AGN analysis and problems tutorial

Stephen Fegan
LLR/Ecole Polytechnique, France

sfegan@llr.in2p3.fr

Overview

- AGN analysis is much the same as for other source types, but some tasks more relevant, so certain “problems” more common
- Light curve calculation - tutorial / problems
- Variability testing - 1FGL and 2FGL methods
- Absolute “goodness-of-fit” measure in spectral modeling
- Flux / Index correlations

Light curve tutorial

- Choices of light curve
  - Regular binning with likelihood analysis
  - Adaptive binned with likelihood analysis
  - Aperture photometry
  - Bayesian blocks (constant rate segments)
  - *Others?*
- Choice depends on your needs
- Flux vs Flux/Index light curves

Light curve tutorial

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Regular flux LC w/Likelihood

1. Perform a standard likelihood analysis of the full time period - denote “DC analysis”
   - Identify sources in the ROI
   - Measure spectra over the full time range
   - Get best-fit “DC XML model”

Regular flux LC w/Likelihood

2. Determine what binning is reasonable

- Based on science goals and strength of source of interest
- Number of bins should not be much larger than $TS_{DC}/25$
- Consider how the presence of upper limits will affect your analysis
- Avoid periods which are close to being integer fractions of the orbital precessional period of 53.7day.

Regular flux LC w/Likelihood

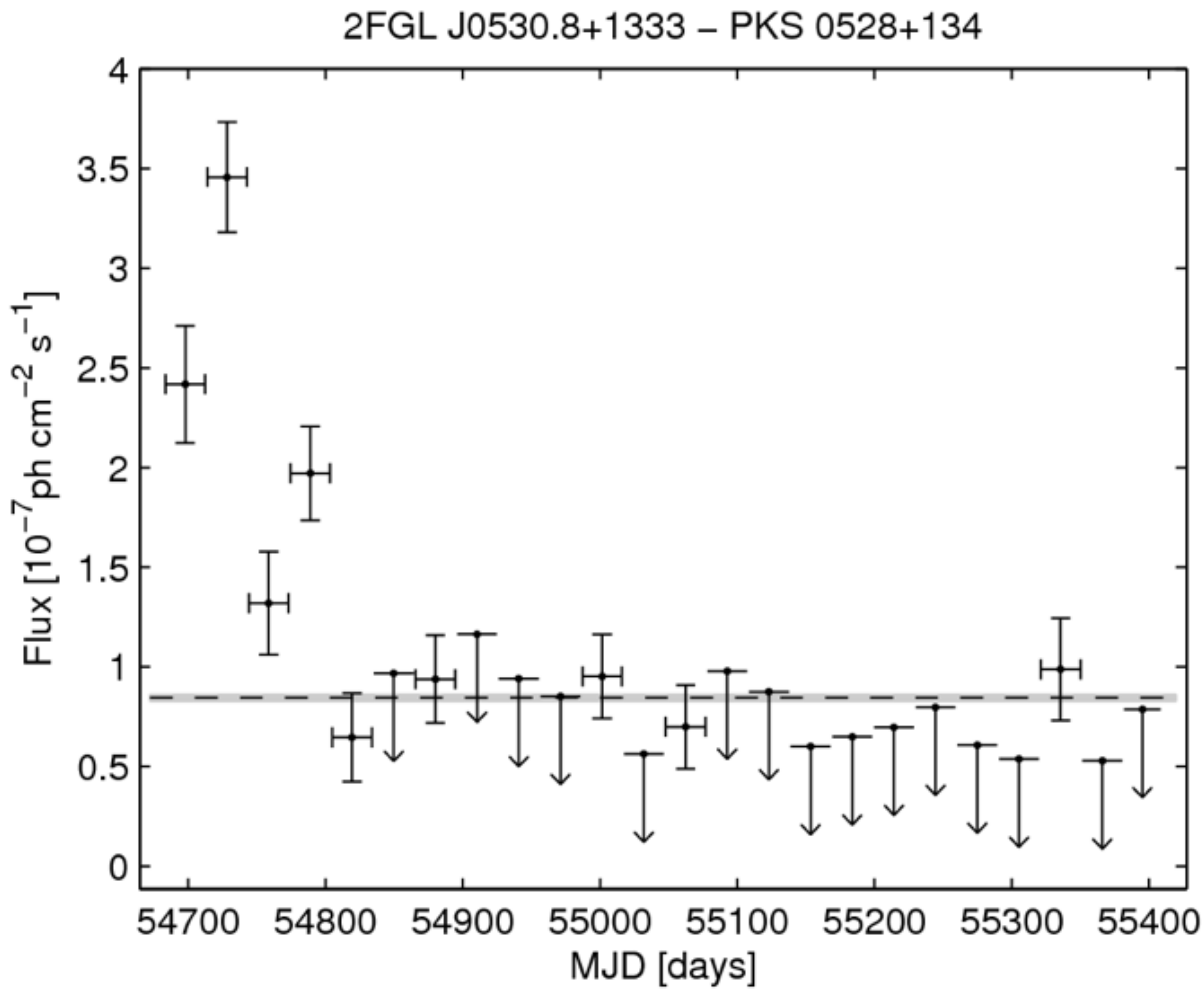
3. Prepare an ROI model for the time bins

- Freeze spectral shapes of all background sources (PL index, alpha, beta...)
- Freeze *all* parameters of weak background sources - these will cause convergence problems. $TS_{DC}/N_{bin}<4$ or 9
- For flux-only LC, freeze spectral shape of source of interest. Smaller flux errors and better sensitivity to variability.

Regular flux LC w/Likelihood

4. Decide on criteria for upper limits (2FGL?)
   - TS_i<10 or $\Delta F_i/F_i$>0.5 (or $Npred_i$<3)
   - 95% Bayesian UL when TS_i<1.
   - 95% Profile method otherwise
     [*delta = chi2inv(2*(0.95-0.5))/2 = 2.71/2*]
5. Divide data into bins (gtselect)
6. Run likelihood analysis on each
7. Check each analysis for problems (see later)
8. Compute upper limits where necessary

Example from 2FGL

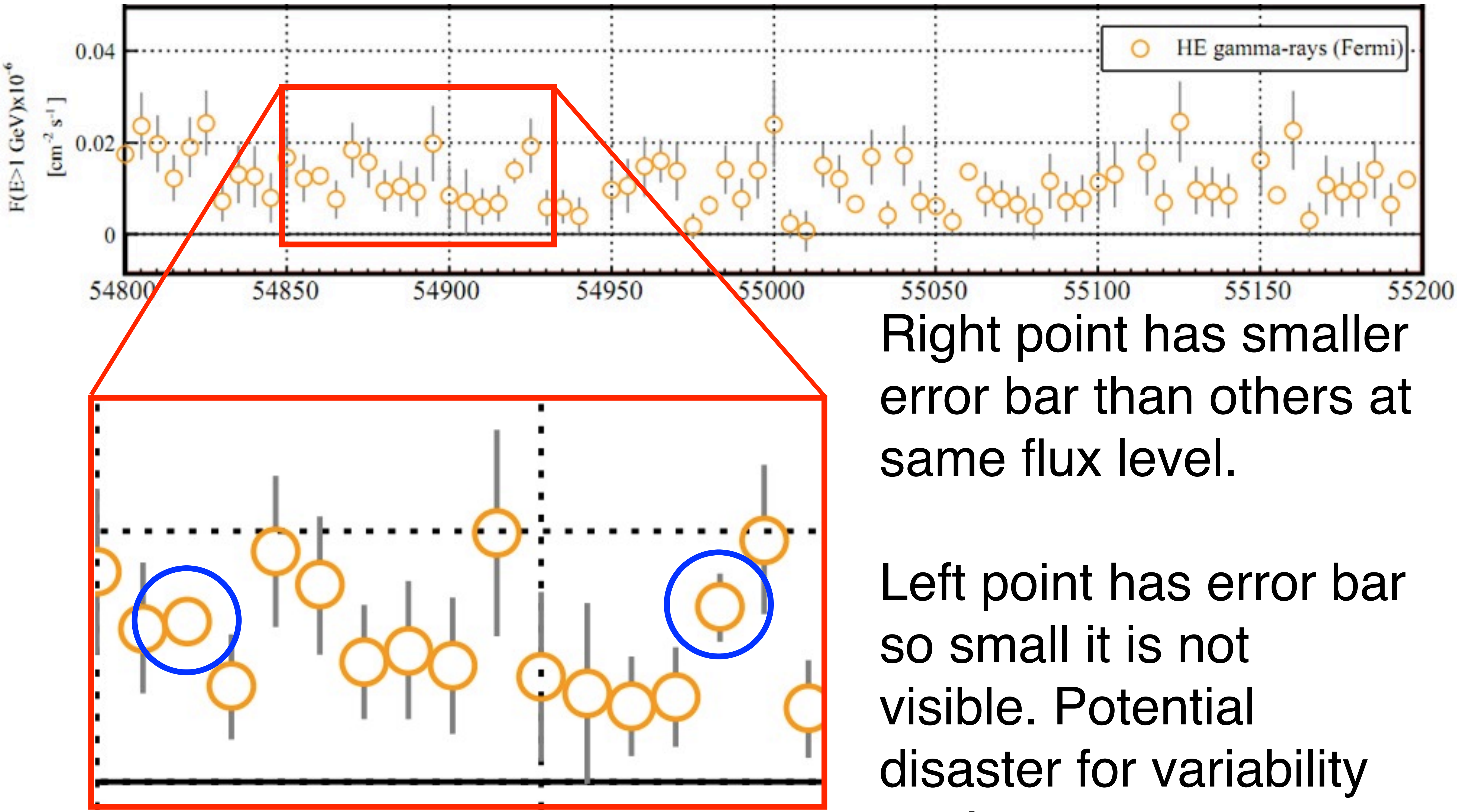


Error-matrix problems in LCs

- A relatively common problem in calculation of LC points is that the minimizer may not calculate the error matrix properly
- Usually the minimizer will converge, and the model parameters will be OK...
- But the errors can be VERY wrong
- Hence a χ^2 fit to a constant can be wrong
- This seems to be related to parameters that are not properly constrained (and hence hit limits).

Sample LAT LC from

http://arxiv.org/abs/1101.2764

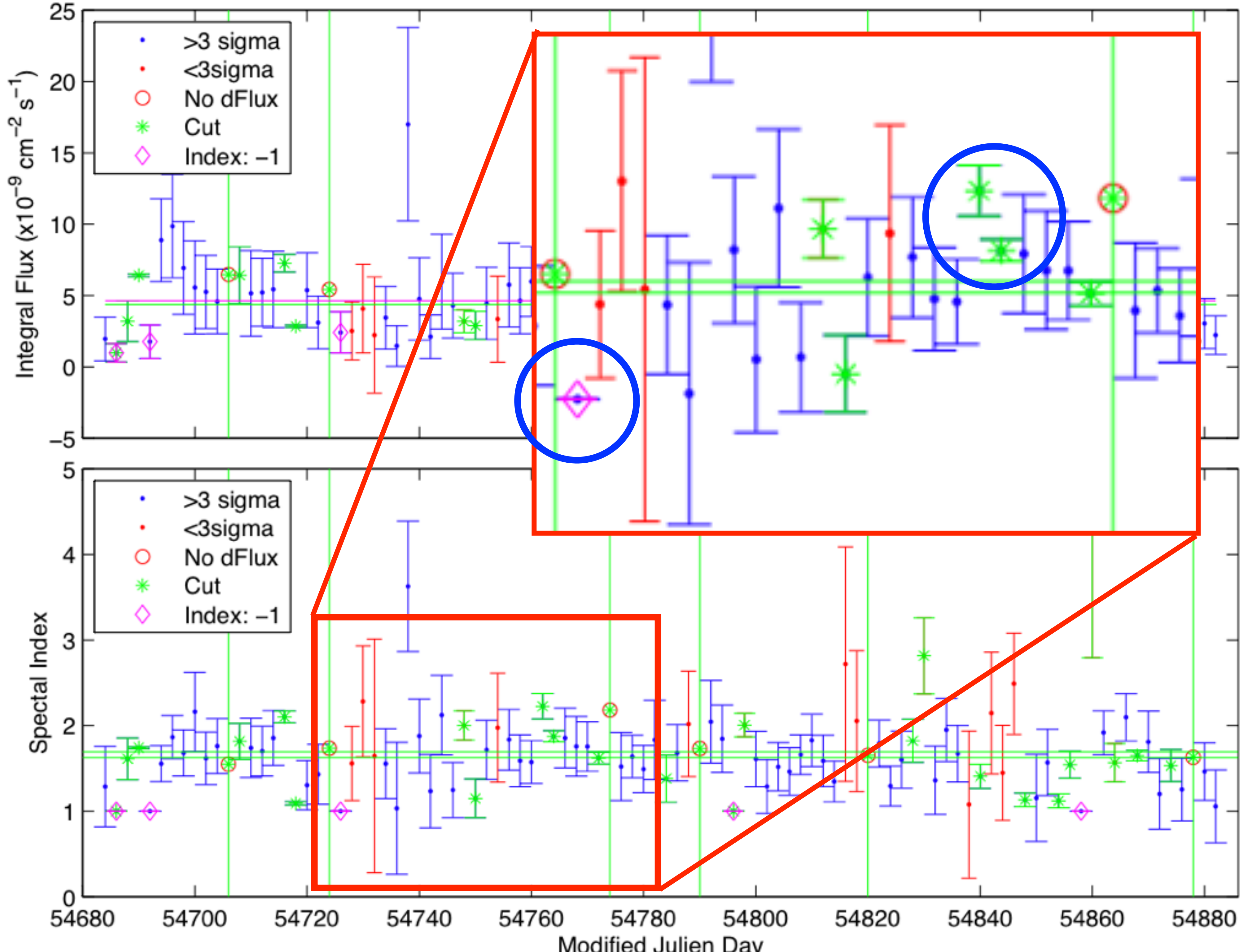


Right point has smaller error bar than others at same flux level.

Left point has error bar so small it is not visible. Potential disaster for variability test!

Aleksić et al.

Sample flux/index LC

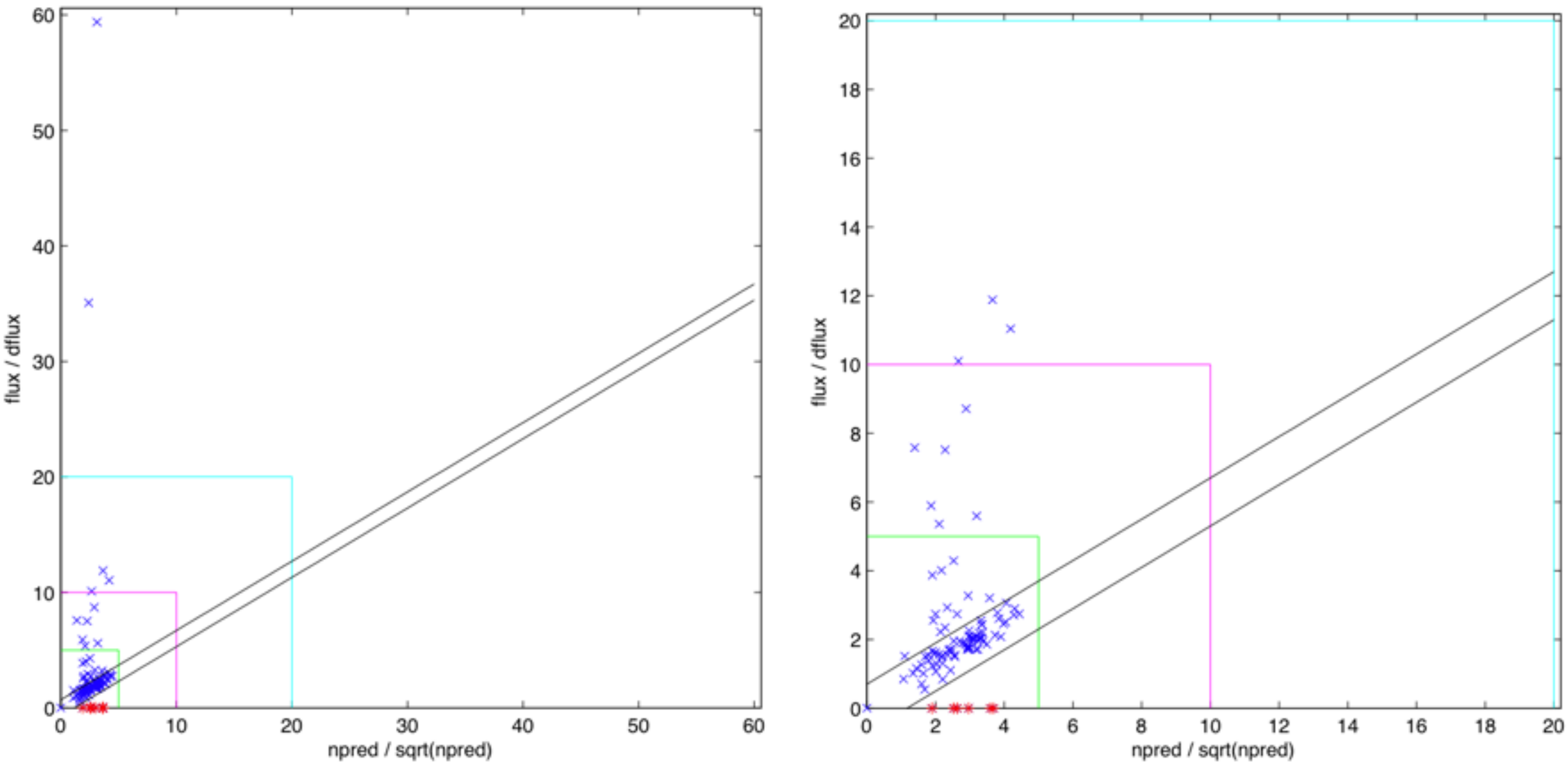


D. Horan

Automated test?

- Need some automatic way to find points whose errors are “too small”
- Doug told us that minimum error (variance) is given by Cramér–Rao bound. But that is not calculated in ST
- Recognize that if source model has converged and predicts Npred counts, then ratio of flux error to flux (σ_F/F) should not be better than that of Poisson counts underlying measurement: sqrt(Npred)/Npred

Simple test to find such points



Plot of flux/flux-error to npred/sqrt(npred) and find outliers. These should be investigated.

D. Horan

Simple test to find such points

```
>>> from math import *
>>> import BinnedAnalysis
>>> obs=...
>>> like=...
>>> like.fit()
>>> src='VER0521'
>>> flux=like.normPar(src).getValue()
>>> error=like.normPar(src).error()
>>> npred=like.NpredValue(src)
>>> print error/flux, sqrt(npred)/npred
0.0281931953422 0.421340108935
>>> print like.optObject.getRetCode()
102
```

← Anything other than zero indicates problems

Check MINUIT status

gtlike with chatter=3

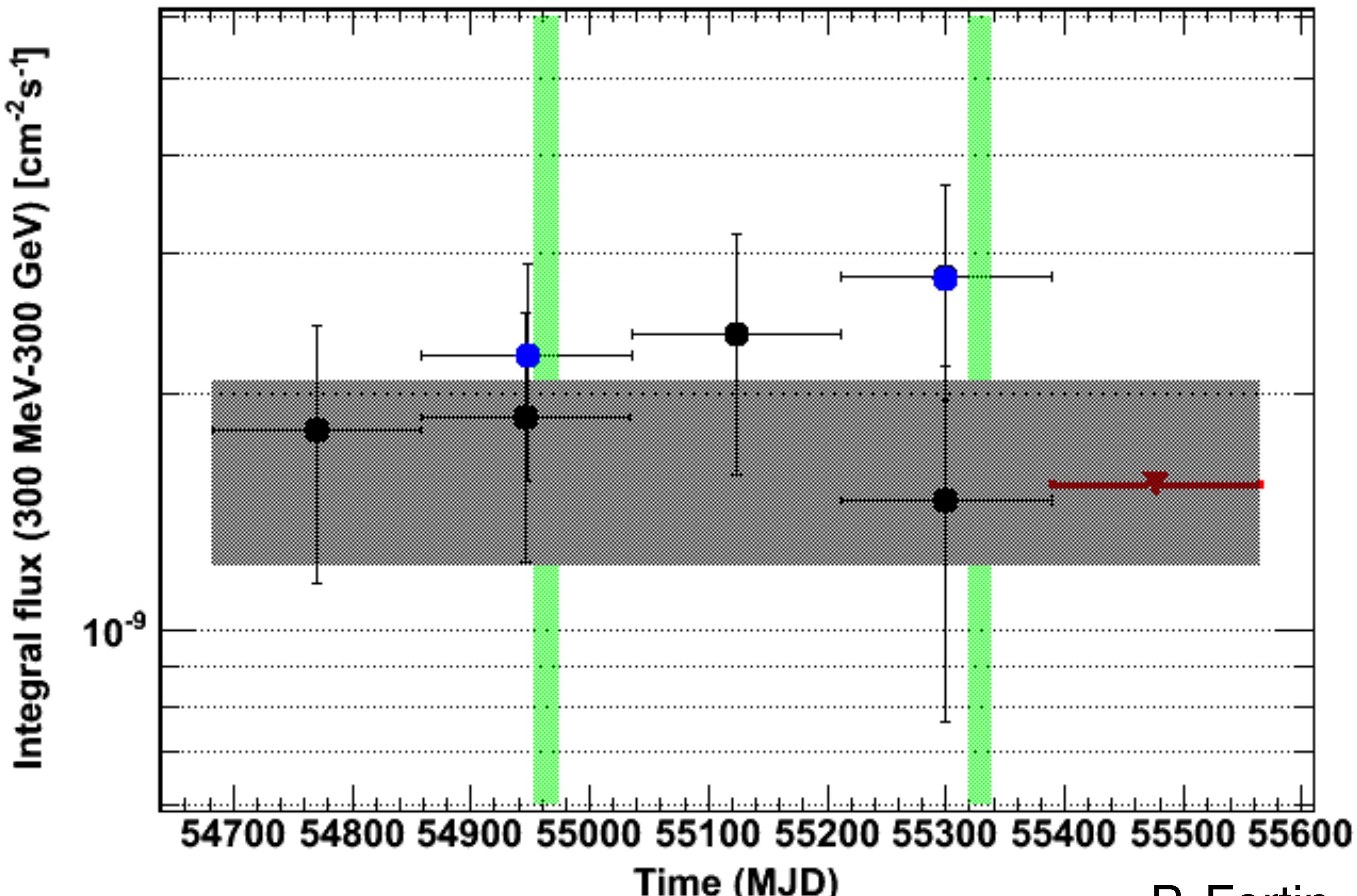
```
 ERR MATRIX NOT POS-DEF
Minuit fit quality: 2   estimated distance: ...
```

pyLikelihood

```
obs = BinnedObs(...)
like = BinnedAnalysis(obs,'model.xml','MINUIT')
minuit_obj = pyLike.Minuit(like.logLike)
like.fit(covar=True,optObject=minuit_obj)
distance_from_minimum=minuit_obj.getDistance()
qual = minuit_obj.getQuality()
```

0: Error matrix not calculated at all
1: Diagonal approximation only, not accurate
2: Full matrix, but forced positive-definite *(i.e. not accurate)*
3: Full accurate covariance matrix (After MIGRAD, this is the indication of normal convergence.)

Conjunctions with the Sun



P. Fortin

Conjunctions with the Sun can lead to contamination of the LC by gamma rays from the Sun.

In 2FGL we flag all periods when the sun-to-source separation is less than 2.5deg.

- Always check the ecliptic coordinates of the source
- For sources with low ecliptic latitude remove time periods where the source-to-Sun separation is small

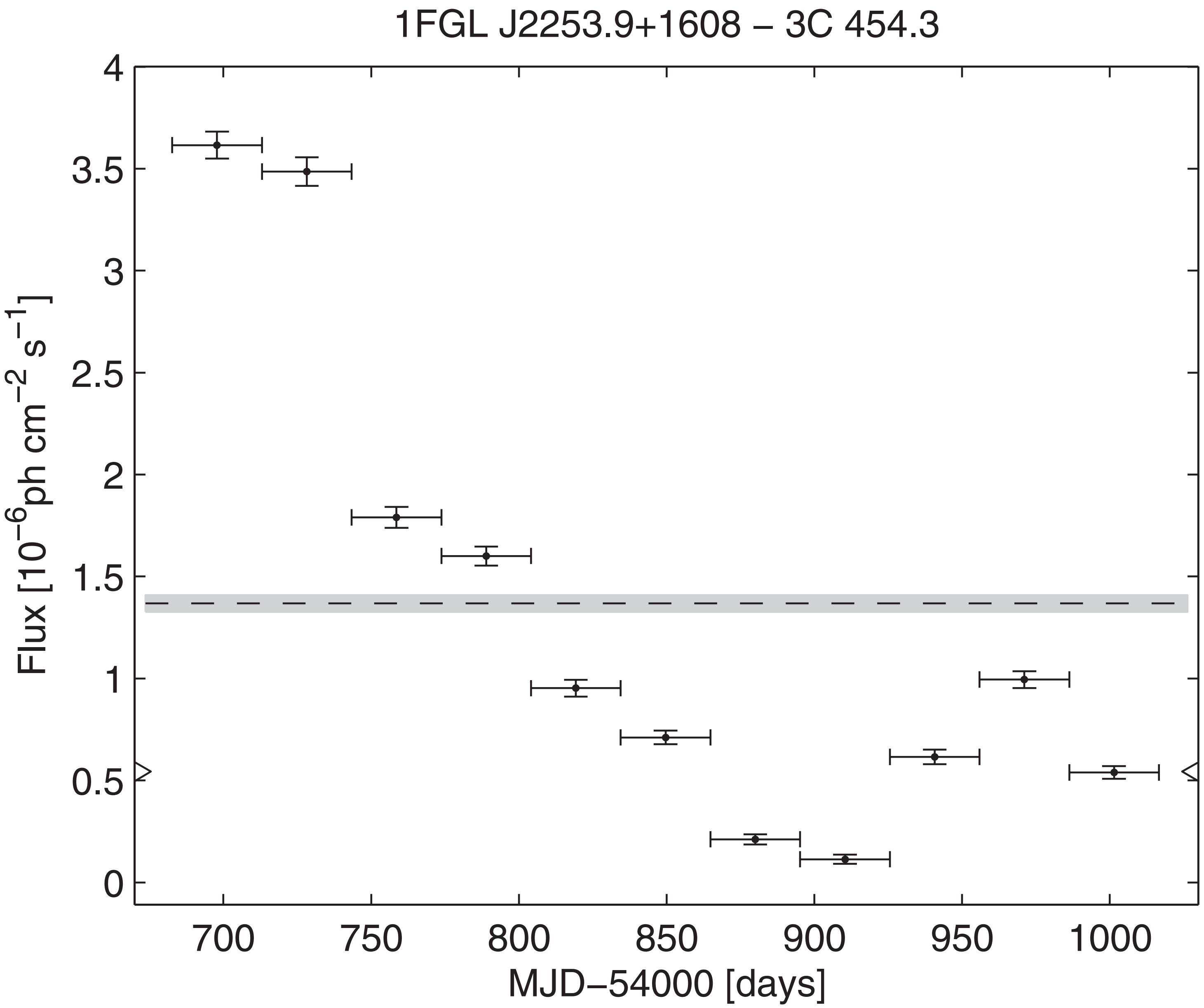
Recommendations for LC

- Follow the strategy of 1FGL / 2FGL
- Freeze spectral shape parameters (PL index, alpha/beta, Ebreak etc...)
- Freeze very weak background sources that have very low TS values
- Set threshold on TS, ΔF/F (and Npred?) and calculate upper limits for weaker sources
- Look for suspicious points! Check MINUIT status.

Variability testing

- If you claim variability (or lack of it) then a quantitate test is a good idea.
- Different variability tests are available and may be useful in certain circumstances
  - 1FGL chi-squared test for a constant flux
  - 2FGL likelihood test for a constant flux
  - Bayesian blocks test for Poisson process
- 2FGL method is more sensitive than 1FGL (but more complex to compute)

1FGL variability index



χ^2 criterion based on best-fit fluxes and flux errors in LC.

$$w_i = \frac{1}{\sigma_i^2 + (f_{\rm rel} F_i)^2}$$

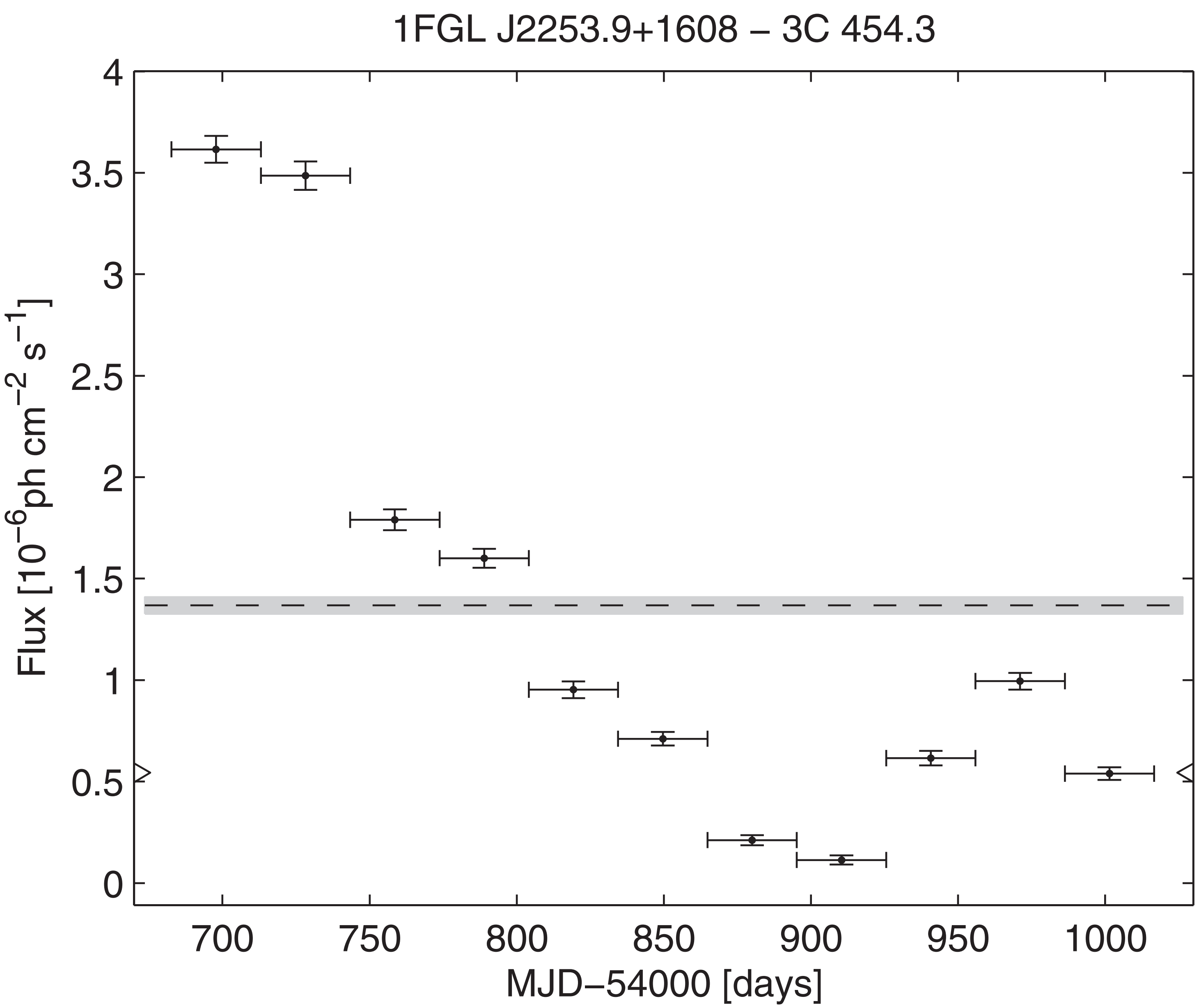
$$F_{\rm wt} = \frac{\sum_i w_i F_i}{\sum_i w_i}$$

$$V = \sum_i w_i (F_i - F_{\rm wt})^2,$$

No variability: V ~ χ^2(N-1)

Prescription to include upper limits and systematic errors in exposure.

1FGL variability index



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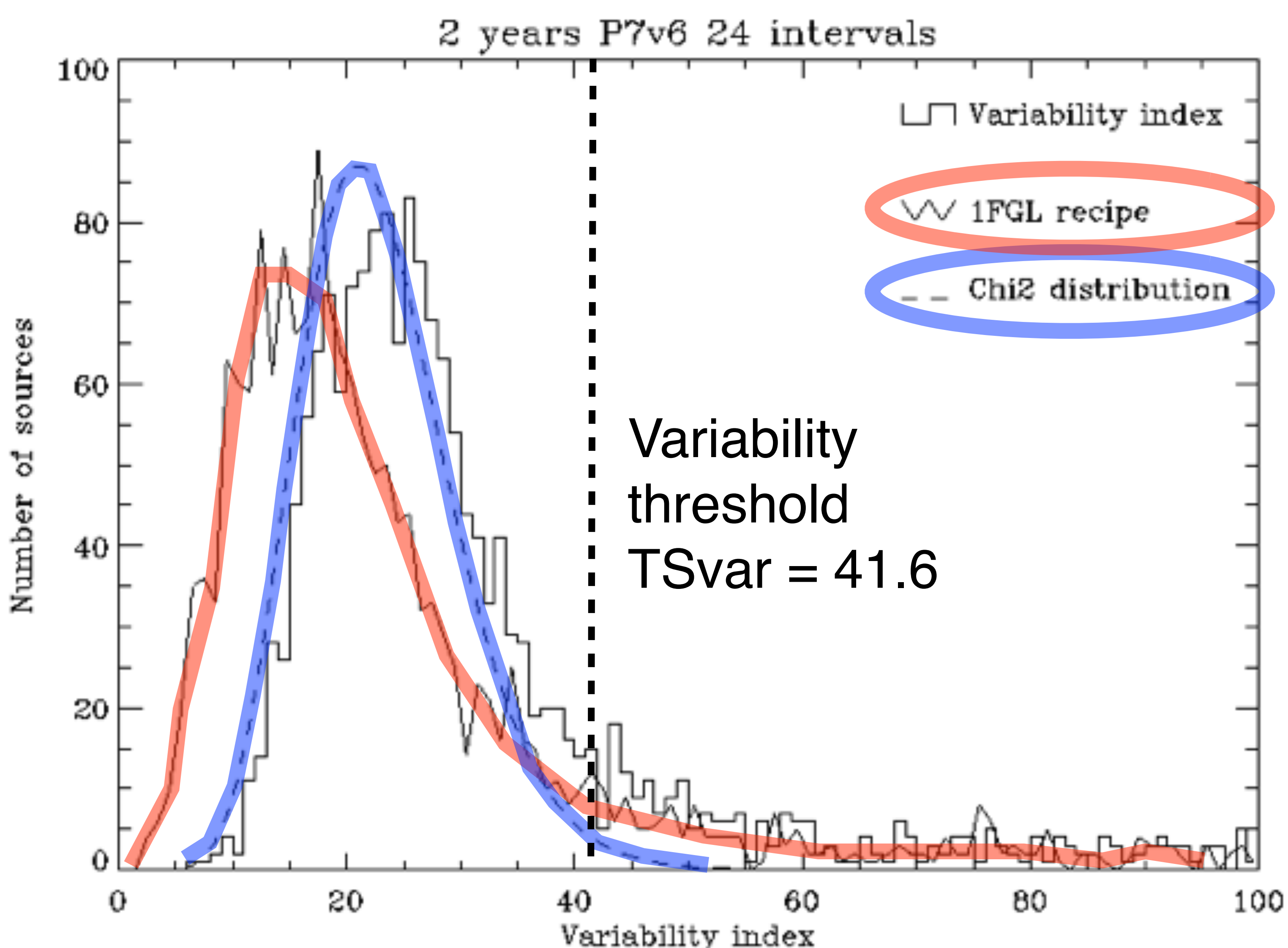
Problems with 1FGL method



J. Ballet

The 1FGL method gives significantly smaller values than expected from χ^2 statistics.

Method is therefore less sensitive to variable sources than desired.

In this method, likelihood is assumed Gaussian. But not true for weak fluxes.

In 2FGL variability index is based on actual likelihood.

2FGL variability index

- The 2FGL variability index is based on a comparison of the log likelihood values for the time bins under two hypothesis:

  0. Null hypothesis: the flux is constant in all time bins - F_{Const} - found by ML over all bins

  1. Alternate hypothesis: the source flux in each bin is different - F_i - found by ML in each bin

- Sum log likelihoods in each - $logL^{(0)}$ & $logL^{(1)}$
- Wilks' theorem: $TS_{VAR} = 2\Delta logL \sim \chi^2(N\text{-}1)$

2FGL variability index

- How to find F_{Const}? Minimized over all bins?
- Turns out that in many circumstance F_{Const} is close to the value of the source flux from the DC analysis - $F_{Const} \approx F_{DC}$
- This was *assumed* to be true in 2FGL
- This gives a simple recipe - compare the summed likelihood with the flux optimized in each bin to the summed likelihood with F_{DC}
- Can improve on this - e.g. try F_{DC} & $F_{DC} \pm x\Delta F_{DC}$

Recipe: 2FGL variability

1. Analyze full time range - “DC analysis”
2. Determine binning
3. Prepare “DC model”
4. *Optional:* decide criteria for upper limits
    - The 2FGL variability index works whether you calculate ULs or not
5. Divide data into bins

Recipe: 2FGL variability

6. Run likelihood analyses on each bin

   a. using DC model, but with the flux of the source of interest frozen at its DC value. Record total likelihood value - $logL^{(0)}_i$

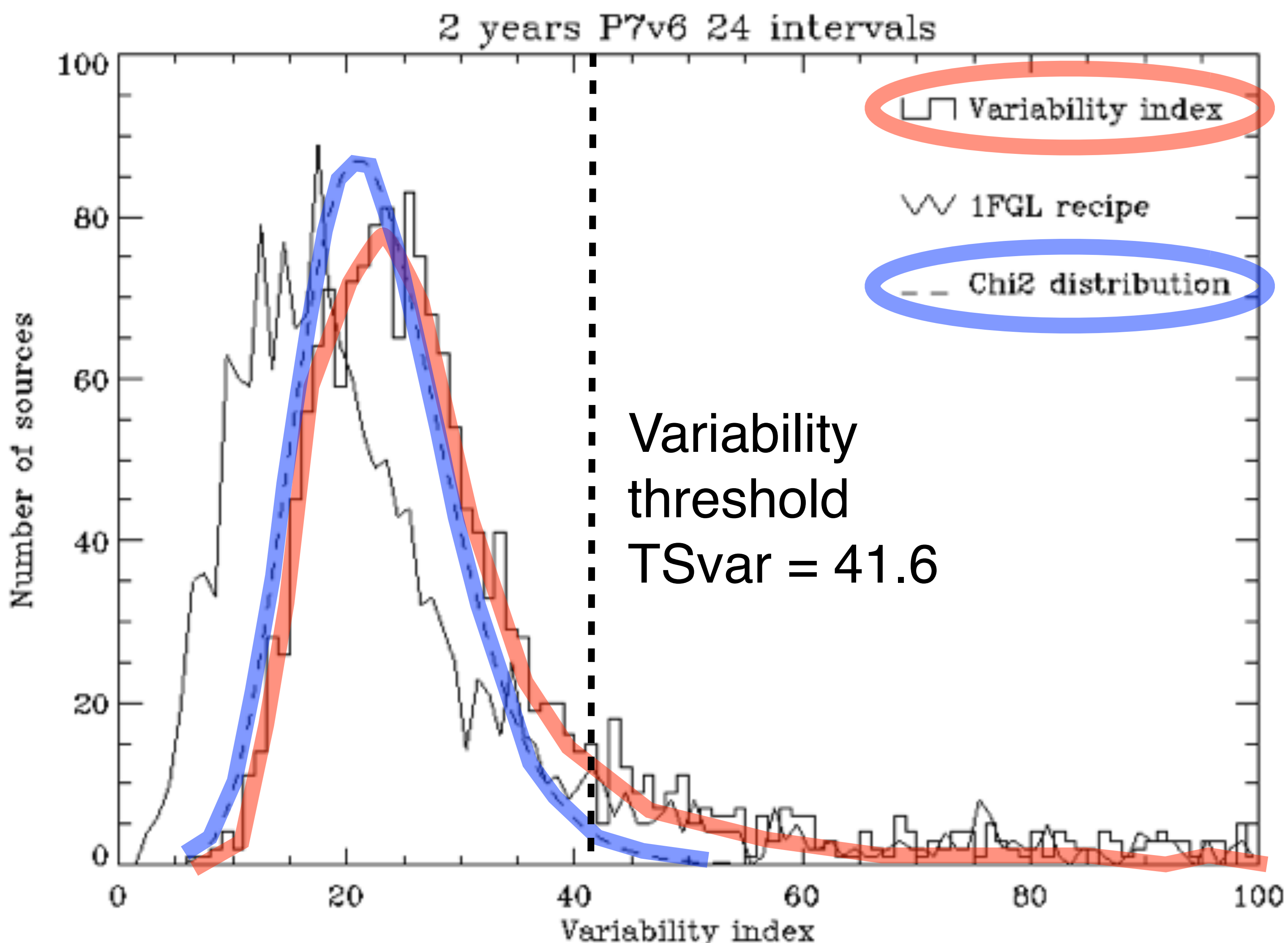
   b. *Optional:* repeat last step with flux frozen at values of say $F_{DC}\pm\frac{1}{2}\Delta F_{DC}$ & $F_{DC}\pm\Delta F_{DC}$ - in this case there are multiple test values of $logL^{(0)}_i$ for the different fluxes tried.

   c. as normal with source flux free - $logL^{(1)}_i$

Recipe: 2FGL variability

7. Check each analysis for problems
8. *Optional:* compute upper limits
9. Calculate the summed likelihood under the two hypotheses: $logL^{(0)}$ and $logL^{(1)}$
   - *Optional:* If multiple test values of $logL^{(0)}$ were calculated (step 6b) then the peak should be found by fitting a parabola to the three points around the highest
10. Calculate TS_{VAR}

Is 2FGL method better?



J. Ballet

Better agreement with χ^2 distribution than 1FGL method. Values slightly larger than expected. But:

- Source variability present to some degree (AGN)
- Small systematic in exposure calculation
- In 2FGL F_{Const} was not optimized explicitly

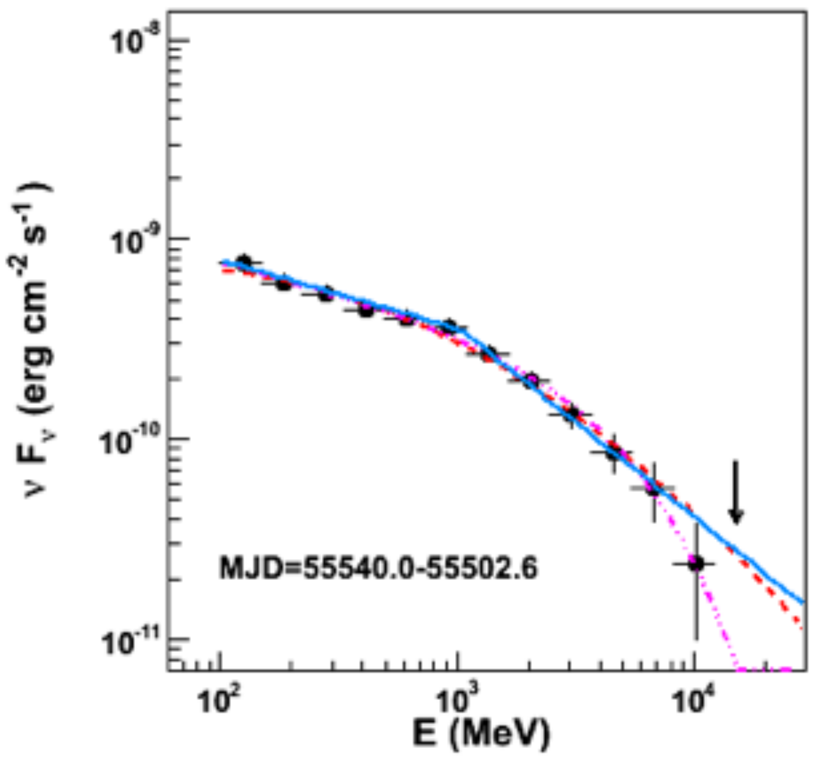
More sensitive than 1FGL method!

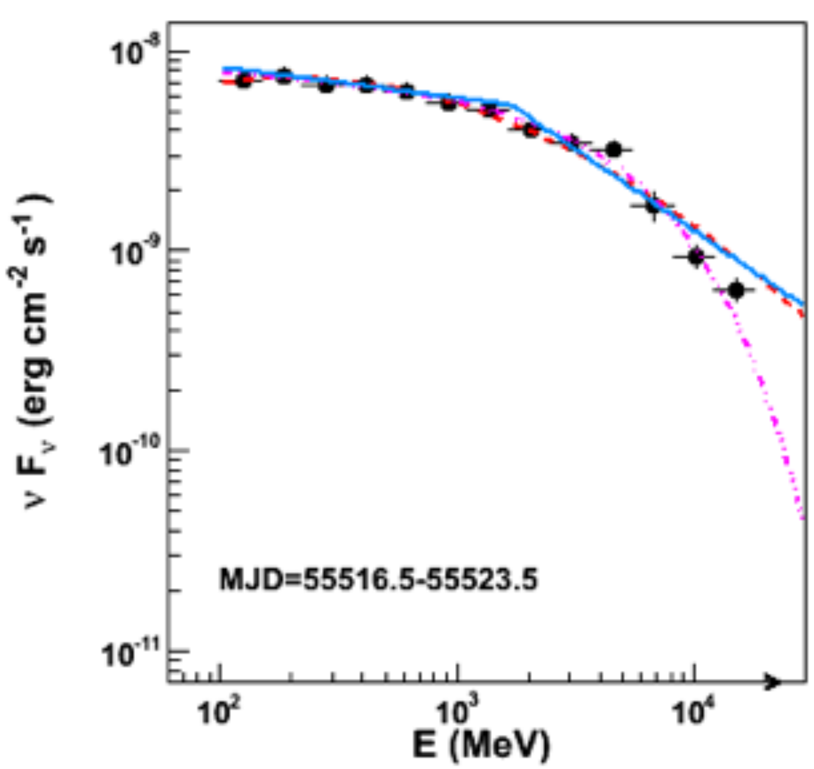
Spectral issues

Spectral model comparisons

- Changes in 2ΔlogL allows “nested” spectral models to be compared statistically
  - Log parabola (LP) to power law (PL)
  - Broken power law (BPL) to PL
  - PL with exponential cutoff (ECO) to PL
- But be careful comparing non-nested models, e.g. trying to choose between BPL and LP

Spectral model comparisons





- **BPL2** (parameters derived from a loglikelihood profile fitting)

	Flux (10^{-6} ph cm^{-2} s^{-1})	Index1	Index2	Ebreak (MeV)	LogL	Delta LogL
Quiet	3.52+/-0.08	2.34+/-0.02	2.95+/-0.07	1000^{+100}_{-100}	100325.3	-31.6
Plateau	11.2+/-0.2	2.28+/-0.02	3.0+/-0.1	2800^{+300}_{-600}	49522.1	-18.1
Flare	43.0+/-0.6	2.15+/-0.01	2.81+/-0.05	1700^{+100}_{-200}	66656.7	-74.5
Post-flare	20.2+/-0.3	2.29+/-0.02	3.2+/-0.1	2300^{+300}_{-300}	62082.8	-44.4

- **LogParabola**

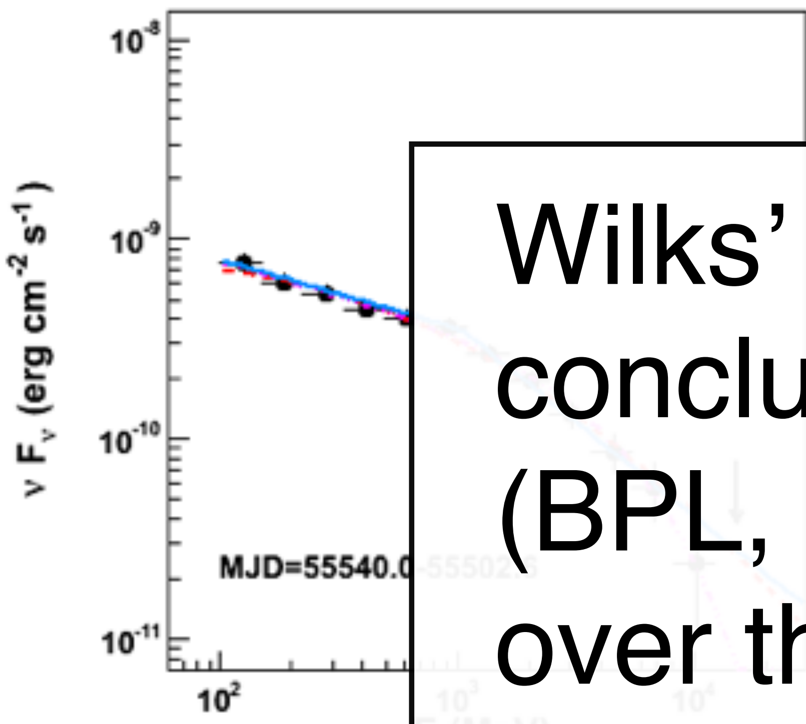
	Normalization (1e-9)	Index	Eb (MeV)	beta	LogL	Delta LogL
Quiet	0.192+/-0.004	2.61+/-0.03	1000	0.11+/-0.01	100330.6	-26.3
Plateau	0.75+/-0.02	2.39+/-0.02	1000	0.06+/-0.01	49526.4	-13.8
Flaring	3.40+/-0.06	2.36+/-0.02	1000	0.109+/-0.009	66657.5	-73.7
Post-flare	1.32+/-0.03	2.49+/-0.02	1000	0.12+/-0.01	62089.8	-37.4

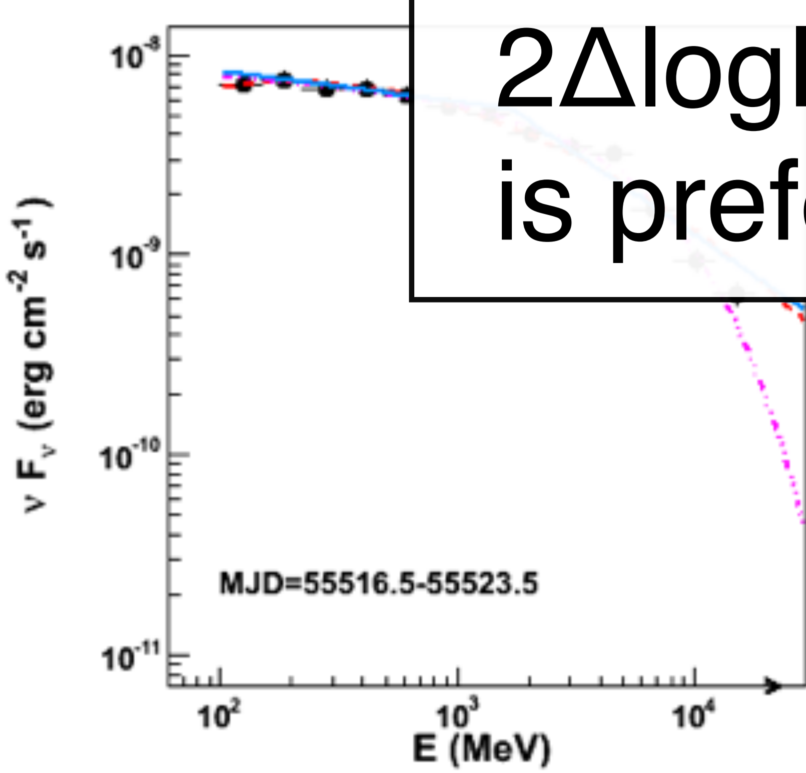
- **PL+Expcutoff**

	Prefactor (1e-7)	Index	Eb	p1	LogL	Delta LogL
Quiet	0.49+/-0.02	2.30+/-0.04	0.01	5000+/-1000	100332.8	-24.1
Plateau	1.41+/-0.05	2.23+/-0.03	0.01	11000+/-2000	49521.9	-18.3
Flaring	5.0+/-0.1	2.09+/-0.02	0.01	6200+/-700	66645.7	-85.5
Post-flare	2.56+/-0.07	2.21+/-0.02	0.01	5900+/-800	62080.5	-46.7

L. Escande & B. Lott

Spectral model comparisons





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	Flux (10^{-6} ph cm^{-2} s^{-1})	Index1	Index2	Ebreak (MeV)	LogL	Delta LogL
Quiet	[illegible]	[illegible]	[illegible]	[illegible]	100325.3	-31.6
Plateau	[illegible]	[illegible]	[illegible]	[illegible]	49522.1	-18.1
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Post-flare	[illegible]	[illegible]	[illegible]	[illegible]	62089.8	-37.4

Wilks' theorem can be used to conclude that all the models (BPL, LP, ECO) are preferred over the simple PL. However it is not correct to use the 2ΔlogL values to say that ECO is preferred over BPL or LP.

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L. Escande & B. Lott

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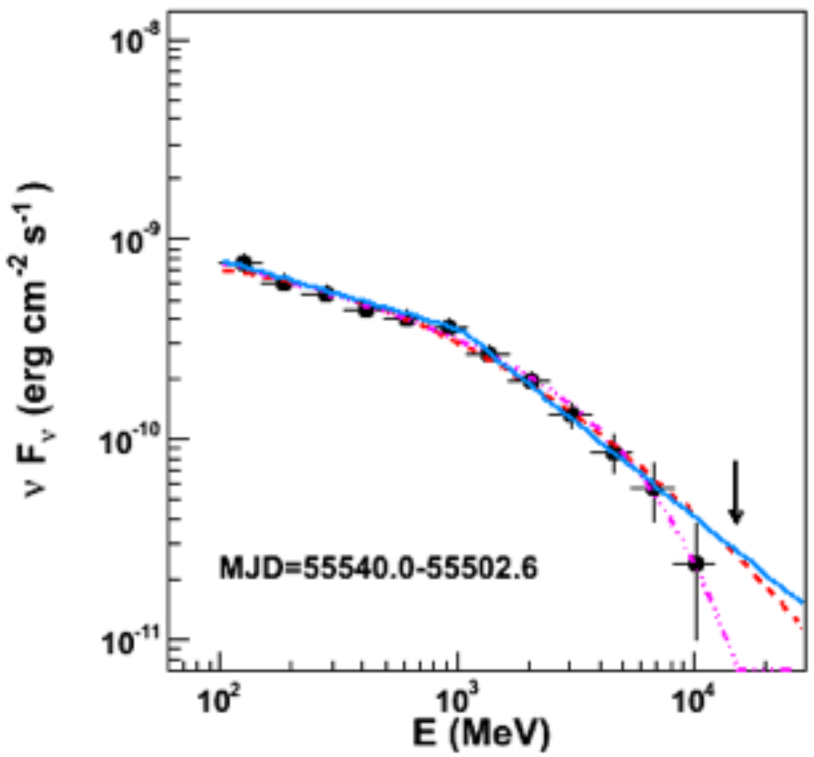

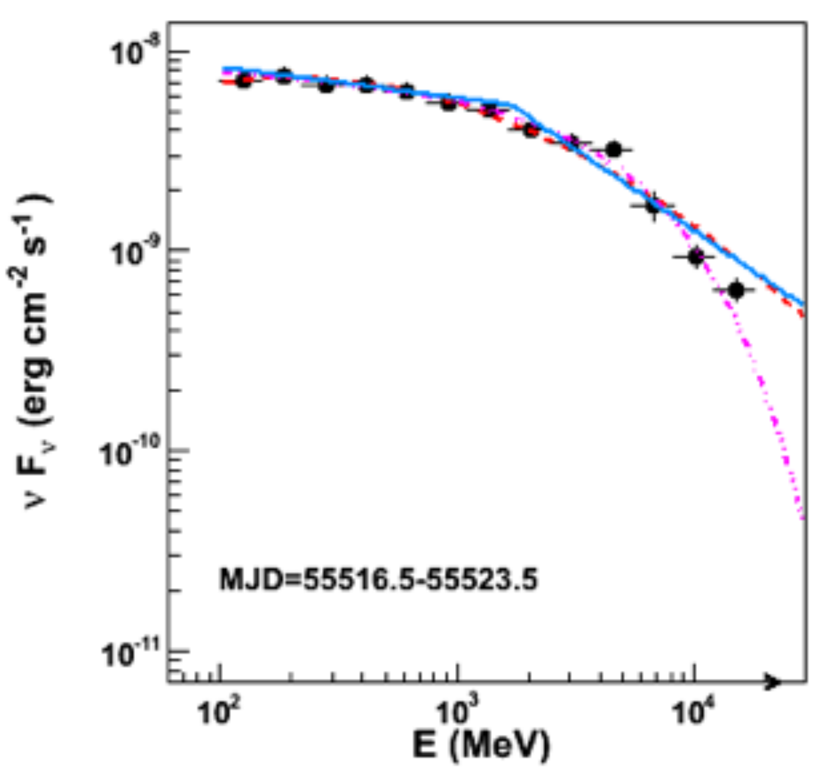

Absolute "goodness of fit" measure using flux in band values and chi-squared fit.

- Calculate flux in band values
- Calculate BPL, LP, ECO model prediction for each band
- Compute X^2 for each model

See 1FGL, 4C21.35, 3C454.3 papers

Can be done with Likelihood also in manner similar to variability test

	Number of points	BPL2	LogPar	Expcutoff
Quiet	12	6.3	19.6	9.4
Plateau	13	6.5	12.5	7.2
Flare	13	45.9	43.9	22.8
Post-flare	12	16.6	13.0	6.3

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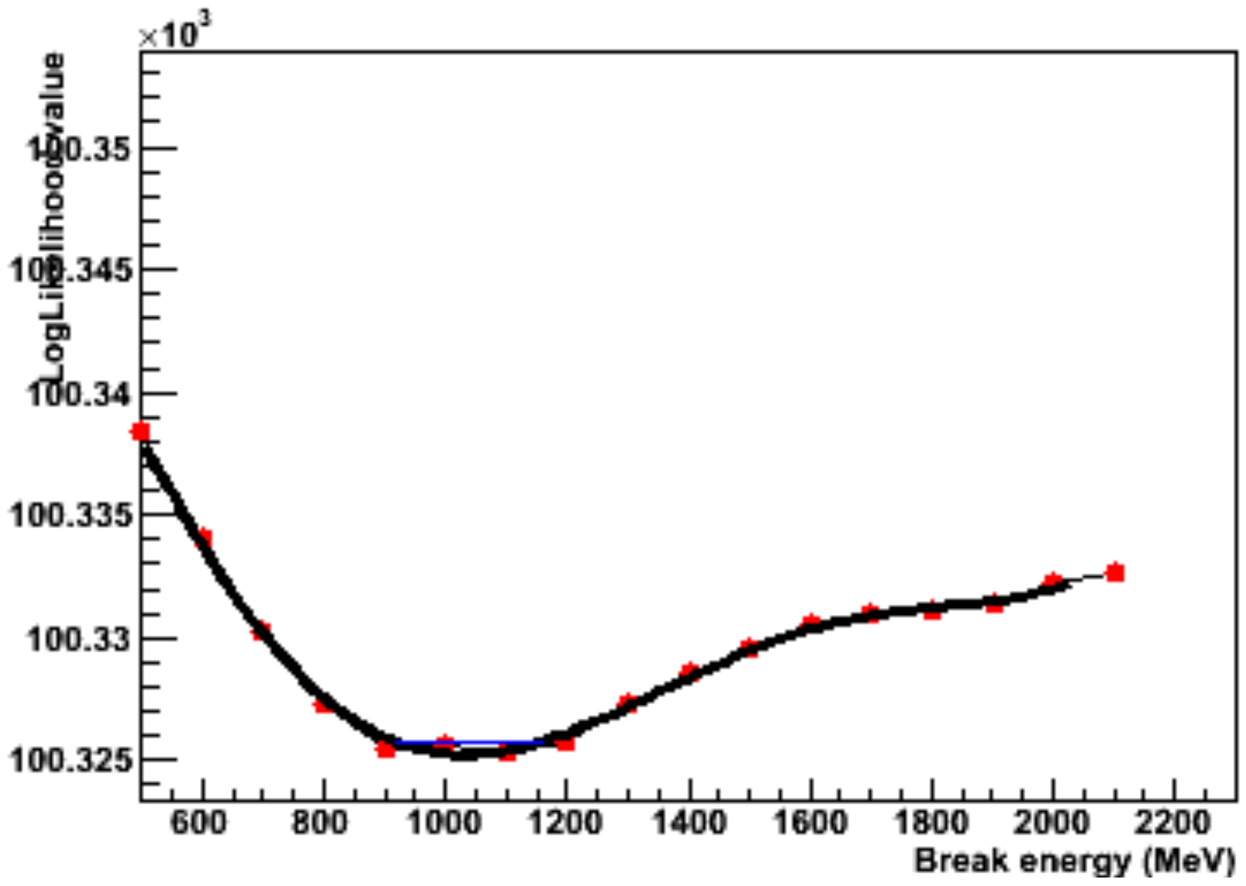
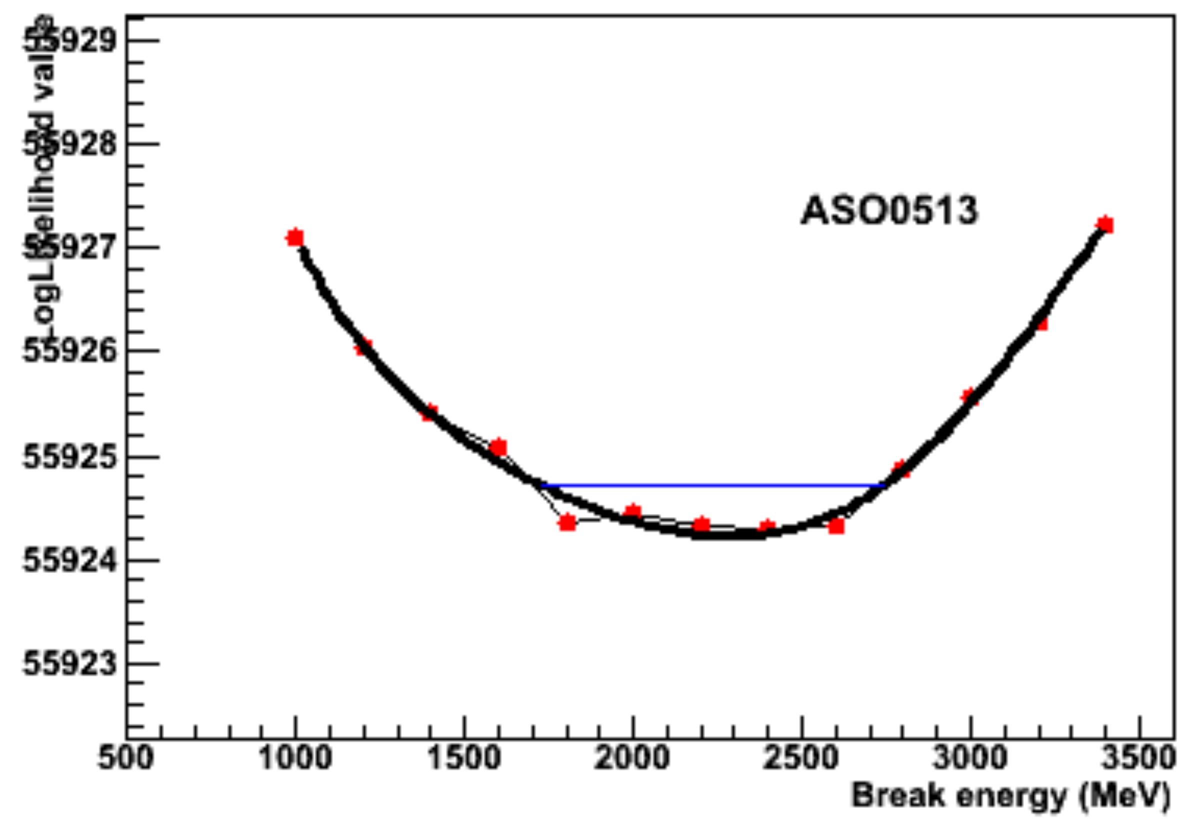
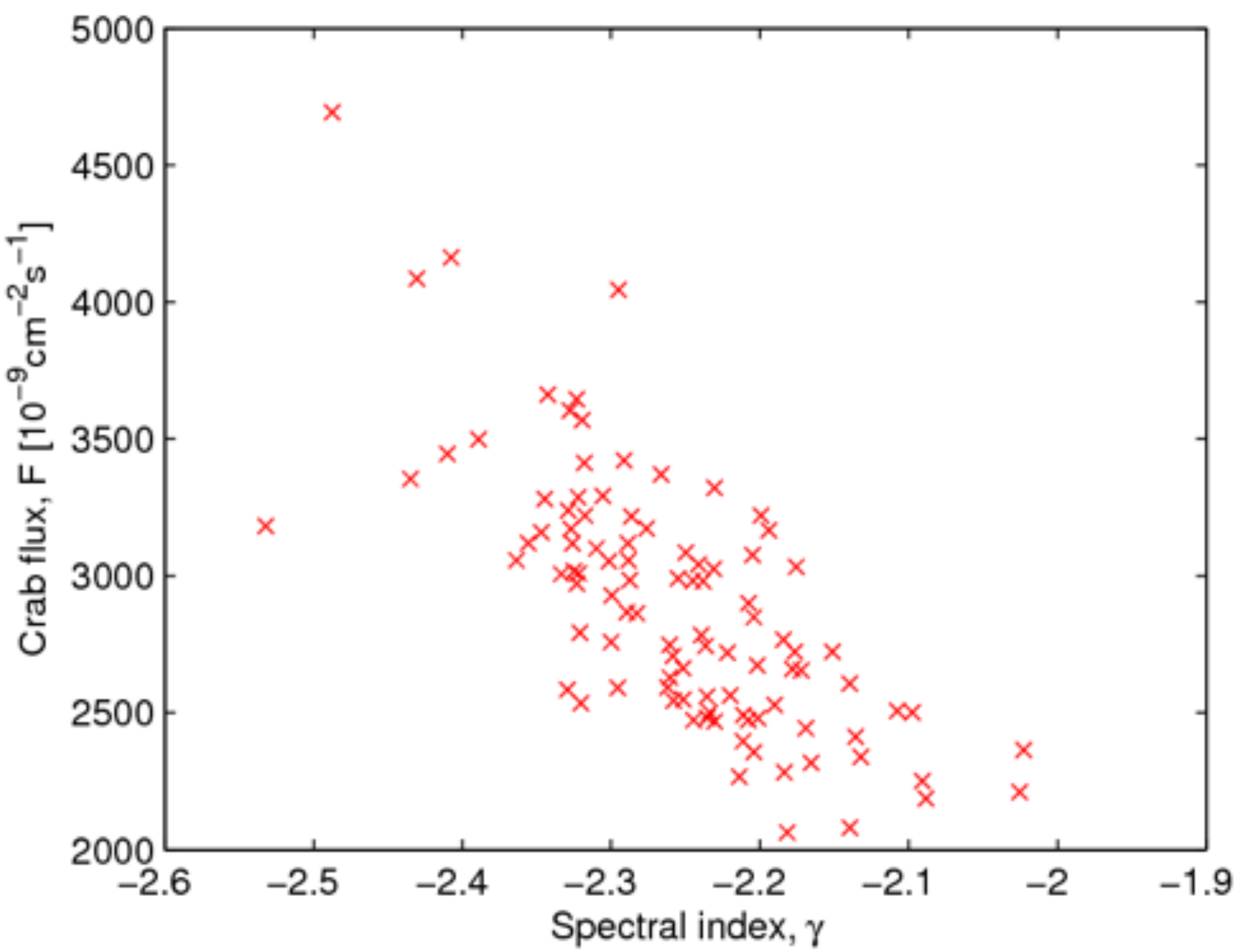
Optimization of BPL break



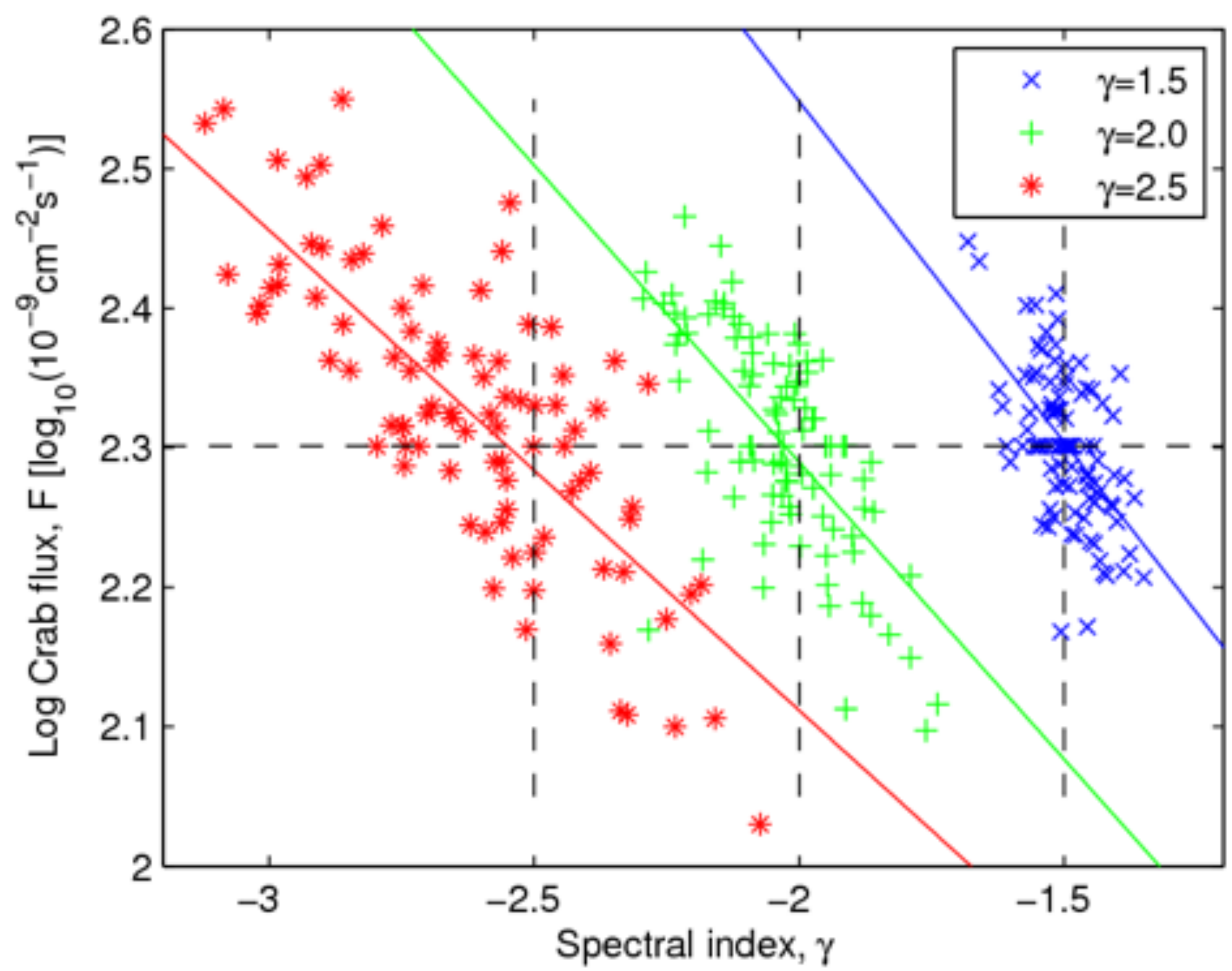


One final thought on BPL models... optimization of break energy with ST minimizers does not work well. Better to calculate the Likelihood profile and find minimum and confidence interval manually (ΔlogL=0.5)

Flux/Index correlations



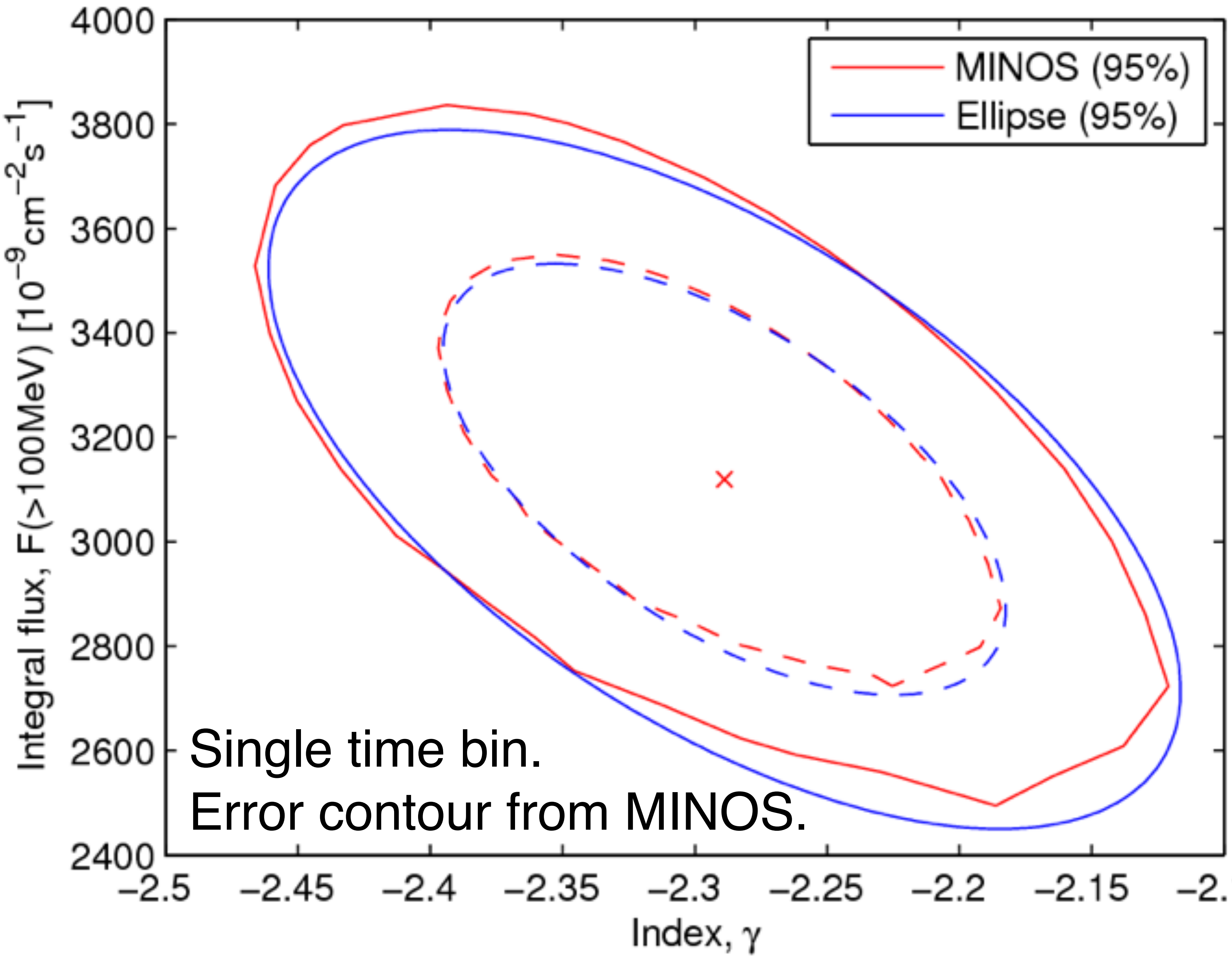
Crab 3-day bins



Simulations of 3 sets of sources with constant spectra

In a PL fit the integral flux and spectral index are intrinsically (mathematically) related

Flux/Index correlations



Beware of Harder/Weaker correlations

“Decorrelation energy”

- The correlation can be eliminated if the integral flux is expressed above the “integral flux decorrelation energy”
(see http://tinyurl.com/LAT-decorrelation)
- Not to be confused with “Pivot energy” or “differential flux decorrelation energy” - E_0
- Optimal integral window has low energy bound of:
$\ln(E_{low}) \approx \ln(E_0) - 1/(\gamma-1)$ [see eq. 21]