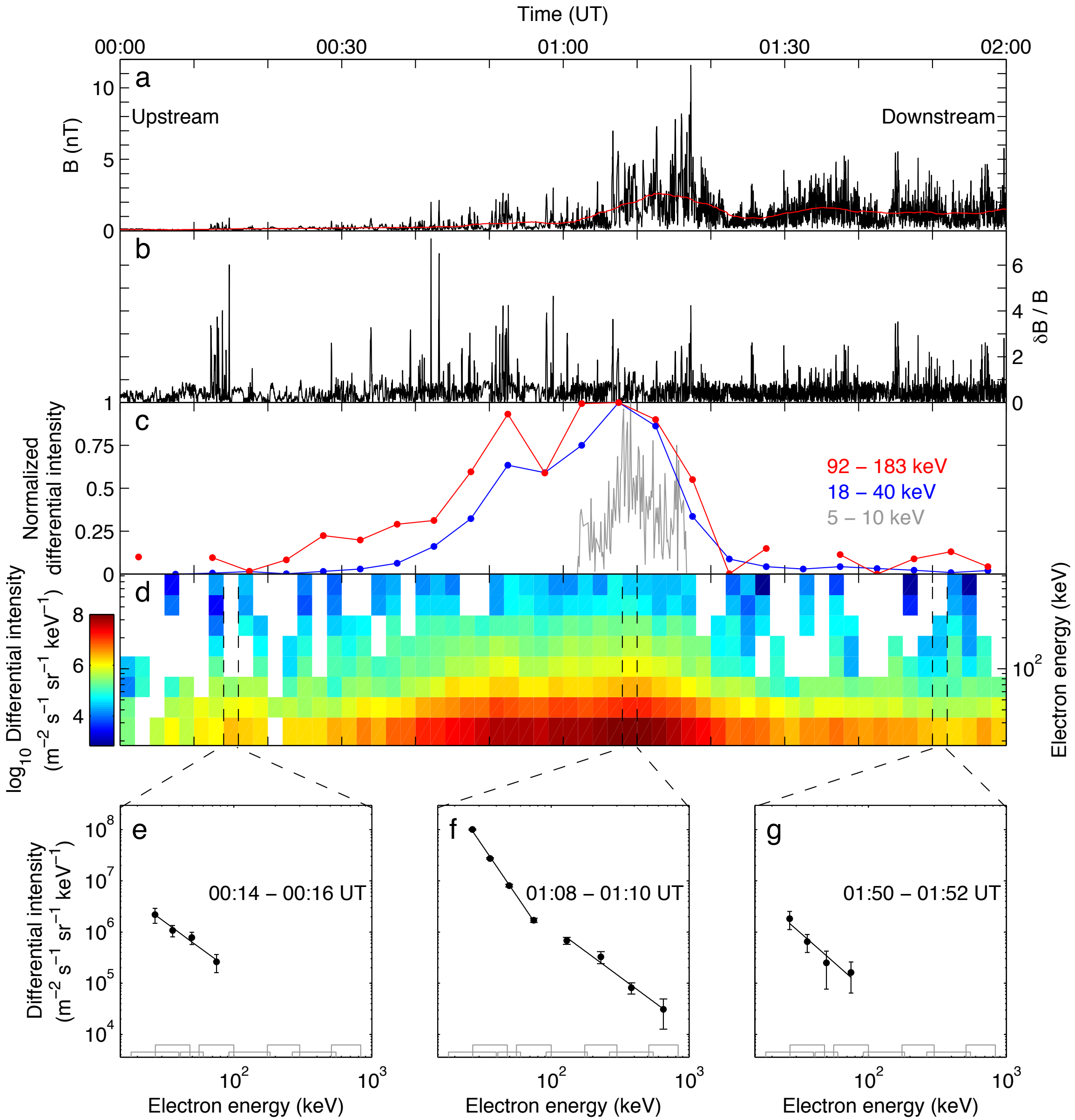
Fast, and result in a universal spectrum (for strong shock: $R=U_-/U_+=4$)

$f(p) \sim p^{-4}$ meaning $n(E) \sim E^{-2}$

But so far we ignored:

i) Injection mechanism (particularly important in the case of electrons)
ii) Magnetic field (global configuration and turbulence conditions; turbulence is needed to scatter particles at both sides of a shock; effective scattering/isotropisation is crucial for the DSA!)
iii) Non-linear effects ("cosmic-ray modified" shocks)
iv) Relativistic shock velocities (relativistic shocks work differently!)

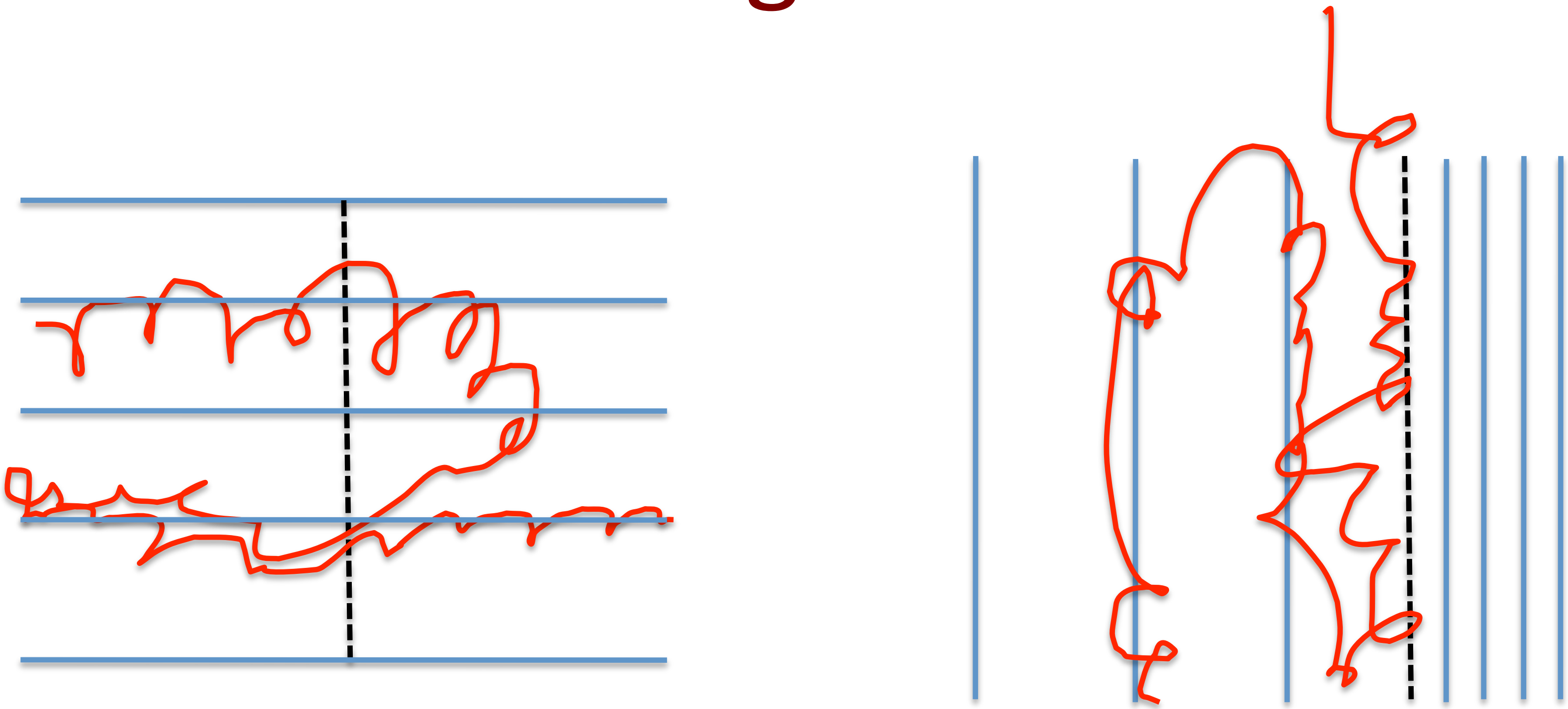
IV. Injection Problem



Masters et al. 2013:

Electron injection due to the interactions with whistler waves at high-M quasi-parallel shock

IV. Magnetic field

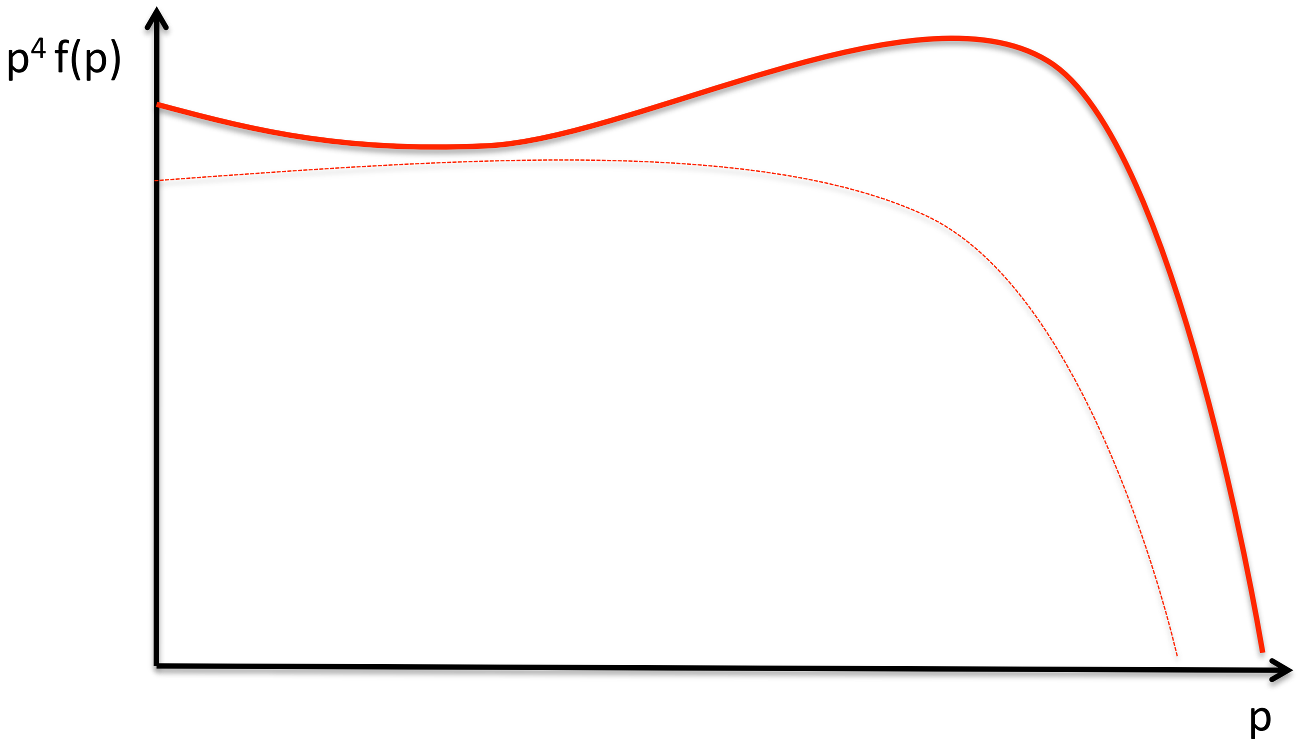


Parallel vs. perpendicular shock configuration.

“Classical” DSA deals with the parallel configuration.

CRs streaming ahead of a parallel shock may excite turbulence and amplify magnetic field in the upstream.

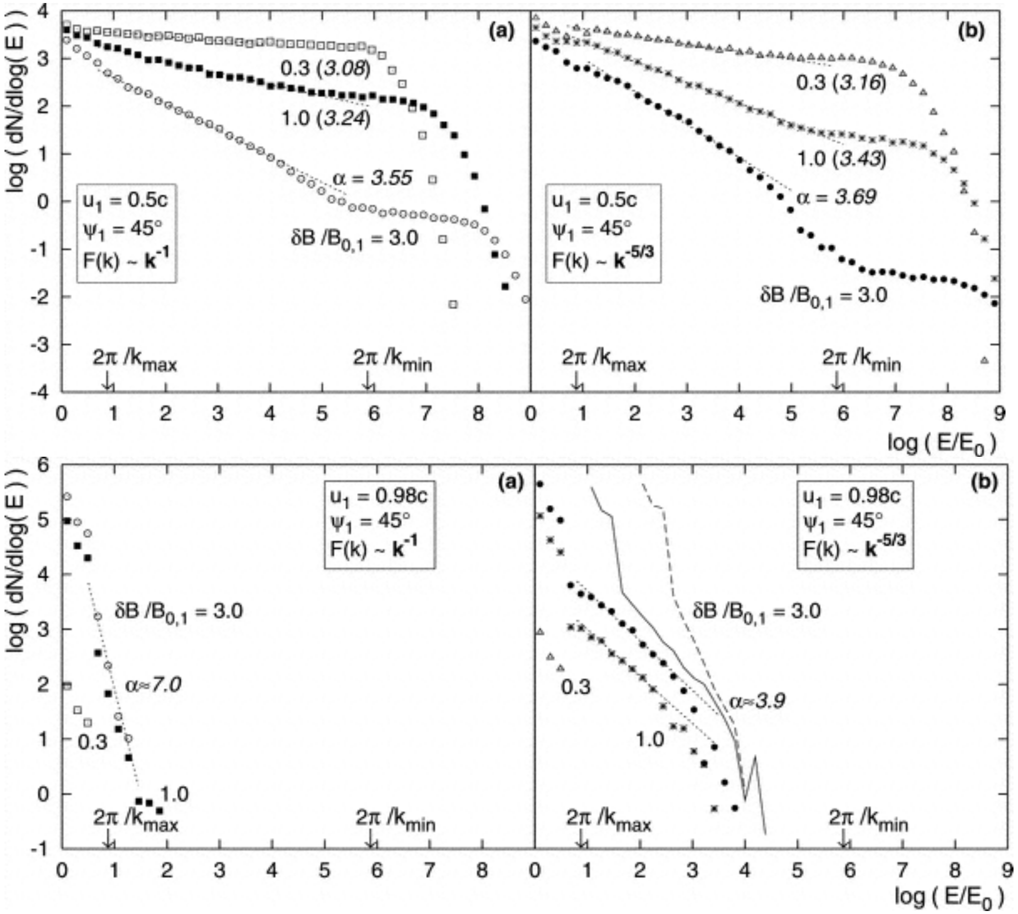
IV. Non-linear Shocks



Concave spectra: lower energy CRs “feel” smaller shock compression ratio.

Conditions expected for young SNRs.

IV. Relativistic Shocks



"Subluminal" shocks:
projection of the upstream fluid velocity
on the upstream magnetic field direction
is less than the light speed

"Superluminal" shocks:
projection of the upstream fluid velocity
on the upstream magnetic field direction
is larger than the light speed

Niemiec & Ostrowski 2004

V. Energy Evolution

$$\frac{\partial n(E)}{\partial t} = -\frac{\partial}{\partial E}\left[\frac{E\, n(E)}{\tau_{rad}}\right] - \frac{n(E)}{\tau_{esc}} + \dot{Q}(E) \quad (42)$$

$$n(E) = \frac{\tau_{rad}(E)}{E} \int_E^\infty dE' \; \dot{Q}_e(E') \; \exp\left[-\int_E^{E'} \frac{dE''}{E''} \; \frac{\tau_{rad}(E'')}{\tau_{esc}(E'')}\right] \quad (43)$$

$$n(E) \sim \tau_{esc} \times \dot{Q} \quad \text{for} \quad \tau_{rad} \gg \tau_{esc} \quad (44)$$

$$n(E) \sim \tau_{rad} \times \dot{Q} \quad \text{for} \quad \tau_{rad} \ll \tau_{esc} \quad (45)$$

For example, $\tau_{syn} \sim E^{-1}$, so the injected spectrum $n(E) \sim E^{-2}$ subjected to synchrotron cooling steepens to $n(E) \sim E^{-3}$

For example, $\tau_{esc} \sim E^{-1/3}$ for a Kolmogorow turbulence, so the injected spectrum $n(E) \sim E^{-2}$ subjected to diffusive propagation steepens to $n(E) \sim E^{-2.3}$