SSC physical parameters from observations

Given the synchrotron peak energy ϵ_s, SSC peak energy ϵ_c (both in the observer's frame), the synchrotron peak luminosity L_s and the Compton dominance κ of a source, find its physical configulation.

Recall : $\epsilon_s = \gamma_b^2 \frac{B\delta}{B_{crit}}$, $\epsilon_c = \gamma_b^4 \frac{B\delta}{B_{crit}}$, B in $B_{crit} = 4.41\, 10^{13}$ G units

This is the slow cooling regime: Here I assumed that the electron index is $p>2$ ($n(\gamma) \propto \gamma^{-p}$) and the peaks of the SED are formed at γ_b where cooling and escape times are equal. I also assumed Thomson scattering forms the SSC peak.

I will include numerical values as an example for the case of aligned HSP sources that are powerdul:

$$\epsilon_s = 2\times 10^{-4}, \quad \epsilon_c = 2\times 10^{5} \quad L_s = 10^{46} \text{ erg/s}, \quad \kappa = 1$$

Find γ_b: $\gamma_b = \left(\frac{\epsilon_c}{\epsilon_s}\right)^{1/2} = 10^{4.5}$

Thompson condition requires $\frac{1}{\delta}\epsilon_s \gamma_b < 1 \Rightarrow \delta > \epsilon_s \left(\frac{\epsilon_c}{\epsilon_s}\right)^{1/2} = (\epsilon_s \epsilon_s)^{1/2} \simeq 6.3$

From $\epsilon_s = \gamma_b^2 B\delta = \frac{\epsilon_c}{\epsilon_s} B\delta \Rightarrow \boxed{B\delta = \frac{\epsilon_s^2}{\epsilon_c} B_{crit} = 8.82 = C_1}$

In Gauss

Also $U_s = \frac{L_s}{4\pi c R^2 \delta^4} = \kappa U_B = \kappa \frac{B^2}{2\,8\pi} \Rightarrow B^2 R^2 \delta^4 = \frac{2}{\kappa}\frac{L_s}{c} \Rightarrow$

$$\boxed{BR\delta^2 = \left(\frac{2L_s}{\kappa c}\right)^{1/2} = \frac{8.16\, 10^{17}}{\kappa^{1/2}} = C_2}$$

Finally, (assuming escape time $\frac{\lambda R}{c}$)

$$\gamma_b = \frac{3mc^2}{4\sigma_T \frac{B^2}{8\pi}(1+\kappa)R\lambda} = \frac{2.32\, 10^{19}}{B^2 R\lambda(1+\kappa)} \Rightarrow \boxed{B^2 R = \frac{2.32\, 10^{19}}{\lambda(1+\kappa)\gamma_b} = \frac{7.34\, 10^{14}}{(1+\kappa)\lambda} = C_3}$$

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We now have three equations with three unknowns

$$\left.\begin{aligned} B\delta &= c_1 \\ BR\delta^2 &= c_2 \\ B^2R &= c_3 \end{aligned}\right\} \quad \left.\begin{aligned} &\delta = \frac{c_1}{B} \\ &BR\frac{c_1^2}{B^2} = c_2 \Rightarrow \frac{R}{B} = \frac{c_2}{c_1^2} \\ &B^2R = c_3 \end{aligned}\right\} \quad B^3 = \frac{c_3 c_1^2}{c_2} = \frac{7.34\,10^{14}\,8.82^2\sqrt{k}}{\lambda(1+k)\,8.16\,10^{17}}$$

$$B^3 = 7\times10^{-2}\,\frac{\sqrt{k}}{\lambda(1+k)}$$

For our numerical example, $\lambda=1$: $B = 0.33$ G

$$\delta = \frac{c_1}{B} = 26.7 \qquad R = \frac{c_3}{B^2} = 3.43\ 10^{15}\ \mathrm{cm}$$

This corresponds to a variability light crossing time scale

$$t_{var} = \frac{R}{c\delta} = 4.28\ 10^3\ \mathrm{s} \sim 1\,\mathrm{h}$$

If the jet has opening angle $\Theta = \frac{1}{\Gamma} \cong \frac{1}{\delta}$, then for the blob to "fit" in the jet needs to be at a distance at least

$$d_{min} = R\Gamma = 6.5\ 10^{16}\ \mathrm{cm}$$

The poynting flux required is $L_B = \frac{B^2}{8\pi}\pi R^2 c\Gamma^2 = 3.42\ 10^{42}\ \frac{\mathrm{erg}}{\mathrm{s}}$

The radiated power is $L_{rad} = L'_{rad,com}\Gamma^2 = \frac{L_s(1+k)}{\delta^4}\Gamma^2 = \frac{L_s(1+k)}{\delta^2} = 1.4\ 10^{43}\ \frac{\mathrm{erg}}{\mathrm{s}}$

The minimum relativistic electron power is $L_{e,min} = L_{rad}$,

So the source is particle dominated at least by $\frac{L_{e,min}}{L_B} = 4.1$

NOTE: For $p>2$ $\quad L'_{inj} = \frac{Qmc^2}{p-2}\gamma_{min}^{2-p}$, $\quad L'_{rad} = \frac{Qmc^2}{p-2}\gamma_b^{2-p}$

$$L'_{inj} = L'_{rad}\left(\frac{\gamma_{min}}{\gamma_b}\right)^{2-p} = \frac{L_s(1+k)}{\delta^4}\left(\frac{\gamma_{min}}{\gamma_b}\right)^{2-p}. \quad L_{inj} = L'_{inj}\Gamma^2 = \frac{L_s(1+k)}{\delta^4}\Gamma^2\left(\frac{\gamma_{min}}{\gamma_b}\right)^{2-p}$$

Scaling and Paper form for B:

$$B^3 = \frac{C_3 C_1^2}{C_2} = \frac{c'}{1+K}\left(\frac{\epsilon_s}{\epsilon_c}\right)^{1/2} \frac{\epsilon_s^4}{\epsilon_c^2} B_{cr}^2 \frac{K^{1/2} c^{1/2}}{2^{1/2} L_s^{1/2}}$$

$$\boxed{B = \mathrm{const}^{1/3} \frac{\epsilon_s^{3/2}}{\epsilon_c^{5/6} L_s^{1/6}} \frac{K^{1/6}}{(1+K)^{1/3}}}, \quad \mathrm{const} = \frac{c' c^{1/2} B_{cr}^2}{2^{1/2}},$$

$$c' = \frac{6\pi mc^2}{\sigma_T}$$

Substituting $\epsilon_s = \frac{h\,10^{17}}{mc^2} \nu_{17}$, $\epsilon_c = \frac{10^{11}}{5.11\ 10^5} E_{100}$ (↑ 100 GeV), $L_s^{1/6} = \left(10^{45}\right)^{1/6} L_{45}^{1/6}$

We obtain $$B = \boxed{5.01 \frac{\nu_{17}^{3/2}}{\epsilon_{100}^{5/6} L_{45}^{1/6}} \frac{K^{1/6}}{(1+K)^{1/3}} G}$$

Scaling and Paper form for δ

$$\delta = \frac{C_1}{B} = \frac{\epsilon_s^2}{\epsilon_c} B_{crit} \left[\frac{r_2}{c' c^{1/2} B_{cr}^2}\right]^{1/3} \frac{\epsilon_c^{5/6} L_s^{1/6}}{\epsilon_s^{3/2}} \frac{(1+K)^{1/3}}{K^{1/6}}$$

$$\boxed{\delta = \left[\frac{r_2 \ B_{crit}}{c' c^{1/2}}\right]^{1/3} \frac{\epsilon_s^{1/2}}{\epsilon_c^{1/6}} L_s^{1/6} \frac{(1+K)^{1/3}}{K^{1/6}}}$$

$$\boxed{\delta = 29.42 \frac{\nu_{17}^{1/2}}{\epsilon_{100}^{1/6}} L_{45}^{1/6} \frac{(1+K)^{1/3}}{K^{1/6}}}$$

Scaling and paper form for R

$$R = \frac{c_3}{B^2} = \frac{c'}{1+\kappa} \frac{1}{\mathrm{Const}^{2/3}} \frac{\epsilon_s^{1/2}}{\epsilon_c^{1/2}} \frac{\epsilon_c^{10/6} L_s^{1/3}}{\epsilon_s^{3/2}} \frac{(1+\kappa)^{2/3}}{\kappa^{1/3}}$$

$$\boxed{R = \frac{c'}{\mathrm{Const}^{2/3}} \frac{\epsilon_c^{7/6} L_s^{1/3}}{\epsilon_s^{5/2}} \frac{1}{(\kappa(1+\kappa))^{1/3}}}$$

$$\boxed{R = 5.95 \; 10^{13} \frac{\epsilon_{100}^{7/6} L_{45}^{1/3}}{v_{17}^{5/2}} \frac{1}{(\kappa(\kappa+1))^{1/3}} \; \mathrm{cm}}$$

Finally $\boxed{t_{var} = 67.4 \frac{\epsilon_{100}^{4/3} L_{45}^{1/6}}{v_{17}^{3}} \frac{1}{\kappa^{1/6} (1+\kappa)^{2/3}} \; \mathrm{s}}$

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Assuming a jet power of $10^{44.5}$ from the cavity work,

the poynting flux is $\frac{3.42\ 10^{42}}{10^{44.5}} = 10^{-2}$ of the jet power

the radiative efficiency is $\frac{L_{rad}}{L_{jet}} = \frac{1.4\ 10^{43}}{10^{44.5}} = 4.4 \times 10^{-2}$

To obtain the lowest γ_{min} to which the injected power-law can extend we equate $L_{inj} = L_{jet} = 10^{44.5}$

$$L_{jet} = \frac{L_s}{\delta^4} r^2 \left(\frac{\gamma_{min}}{\gamma_b}\right)^{2-p} \Rightarrow \left(\frac{\gamma_{min}}{\gamma_b}\right)^{2-p} = \frac{L_{jet}}{L_s} \delta^2$$

$$\gamma_{min} = \gamma_b \left(\frac{L_{jet}\delta^2}{L_s}\right)^{\frac{1}{2-p}} \quad \text{For } p = 2.5$$

$$\gamma_{min} = 10^{4.5} \left(\frac{10^{44.5}\ 26.7^2}{10^{46}}\right)^{-2} = 62$$

This corresponds to $\nu_{min} = 2 \times 10^6 B \gamma^2 \Gamma \approx 2 \times 10^{12}$ Hz