Plasma effect on the lower limit of the quantum gravity mass?

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Lorentz invariance violation

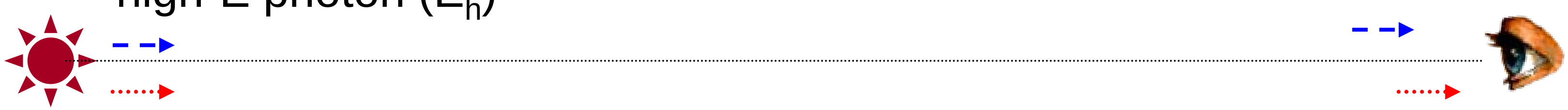
Lorentz invariance, a postulate of Einstein's special relativity:

the relevant laws of physics of a non-accelerated physical system are not affected when this system undergoes Lorentz transformation (rotation and/or boost).

Lorentz invariance violation (LIV) allowed by some quantum gravity models results
$v(E) \neq c$

$$E^2 - p^2c^2 \simeq \pm p^2c^2 \left(\frac{pc}{\xi E_{\rm pl}}\right)^n ,$$ where $E_{Planck}=1.22\times10^{19}$ GeV

high-E photon (E_h)



z=0

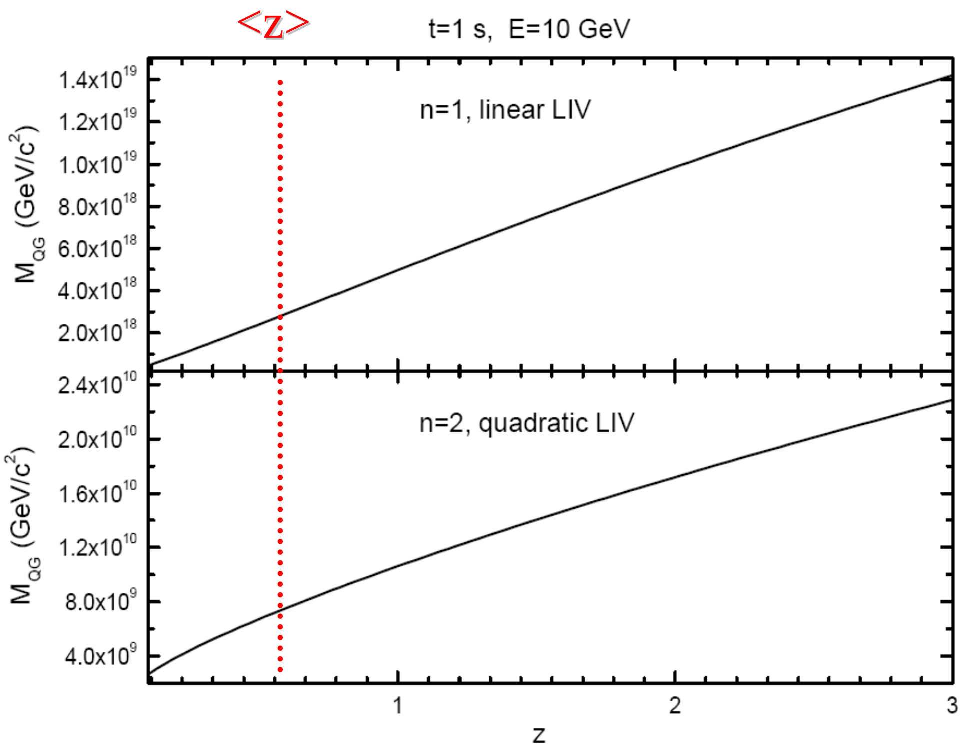
a GRB @ redshift z

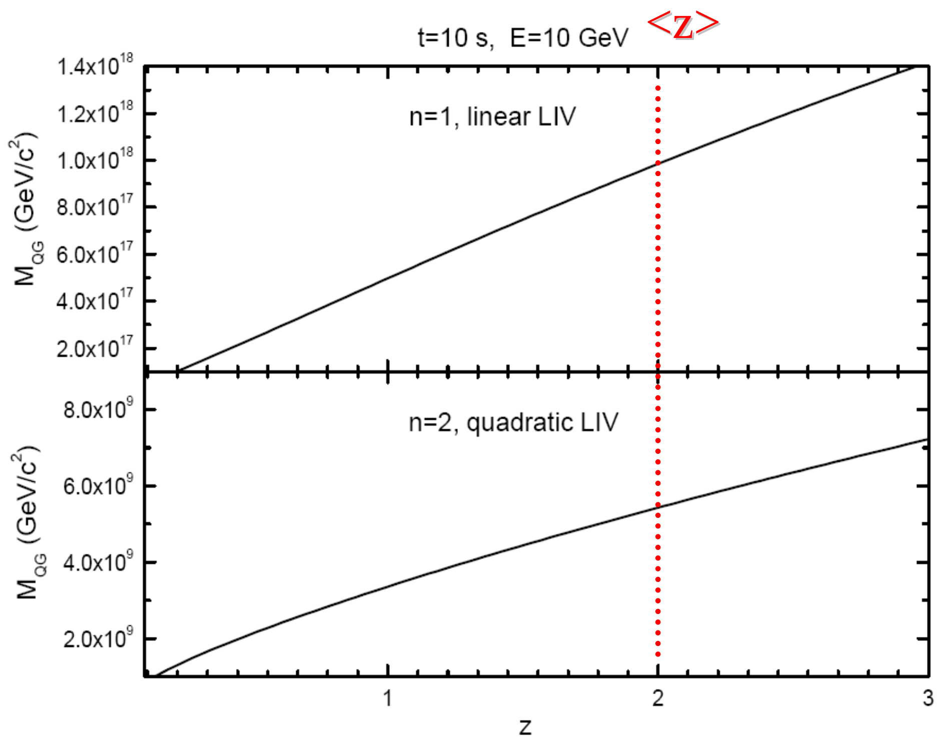
low-E photon (E_l)

$$\Delta t = \frac{(1+n)}{2H_0}\frac{E_h^n - E_l^n}{(M_{{\rm QG},n}c^2)^n}\int_0^z \frac{(1+z')^n}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}\,dz'$$

n=1 for linear LIV, n=2 for quadratic LIV

short GRBs vs long GRBs





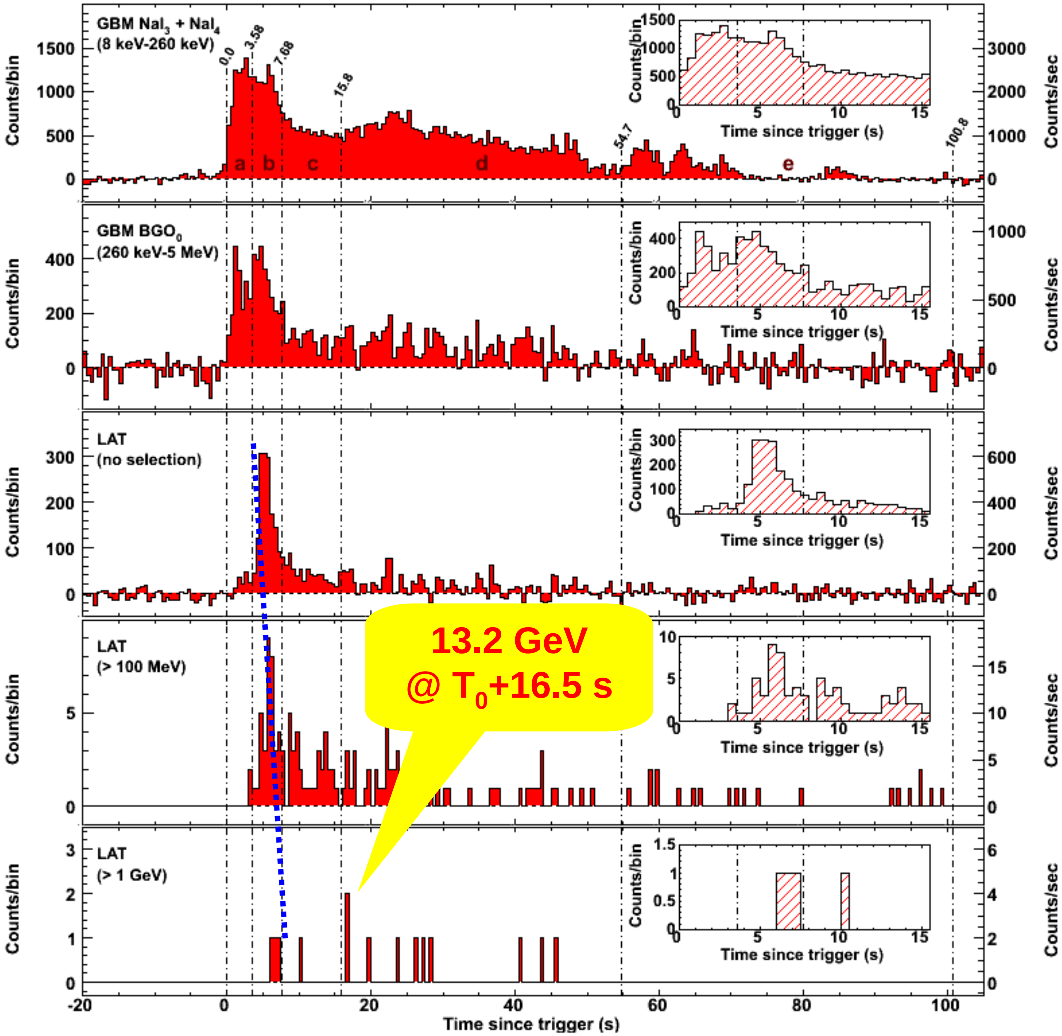
- ❑ **advantage: short duration, short lag;**
- ❑ **disadvantage: low <z>**

- ❑ **advantage: high <z>**
- ❑ **disadvantage: long duration, long lag**

comparable limits on LIV

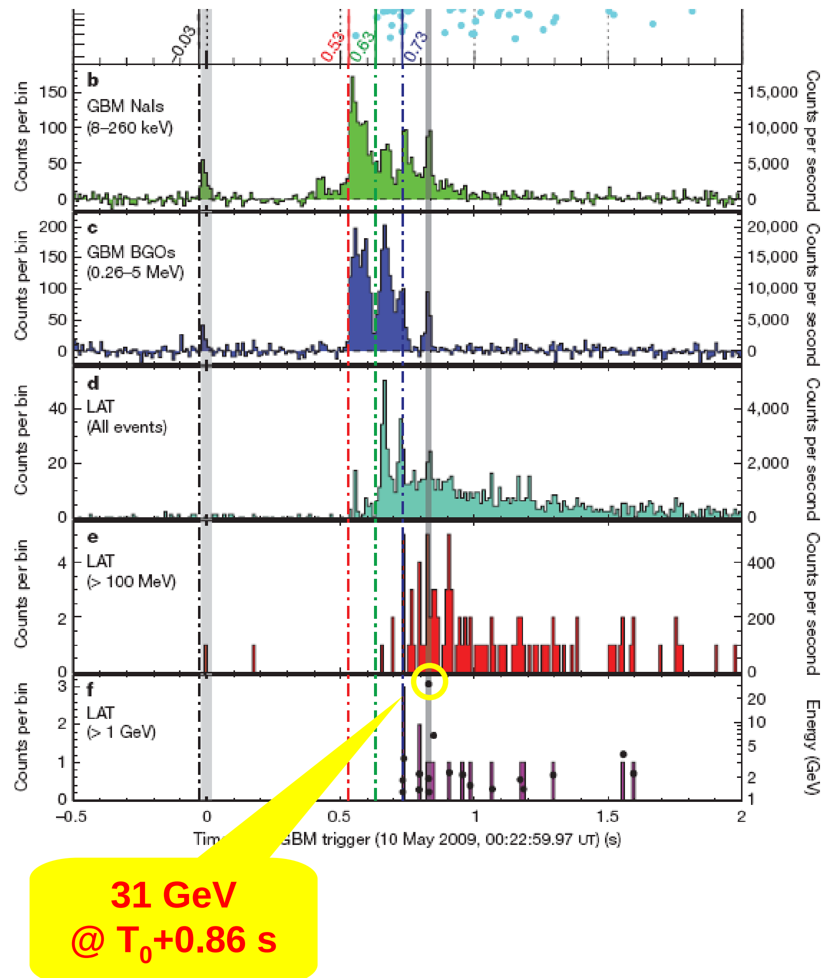
hard (>10 MeV) lags discovered by LAT

long GRB 080916C



Fermi collaboration, 2009, Science, 323, 1688

short GRB 090510

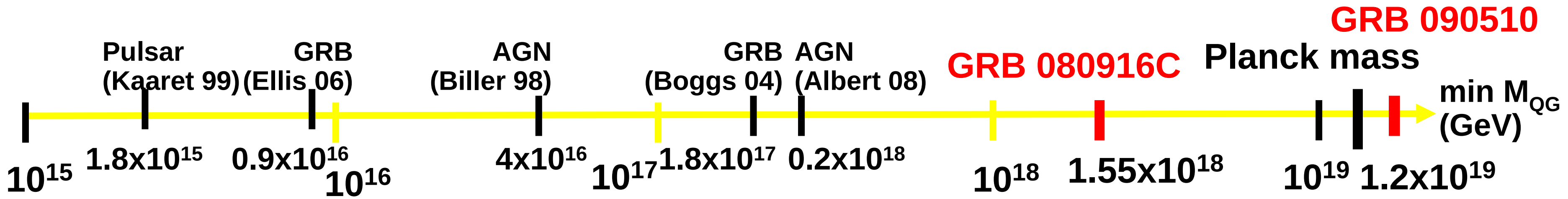


Fermi collaboration, 2009, Nature in press

conservative lower limit on E_{QG}

	GRB 080916C	GRB 090510
E_h	13.2 GeV	31 GeV
Δt	16.5 s	0.86 s
$E_{QG,1}$ (n=1)	$> 0.1\ E_{Planck}$	$>1.2\ E_{Planck}$
$E_{QG,2}$ (n=2)	$> 7.9\times10^{-10}\ E_{Planck}$	$> 2.7\times10^{-9}\ E_{Planck}$

linear (n=1) LIV can be excluded by GRB 090510



dispersion by the plasma effect

dispersion relation of the plasma effect

$$v_p(E) = \frac{c}{\sqrt{1 - \left(\frac{E_p(z)}{E}\right)^2}} \simeq c\left[1 + \frac{1}{2}\left(\frac{E_p(z)}{E}\right)^2\right], \quad \text{for} \;\; E > E_p(z)$$

$$E_p(z) = \hbar\omega_p(z) = 1.97 \times 10^{-23}(1+z)^{3/2}\left(\frac{\Omega_b}{0.05}\right)^{1/2} h_{71}\ \text{GeV}$$

$$\omega_p = \sqrt{4\pi e^2 n_e/m_e} = 5.64 \times 10^4 n_e^{1/2}\ \text{s}^{-1}$$

the number density of electrons in inter-galactic medium (IGM): $n_e = n_e(0)(1+z)^3$,

$$n_e(0) = 3\Omega_b H_0^2/(8\pi G m_p) = 5.66 \times 10^{-6}\Omega_b h_{71}^2\ \text{cm}^{-3}$$

hard lag by the plasma effect

harder photons have a time lag relative to softer photons by

$$\Delta t = \frac{\Delta z}{H_0} = \frac{1}{2H_0}\left[\left(\frac{E_p(0)}{E_l}\right)^2 - \left(\frac{E_p(0)}{E_h}\right)^2\right]\int_0^z \frac{(1+z')dz'}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}$$

$$\approx 22.2\ \mathrm{s}\ h_{71}\left(\frac{\Omega_b}{0.05}\right)\left(\frac{\nu_l}{\mathrm{GHz}}\right)^{-2}\int_0^z \frac{(1+z')dz'}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}$$

which is a few seconds even for a soft photon with radio frequency.

For a soft photon with energy of a few keV, then the lag time is extremely short.

joint effects of the plasma & LIV

dispersion relation by the plasma and LIV effects:

$$v(E) \simeq c\left[1+\frac{1}{2}\left(\frac{E_p(z)}{E}\right)^2 + s_{\rm n}\frac{n+1}{2}\left(\frac{E}{E_{\rm QG}}\right)^n\right]$$

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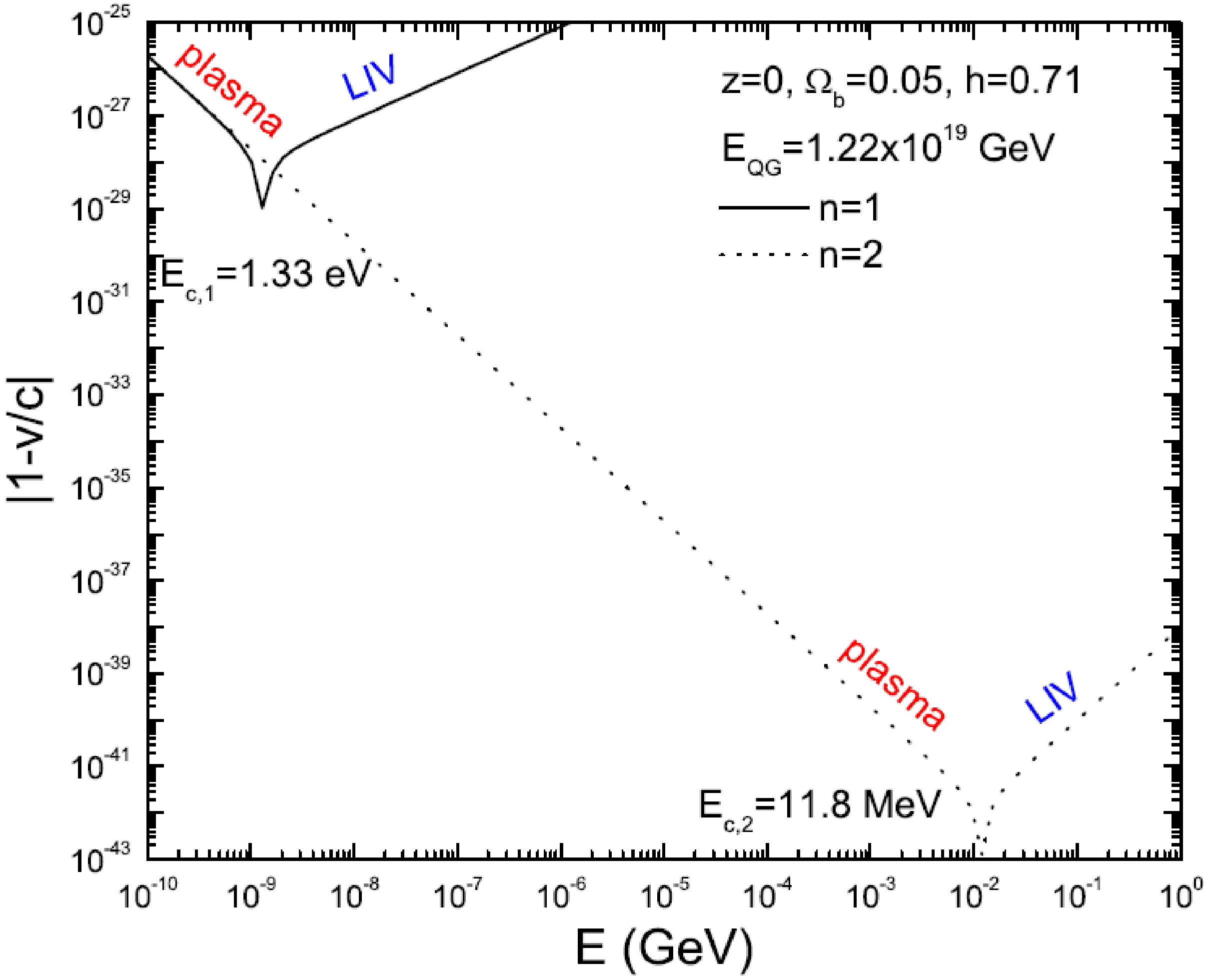
E_c : when plasma = LIV

$$E_{c,n} = \left(\frac{E_p^2 E_{\rm QG}^n}{n+1}\right)^{1/(2+n)} = \begin{cases} 1.33(1+z)h_{71}^{2/3}\left(\frac{\Omega_b}{0.05}\right)^{1/3}\left(\frac{E_{\rm QG}}{E_{\rm pl}}\right)^{1/3} \ {\rm eV}, & n=1, \\ 11.8(1+z)^{3/4}h_{71}^{1/2}\left(\frac{\Omega_b}{0.05}\right)^{1/4}\left(\frac{E_{\rm QG}}{E_{\rm pl}}\right)^{1/2} \ {\rm MeV}, & n=2. \end{cases}$$

plasma effect: hard lag

LIV effect: s_n=+1, soft lag; s_n=-1, hard lag

plasma effect vs. LIV effect



the plasma effect dominates at lower energies $E<E_c$
the LIV effect dominates at high energies E>Ec

plasma effect << LIV effect

$$E_{c,n} = \left(\frac{E_p^2 E_{QG}^n}{n+1}\right)^{1/(2+n)} = \begin{cases} 1.33(1+z)h_{71}^{2/3}\left(\frac{\Omega_b}{0.05}\right)^{1/3}\left(\frac{E_{QG}}{E_{pl}}\right)^{1/3} \text{eV}, & n=1, \\ 11.8(1+z)^{3/4}h_{71}^{1/2}\left(\frac{\Omega_b}{0.05}\right)^{1/4}\left(\frac{E_{QG}}{E_{pl}}\right)^{1/2} \text{MeV}, & n=2. \end{cases}$$

current constraints:

$n=1$, $E_{QG} \sim E_{Planck}$, $E_c \sim 1$ eV

$n=2$, $E_{QG} \sim 10^{-9} E_{Planck}$, $E_c \sim 1$ keV

therefore, plasma effect can be neglected in the analysis of Fermi data for both n=1 and n=2

(Fermi GBM-LAT: 8 keV -300 GeV, > E_c)

outlook: upper limit on $E_{QG,2}$ (I)

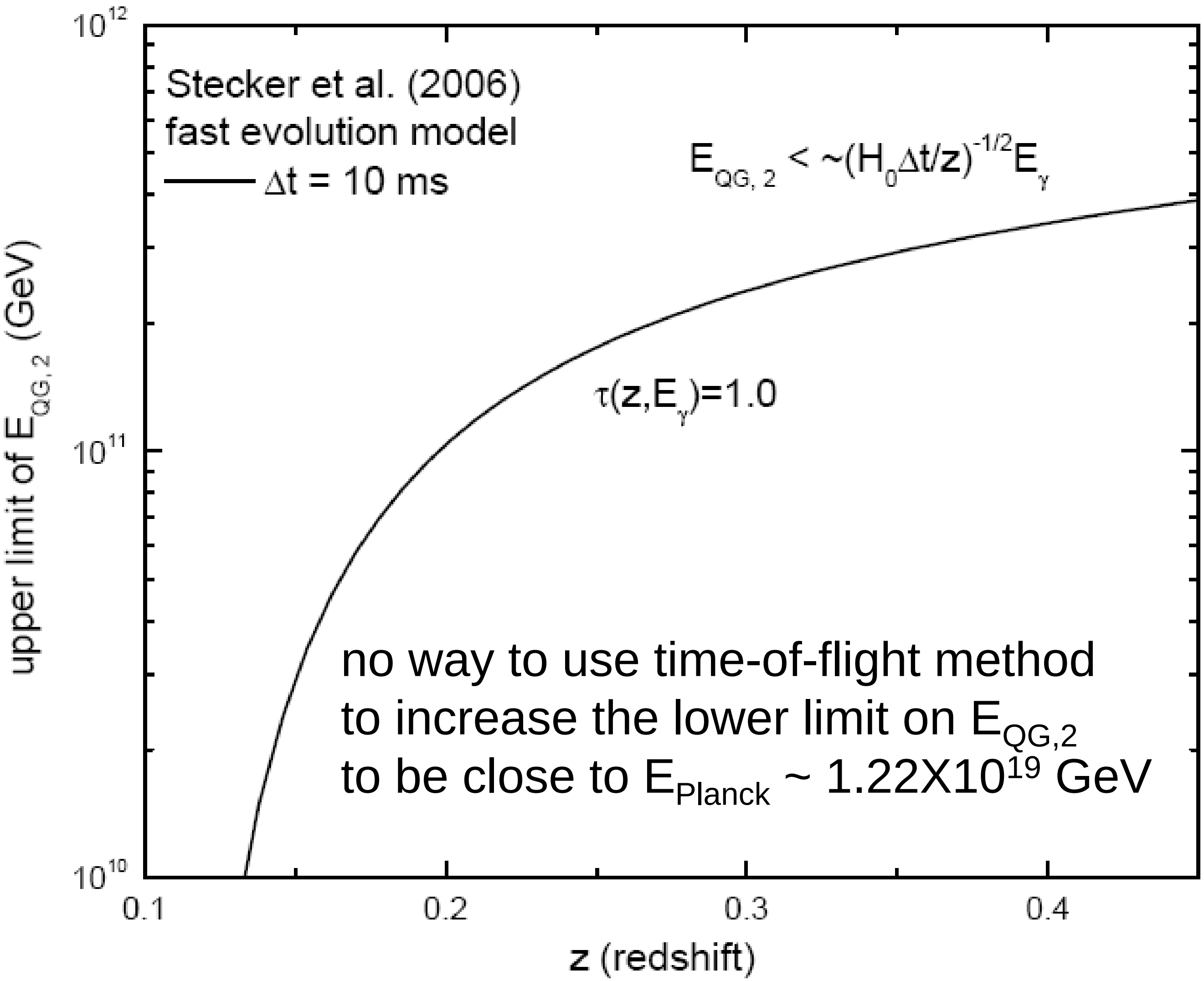
quadratic LIV (n=2):

$$\Delta t = \frac{1}{H_0}\left(\frac{E_h}{E_{\mathrm{QG},2}}\right)^2 \int_0^z \frac{(1+z')^2 dz'}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}},$$

obtain a higher UL on $E_{QG,2}$: small lag, high E_h

higher E_h (e.g., TeV): only detected at smaller redshift (gamma-gamma absorption by background light)

outlook: upper limit on $E_{QG,2}$ (II)



no way to use time-of-flight method
to increase the lower limit on $E_{QG,2}$
to be close to E_{Planck} ~ 1.22X10^{19} GeV