Time-dependent multi-zone modelling of pulsar wind nebulae

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Overview

Noteworthy background.

Problems with current models and motivation for our study.

Spectral Energy Density (SED) Model.

Calibration of Model.

Spatial results.

Conclusions and future work.

PWN Characteristics

- Called a Plerion "filled bag".
- Structured magnetic field.
- Hard radio spectrum.
- Particle re-acceleration at termination shock.
- 2 Component injection spectrum.

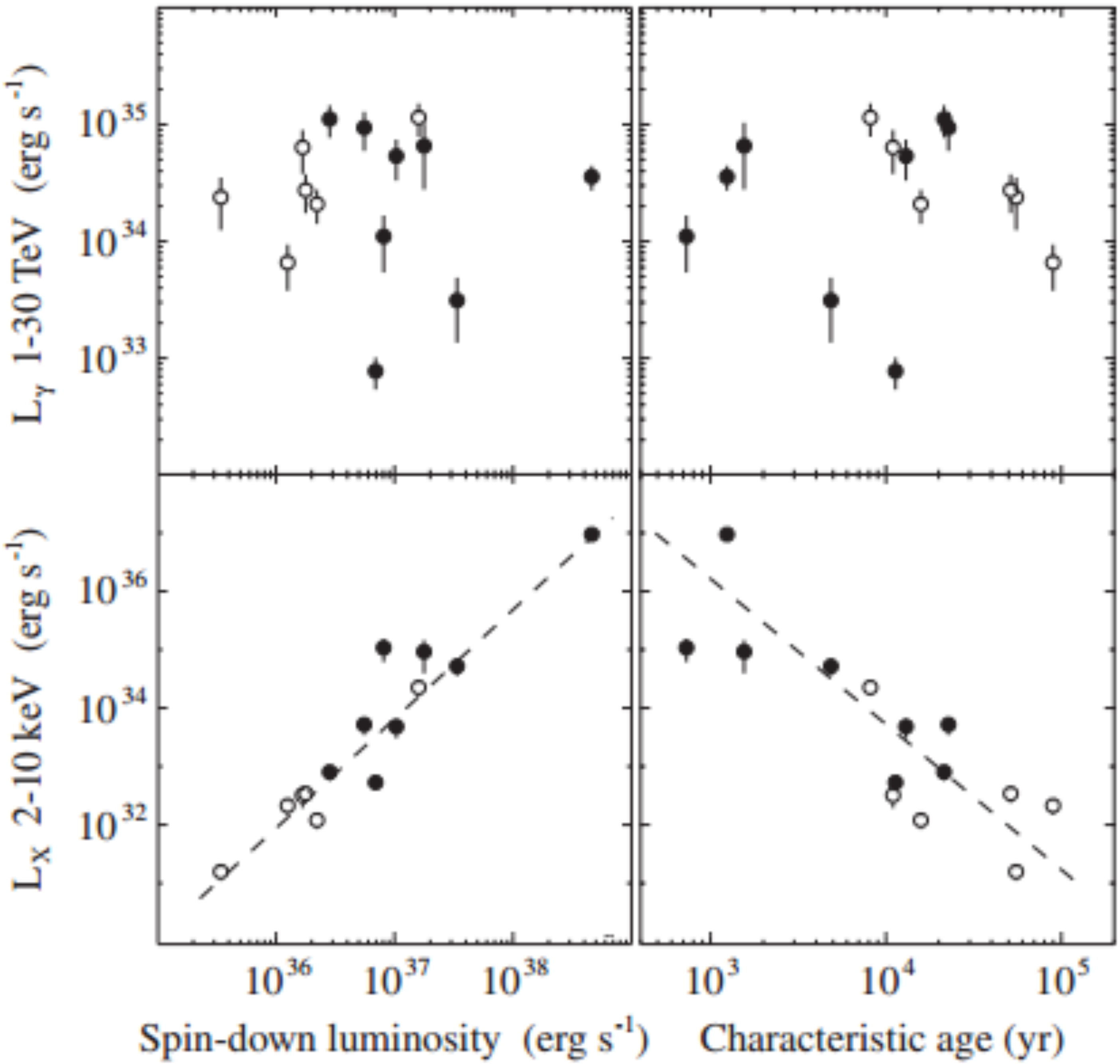
VHE Characteristics

- σ (magnetisation parameter) is small.
- Thus particle dominated, opposite than the magnetosphere of a pulsar.
- Magnetic field of PWN is very small in early epochs due to rapid expansion of the PWN.

Motivation (I)

- No correlation between PWN TeV flux and the $\dot{E}$ or the characteristic age of pulsar.
- X-ray emission correlated with pulsar $\dot{E}$ and anti-correlated with characteristic age.
- Is there any correlation between TeV surface brightness and $\dot{E}$?

Mattana et al. (2009)



Motivation (II)

- Multi-wavelength morphological data becoming available
- CTA: Improved angular resolution
- Spatially-dependent PWN model needed
  - Size vs. Energy
  - Constrain *B(r)*, *k(r)*, and *V(r)* profiles

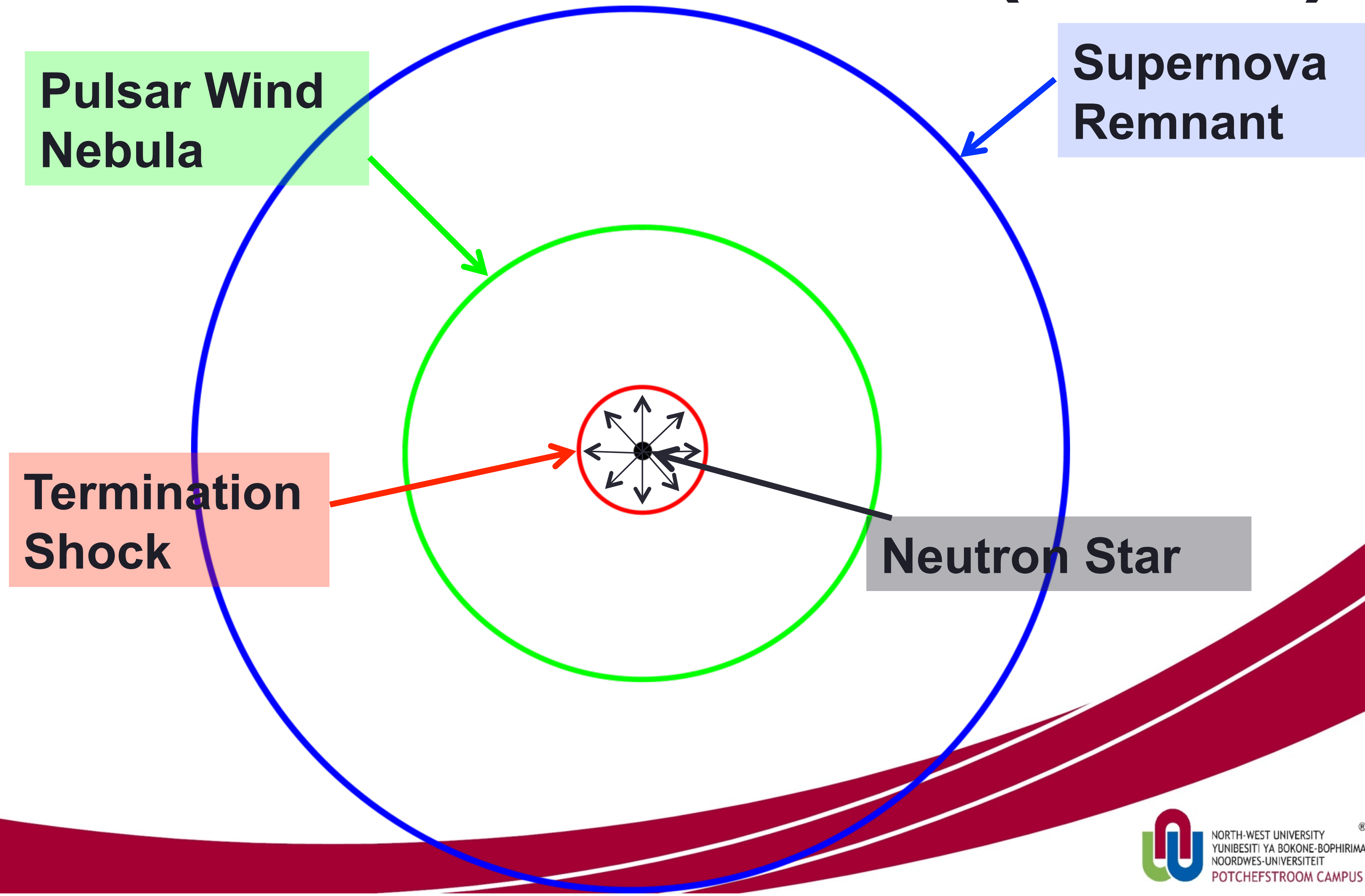
Research goal

- Main aim – Creating a time-dependent, multi-zone model of a PWN
- To yield spatial morphology of a PWN.

Model

Van Rensburg, Krüger & Venter, *in prep.*

Pulsar Wind Nebulae (PWNe)



Transport Equation (I)

Momentum space:

$$\frac{\partial f}{\partial t} = -\nabla \cdot \mathbf{S} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 \left\langle \dot{p} \right\rangle_{\mathrm{tot}} f\right) + Q(\mathbf{r}, p, t),$$

$$\nabla \cdot \mathbf{S} = \nabla \cdot (\mathbf{V} f - \underline{\mathbf{K}} \nabla f)$$

See Parker (1965)

Energy space:

$$\frac{\partial N_{\mathrm{e}}}{\partial t} = -\mathbf{V} \cdot (\nabla N_{\mathrm{e}}) + \kappa \nabla^2 N_{\mathrm{e}} + \frac{1}{3}(\nabla \cdot \mathbf{V}) \left(\left[\frac{\partial N_{\mathrm{e}}}{\partial \ln E} \right] - 2N_{\mathrm{e}} \right) + \frac{\partial}{\partial E}(\dot{E}_{\mathrm{rad}} N_{\mathrm{e}}) + Q(\mathbf{r}, E, t)$$

Transport Equation (II)

It all starts here™

Injection spectrum: broken power law

$$Q(E_{\mathrm{e}},t)=\begin{cases}Q_0(t)\left(\frac{E_{\mathrm{e}}}{E_{\mathrm{b}}}\right)^{\alpha_1} & E_{\mathrm{e}}<E_{\mathrm{b}}\\ Q_0(t)\left(\frac{E_{\mathrm{e}}}{E_{\mathrm{b}}}\right)^{\alpha_2} & E_{\mathrm{e}}\geq E_{\mathrm{b}}.\end{cases}$$

Venter & de Jager (2007)

Time-dependent normalization:

$$\epsilon L(t)=\int_{E_{\mathrm{min}}}^{E_{\mathrm{b}}} QE_{\mathrm{e}}dE_{\mathrm{e}}+\int_{E_{\mathrm{b}}}^{E_{\mathrm{max}}} QE_{\mathrm{e}}dE_{\mathrm{e}}$$

$$L(t)=L_0\left(1+\frac{t}{\tau_c}\right)^{-(n+1)/(n-1)}$$

Pacini & Salvati (1973)

Transport Equation (III)

- SR & IC radiative losses
- Adiabatic losses
- Convection
- Diffusion

$$V(r) = V_0 \left(\frac{r}{r_0}\right)^{\alpha_V}$$

$$\kappa = \kappa_0 \left(\frac{E}{E_0'}\right)^{q}$$

$$B(r,t) = B_{\rm age} \left(\frac{r}{r_0}\right)^{\alpha_B} \left(\frac{t}{t_{\rm age}}\right)^{\beta_B}$$

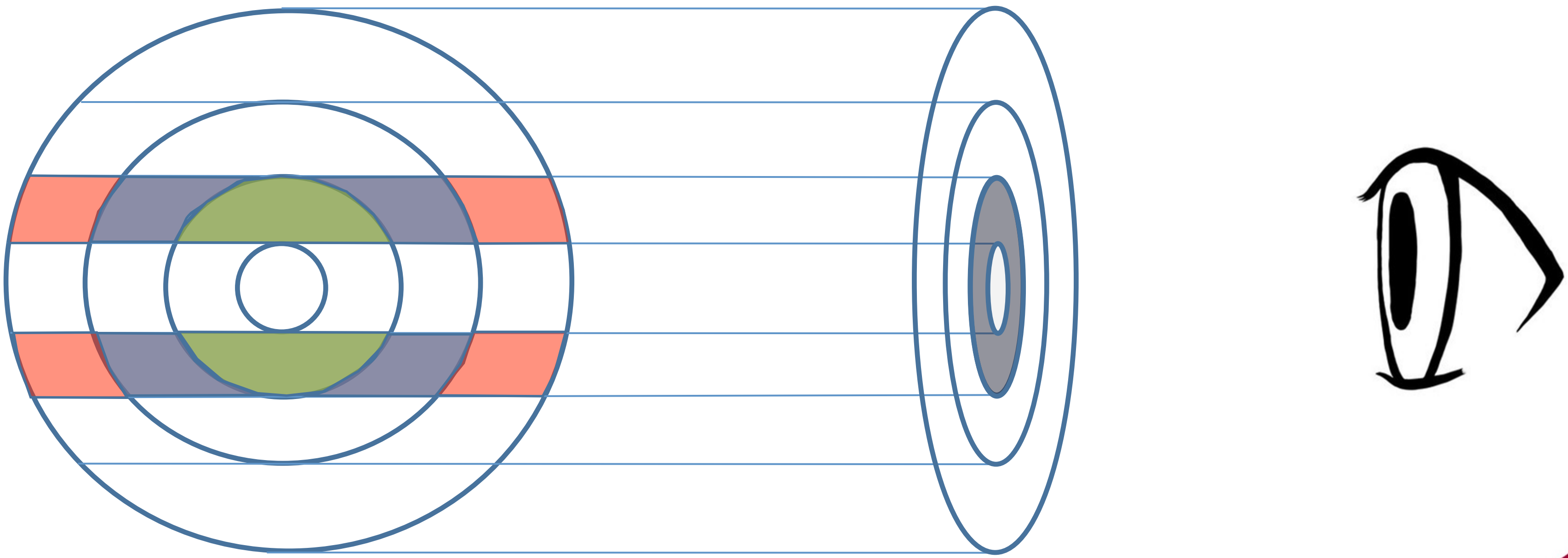
Boundary Conditions

- At $t = 0$ all zones devoid of particles.
- Set of 'ghost points' outside the time boundary for DuFort – Frankel scheme.
- Reflective inner boundary R_{min}, escape at outer boundary R_{max}.
- Limiting energy

Venter & de Jager (2007)

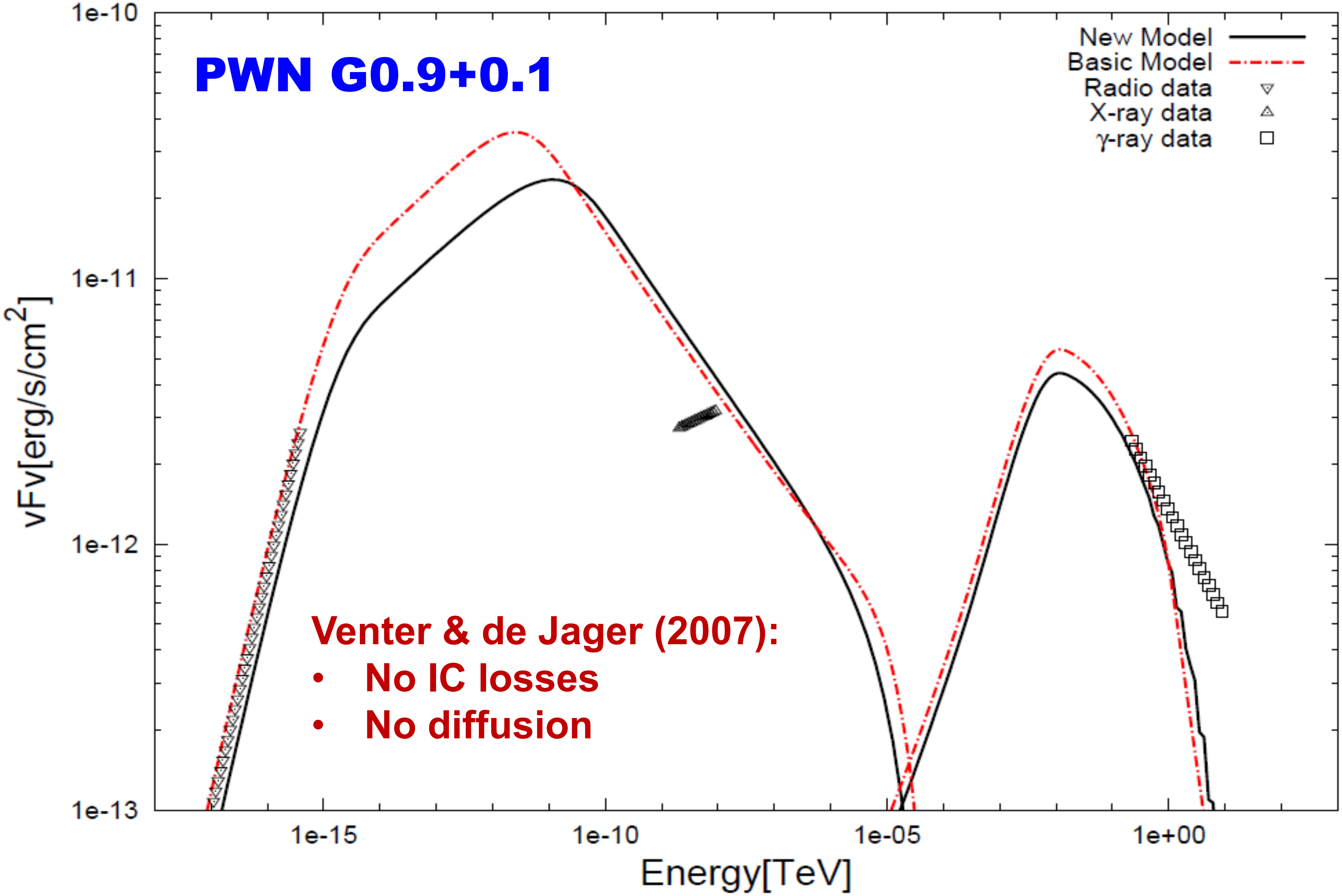
$$E_{\max} = \frac{e}{2}\sqrt{\frac{L(t)\sigma}{c(1+\sigma)}}$$

Line of Sight calculation (LOS)

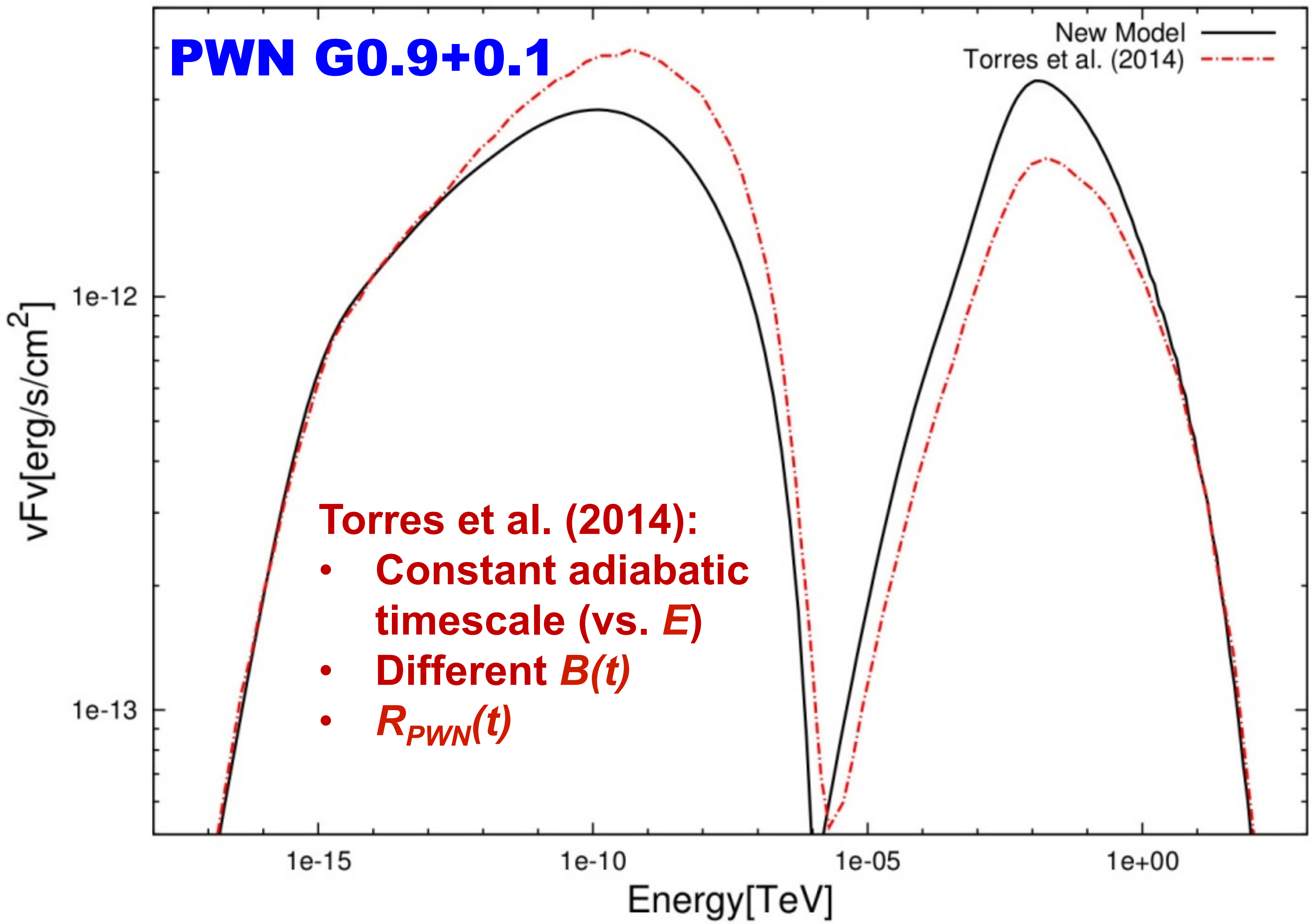


Code Comparison

Code Comparison



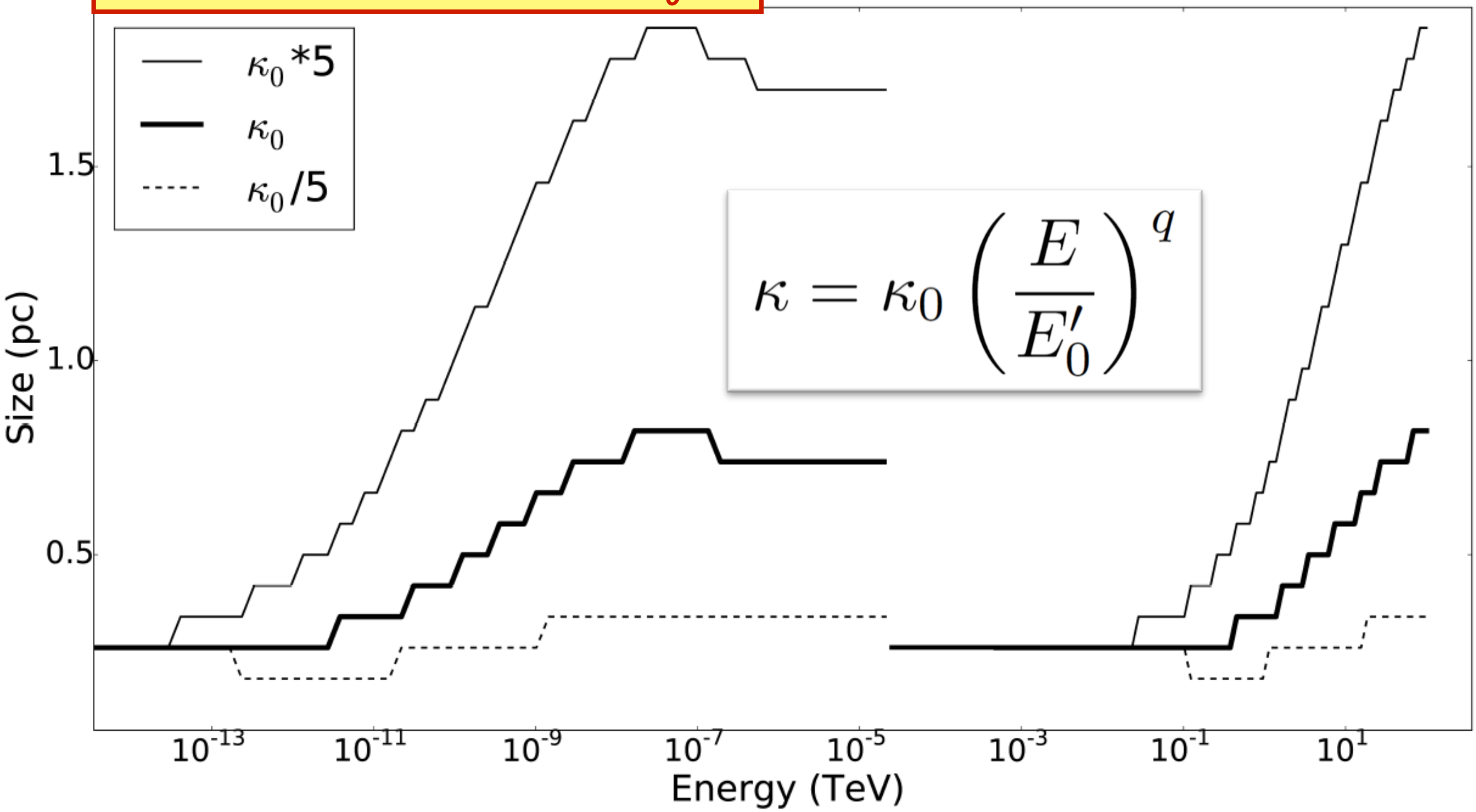
Code Comparison



Parameter Study: Spatially-dependent Results

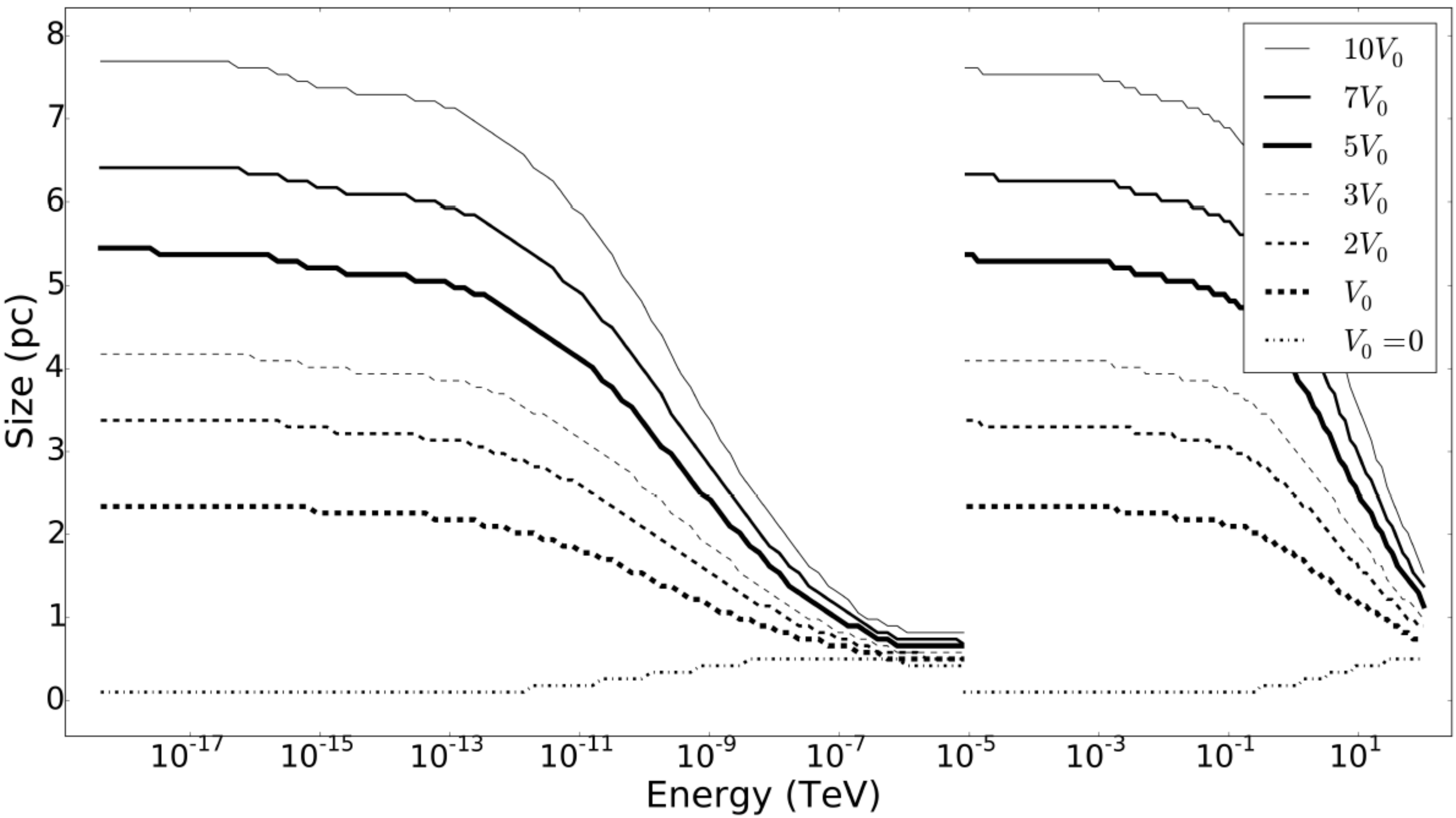
Changes to k_0

PWN size increases with k_0

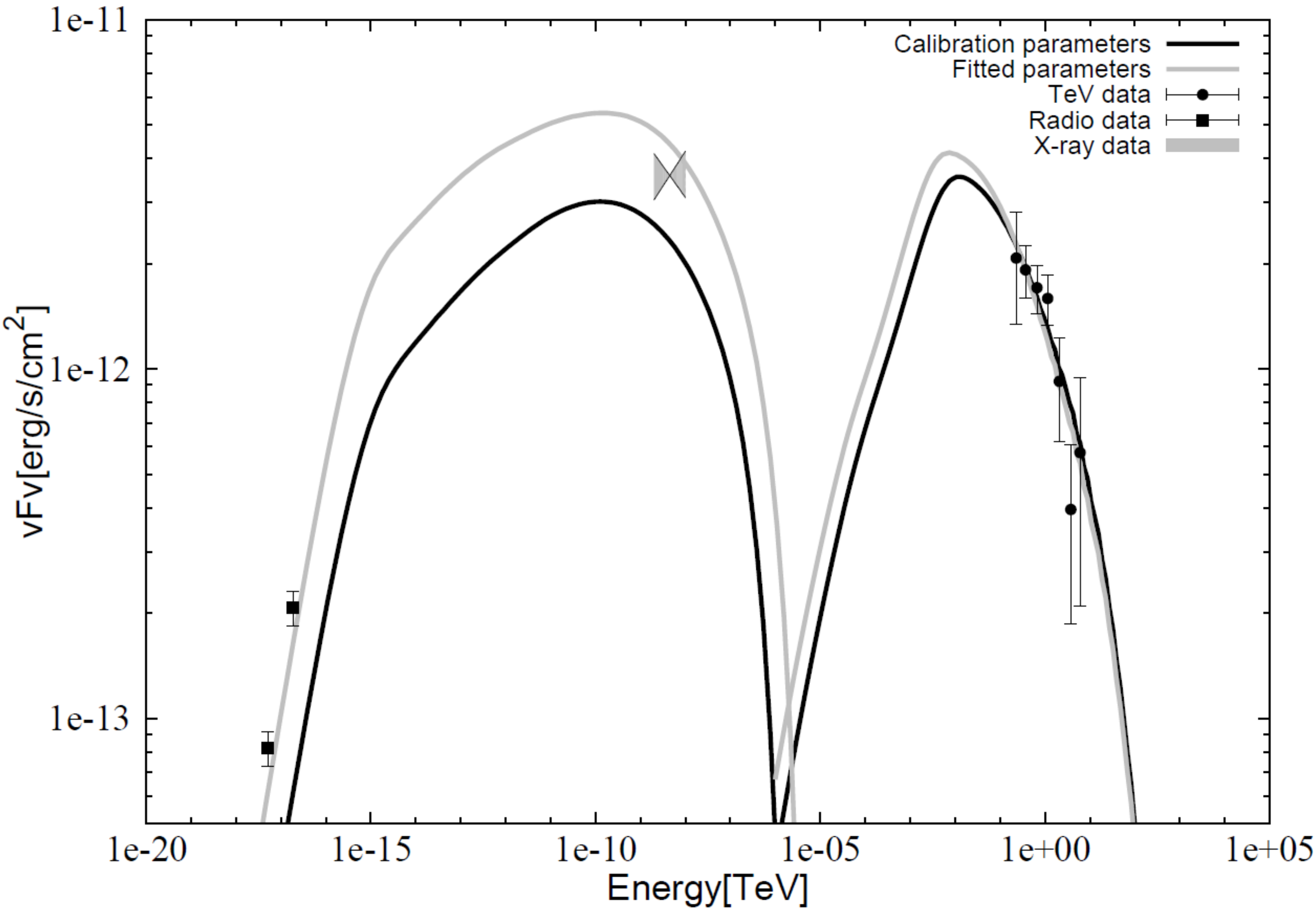


$$\kappa = \kappa_0 \left(\frac{E}{E'_0} \right)^q$$

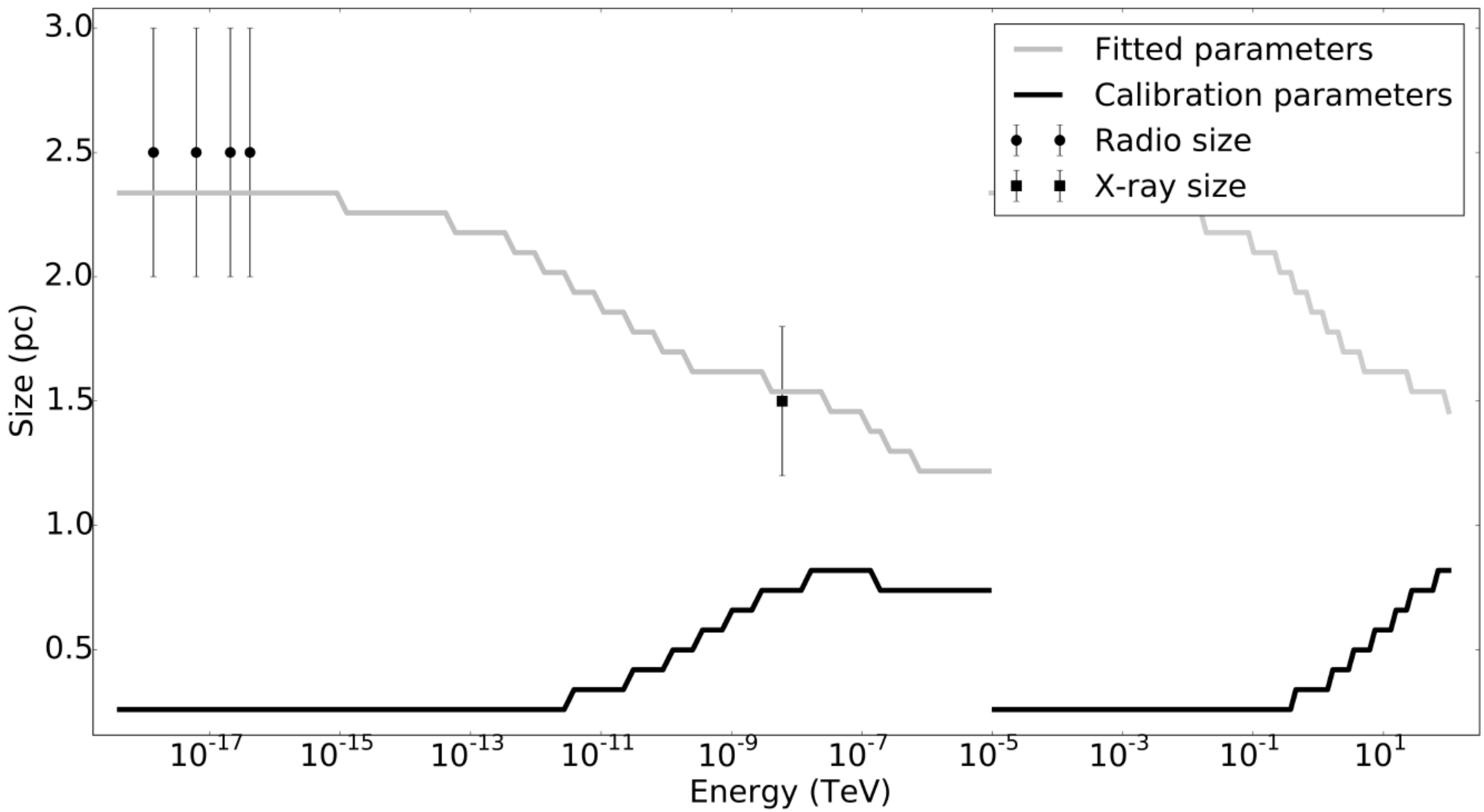
Changes to V_0



Fitting Both PWN Spectrum and Size



1e-11
1e-12
1e-13
vFv[erg/s/cm2]
1e-20
1e-15
1e-10
1e-05
1e+00
1e+05
Energy[TeV]
Calibration parameters
Fitted parameters
TeV data
Radio data
X-ray data



Fitted parameters
Calibration parameters
Radio size
X-ray size
Size (pc)
Energy (TeV)
3.0
2.5
2.0
1.5
1.0
0.5
10^{-17}
10^{-15}
10^{-13}
10^{-11}
10^{-9}
10^{-7}
10^{-5}
10^{-3}
10^{-1}
10^{1}

Model Parameter	Symbol	Value	Torres et al. (2014)
Present-day B-field	$B(t_{\rm age})$	20.0 μG	14.0 μG
Initial spin-down power (10^{38}erg s^{-1})	L_0	1.87	1.0
B-field parameter	α_B	0.0	0
B-field parameter	β_B	-1.0	-1.3
V parameter	α_V	-1.0	1.0
Particle bulk motion	V_0	0.098 c	1.63×10^{-4} c
Diffusion	κ_0	$6.5\kappa_0$	κ_0

Conclusions

- Exploit increase in morphological data
- Model should predict both spectrum and energy-dependent PWN size
- Constrain *B(r)* and *V(r)*

Future Work

It all starts here™

- Investigate boundary condition at R_{max} (particle build up) – $R_{PWN}(t)$
- Add convective term to transport equation to handle $k(r,E)$ correctly
- Fitting more PWNe
- Investigate population trends
- Older PWNe: $B(t)$ and reverse shock
- 2D, 3D? MHD?

THANK YOU!

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