The electron distribution in the acceleration and in the radiation zone.

- The kinetic equation in the acceleration zone and its steady state solution *(or, how much power one gains from acceleration)*

- The kinetic equation in the radiation zone and its steady state solution *(or, does radiation from the radiation zone dominates?)*

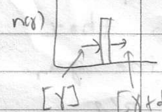
①

Understanding variability in a self consistent way

Physical picture: particles are injected into the acceleration zone where they stochastically accelerate and escape into the radiation zone. There they radiatively cool before they eventually escape. (First paper by Kirk, Rieger, Mastichiadis 98

from Higuera and Georganopoulos, in prep

In the acceleration zone:



$$\frac{\partial N(\gamma,t)}{\partial t} + \frac{\partial}{\partial\gamma}\left[\left(\frac{\gamma}{t_{acc}} - C\gamma^2\right)N\right] + \frac{N}{t_{esc}} = Q\,\delta(\gamma-\gamma_0)$$

$$C \simeq \frac{4}{3}\frac{\sigma_T}{mc}\left(U_B + \int_0^{1/\gamma} U(\epsilon)\,d\epsilon\right),$$ assuming Inverse Compton losses cut off in the KN regime

photon field

Clearly $\gamma_{max} = \frac{1}{t_{acc}\,C}$ (balance acceleration and losses)

SEE P. 1A ⟶

QUESTION: In the steady-state, a power law develops in the acceleration zone:

$$N(\gamma) = K\gamma^{-p}, \quad p = 1 + \frac{t_{acc}}{t_{esc}}, \quad K = \underbrace{\frac{t_{esc}Q\,(1-p)}{\gamma_{max}^{1-p} - \gamma_{min}^{1-p}}}_{\text{this comes from particle cons.}} \quad p \neq 2$$

QUESTION: How much power does the acceleration mechanism provide?

Energy is injected in the acceleration zone at a rate of $Q\gamma_0 mc^2$.

Energy escapes from the acceleration zone at a rate

$$mc^2\frac{1}{t_{esc}}\int^{\gamma_{max}}\gamma N(\gamma)\,d\gamma$$

1a

Question: What is the time evolution of the Lorentz factor of an electron in the acceleration zone. Consider an extremely lucky one that does not escape.

START WITH

ANS: $\frac{d\gamma}{dt} = \frac{\gamma}{t_\alpha} - c\gamma^2$. For $\frac{\gamma}{t_\alpha} << c\gamma^2$, radiative losses can be ignored and

$\frac{d\gamma}{dt} = \frac{\gamma}{t_\alpha} \Rightarrow \gamma = \gamma_0 e^{\frac{t}{t_\alpha}}$. The general case is:

$$\int_{\gamma_0}^{\gamma} \frac{d\gamma}{\frac{\gamma}{t_\alpha} - c\gamma^2} = \int_0^t dt \Rightarrow t_\alpha \log \frac{\gamma}{1 - c t_\alpha \gamma} = t + K \Rightarrow$$

$\log \frac{\gamma}{1 - c t_\alpha \gamma} = \frac{t}{t_\alpha} + K$ Obtain $K = \log \frac{\gamma_0}{1 - c t_\alpha \gamma_0}$ from $\gamma = \gamma_0$ at $t = 0$

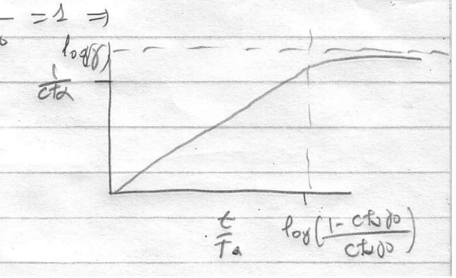
exponentiate to obtain $$\boxed{\gamma = \frac{e^{\frac{t}{t_\alpha}} \frac{\gamma_0}{1 - c t_\alpha \gamma_0}}{1 + e^{\frac{t}{t_\alpha}} \frac{\gamma_0}{1 - c t_\alpha \gamma_0} c t_\alpha}}$$

As $t \to \infty$ $\gamma \to \frac{1}{c t_\alpha}$

The transition from exponential increase to saturation happens at:

$$e^{\frac{t}{t_\alpha}} \frac{c t_\alpha \gamma_0}{1 - c t_\alpha \gamma_0} = 1 \Rightarrow$$

$$\frac{t}{t_\alpha} = \log\left(\frac{1 - c t_\alpha \gamma_0}{c t_\alpha \gamma_0}\right)$$



(1B)

Question: Find the steady state solution the acceleration zone kinetic equation assuming radiative losses appear only as a γ_{max} in the $n(\gamma)$.
Check your solution.

ANS: The equation now is $\frac{\partial}{\partial\gamma}\left[\frac{\gamma}{t_\alpha}N\right] = Q\,\delta(\gamma-\gamma_0) - \frac{N}{t_{esc}}$

$$\Rightarrow \frac{N}{t_\alpha} + \frac{\gamma}{t_\alpha}N' = Q\,\delta(\gamma-\gamma_0) - \frac{N}{t_e} \Rightarrow$$

$$N' + N\frac{p}{\gamma} = \frac{t_\alpha Q\,\delta(\gamma-\gamma_0)}{\gamma}, \quad \text{where } p = 1 + \frac{t_\alpha}{t_e}$$

Multiplying both sides by the integrating factor

$$e^{\int \frac{p}{\gamma}d\gamma} = e^{p\ln\gamma} = \gamma^p \quad \text{we obtain:}$$

$$N'\gamma^p + N\frac{p}{\gamma}\gamma^p = (N\gamma^p)' = t_\alpha Q\,\delta(\gamma-\gamma_0)\,\gamma^{p-1} \Rightarrow$$

$$N\gamma^p = t_\alpha Q\,\gamma_0^{p-1} \Rightarrow \boxed{N(\gamma) = t_\alpha Q\,\gamma_0^{p-1}\,\gamma^{-p}}$$

Check: $\int N(\gamma)\,d\gamma = t_\alpha Q\,\gamma_0^{p-1}\int_{\gamma_0}^{\gamma_{max}} \gamma^{-p}d\gamma \approx t_\alpha Q\,\gamma_0^{p-1}\,\frac{\gamma_0^{1-p}}{p_0 - 1}$

$= \frac{t_\alpha Q}{1+\frac{t_\alpha}{t_e}-1} = t_e Q$ which is how many particles we expect to have in the acceleration zone in the steady-state.

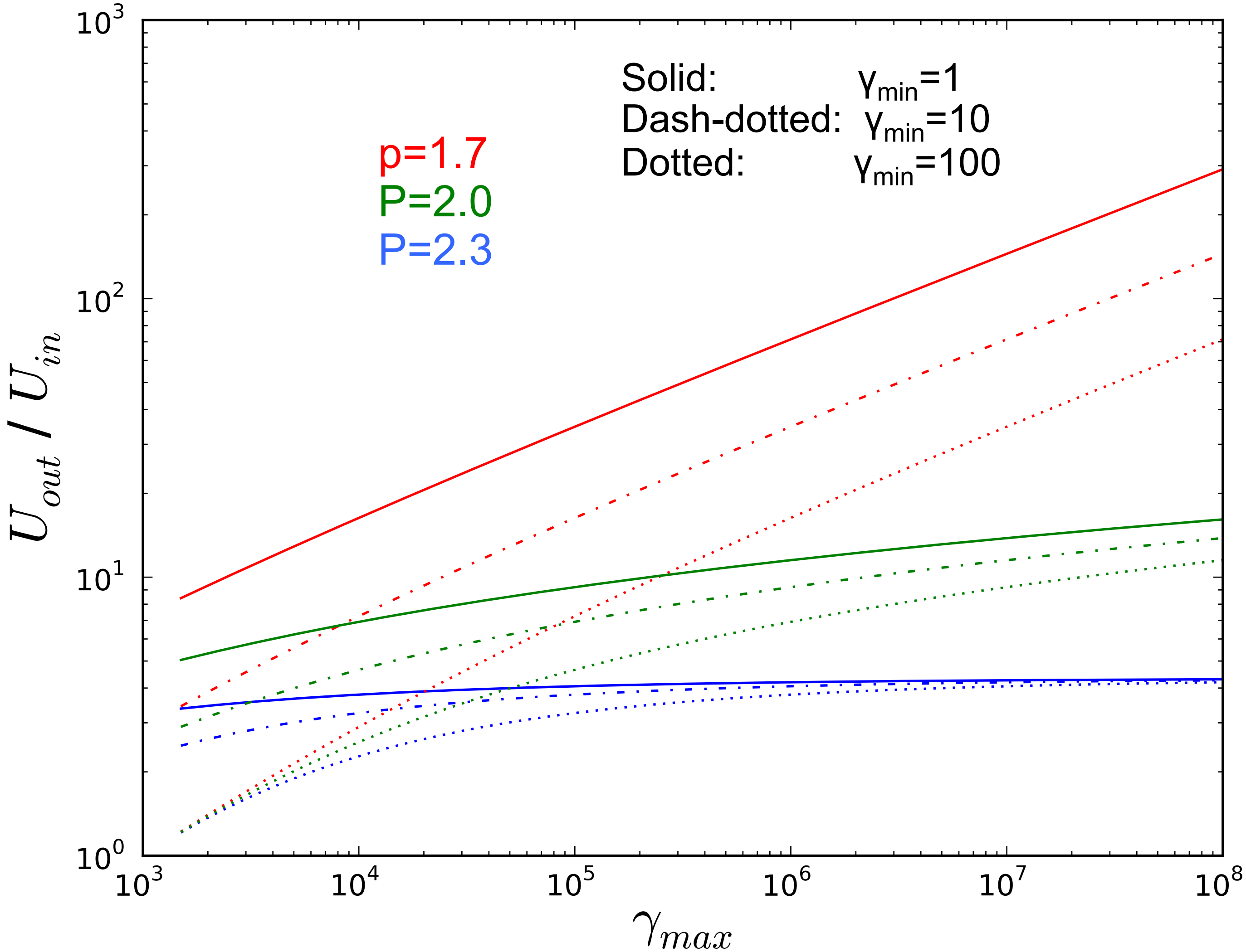
The ratio of electron power injected at γ_0 to electron power escaping from the acceleration zone is

$$G = \frac{P_{out}}{P_{in}} = \frac{1-p}{\gamma_0 \left(\gamma_{max}^{1-p} - \gamma_0^{1-p}\right)} f(\gamma_{max}, \gamma_0) \;, \quad f = \frac{\gamma_{max}^{2-p} - \gamma_0^{2-p}}{2-p} \quad p \neq 2$$

$$f = \log\left(\frac{\gamma_{max}}{\gamma_0}\right) \quad p = 2$$

for $p > 2$, $\displaystyle G_{\gamma_{max} \to \infty} = \frac{p-1}{p-2}$. This is a small gain, (for $p = 2.5$ $G = 3$) a significant part of the power is provided by the pre-acceleration mechanism that brings electrons to γ_0. Even for $p = 2$ the gain is just logarithmic in γ_{max}. Only for $p < 2$ substantial power gains are provided by the acceleration mechanism

Acceleration gains



Notice that for p>=2 we gain only a factor of a few relative to what we injected.

In the radiation zone:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial \gamma}\left(C \gamma^2 n \right) + \frac{n}{t_e} = \frac{N}{t_{esc}}$$

where $\frac{N}{t_{esc}}$ is the injection from the acceleration zone

and $\frac{n}{t_e}$ is the escape from the radiation zone.

In the case of slow cooling $\left(\gamma_b = \frac{1}{C t_e} > \gamma_0 \right)^*$

SEE PAGE 3a

$n(\gamma)$ in the steady state is a broken power law, steepening by 1 at $\gamma = \gamma_b$. The normalization of this is determined by the condition $\int_{\gamma_0}^{\gamma_{max}} n(\gamma)\, d\gamma = Q\, t_e$

To summarize, the electron distribution in the two zones are

$$N(\gamma) = \frac{Q\, t_{esc}}{I_1} \gamma^{-p}$$

$$n(\gamma) = \frac{Q\, t_e}{I_2} \begin{cases} \gamma^{-p} & \gamma \le \gamma_b \\ \gamma_b \gamma^{-1-p} & \gamma \geq \gamma_b \end{cases}$$

Where I_1 and I_2 respectively are integrals of a single and broken power law between $[\gamma_{min}, \gamma_{max}]$ ** see page 4

Below γ_b the ratio $\frac{n}{N} = \frac{I_1}{I_2} \frac{T_e}{t_{esc}} = \frac{I_1}{I_2}(p-1) \frac{\gamma_{max}}{\gamma_b}$

(because $\gamma_{max} = \frac{1}{C\, t_{acc}} = \frac{1}{C(p-1) t_{esc}}$ and *)

3.a

The kinetic equation and its solution

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial \gamma}(\dot{\gamma} n) + \frac{n}{t_e} = Q(\gamma) \qquad (\text{e.g. } Q(\gamma) = \kappa\gamma^{-s}, \; Q(\gamma) = \delta(\gamma-\gamma_0))$$

Cooling time = escape time at $\frac{\gamma}{\dot\gamma} = t_e$

For Thomson or synchrotron cooling $\dot{\gamma} = C\gamma^2$

and $\frac{\gamma_b}{\gamma_b^2 C} = t_e \Rightarrow \gamma_b = \frac{1}{C t_e}$

For $\gamma \ll \gamma_b$ escape dominates and in the steady-state

$n(\gamma) = t_e Q(\gamma)$

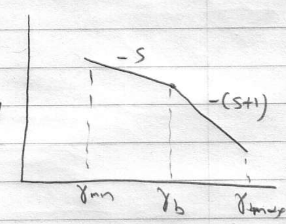
For $\gamma \gg \gamma_b$ cooling dominates

$$\frac{\partial}{\partial\gamma}(\dot\gamma n) = Q(\gamma) \Rightarrow n = \frac{\int Q(\gamma)\, d\gamma}{\dot\gamma}$$

For $Q(\gamma) \propto \gamma^{-s}$, $\dot\gamma \propto \gamma^2$ $\quad n(\gamma) \propto \frac{\gamma^{1-s}}{\gamma^2} \propto \gamma^{-(s+1)}$

Example $\quad Q(\gamma) \propto \gamma^{-s}$, $\gamma_{min} < \gamma < \gamma_{max}$

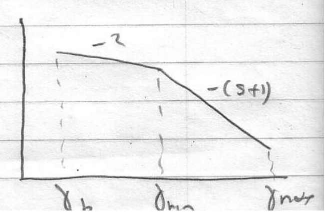
<u>CASE A</u> (SLOW COOLING) $\gamma_{min} < \gamma_b < \gamma_{max}$



<u>CASE B</u> (FAST COOLING) $\gamma_b < \gamma_{min}$

For $\gamma < \gamma_{min}$ the injection can be seen as a δ-function injection at γ_{min}. Then

$$n(\gamma) = \frac{\int \delta(\gamma - \gamma_{min})}{\dot\gamma} \propto \gamma^{-2}$$



If $\gamma_b >> \gamma_0$ the ratio $\frac{I_1}{I_2}$ is very close to unity

and for $\gamma < \gamma_b$ $\frac{n}{N} \simeq \frac{t_e}{t_{esc}} = (p-1)\frac{\gamma_{max}}{\gamma_b}$

Above γ_b.

$$\frac{n}{N} = \frac{Q_0\, t_e\, I_1\, \gamma_b}{I_2\, Q_0\, T_{esc}} \frac{\gamma^{-1-p}}{\gamma^{-p}} = \frac{I_1}{I_2}\frac{t_e}{t_{esc}}\frac{\gamma_b}{\gamma}$$

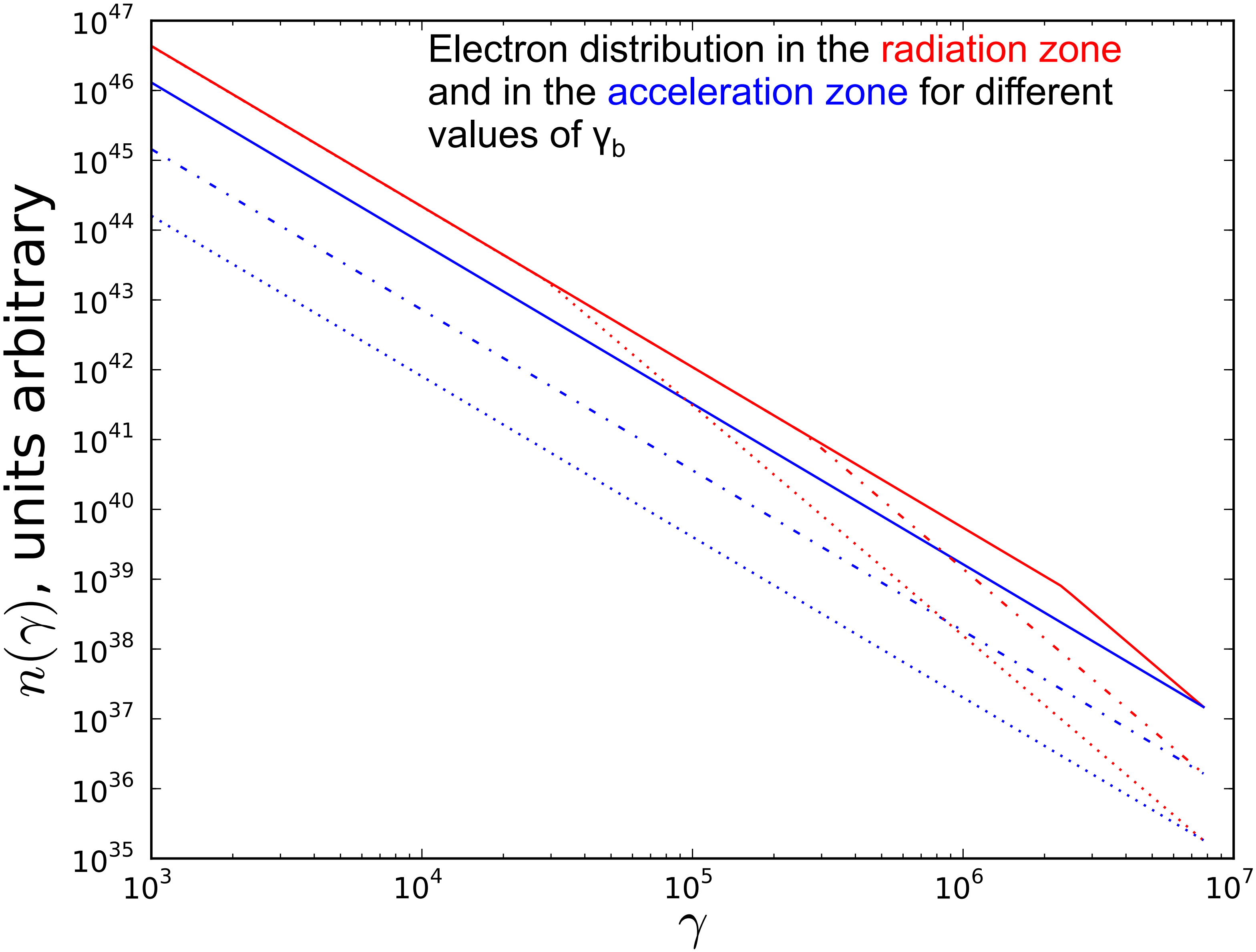
For $\gamma = \gamma_{max}$ $\frac{n}{N} = \frac{I_1}{I_2}\frac{t_e}{t_{esc}}\frac{\gamma_b}{\gamma_{max}} = \frac{I_1}{I_2}\frac{t_e}{t_{esc}}(p-1)\frac{t_{esc}}{T_e}$

$\Rightarrow \frac{n}{N} = \frac{I_1}{I_2}(p-1)$ This is close to unity.

This means that the electron distribution of the radiation zone dominates over that of the acceleration zone, except at $\gamma \sim \gamma_{max}$ where they are practically equal

The situation is similar in the fast cooling regime,

* $I_1 = \int_{\gamma_0}^{\gamma_{max}} \gamma^{-p}\, d\gamma$, $I_2 = \int_{\gamma_0}^{\gamma_b} \gamma^{-p} d\gamma + \gamma_b \int_{\gamma_b}^{\gamma_{max}} \gamma^{-(p+1)}\, d\gamma$



Notice how the radiation zone dominates, with the exception of the highest possible energies, where the two zones are comparable.